Why is the Sasaki Hook not a Static Implication within Quantum Logic?

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Abstract

One encounters certain problems when introducing an implication connective within quantum logic. In the case of an orthomodular lattice, there are five possible connectives which satisfy the strengthened law of entailment (i.e. $a \to b = 1$ iff $a \bullet b$) [6]. All five connectives are internally definable as specific two-variable lattice polynomials and one of these polynomials is referred to as the Sasaki hook, i.e. $a \overset{S}{\to} b = a^{\perp} \vee (a \wedge b)$. The Sasaki hook, viewed as a connective, behaves badly in a non-boolean orthomodular logic due to the failure of a deduction theorem, transitivity, weakening and contraposition (see [3, 4, 5, 7]). Following [9], we will argue that:

> From an operational perspective (i.e. within "operational quantum logic"), the counterfactual nature of the Sasaki hook shows explicitly that $(a \overset{S}{\to} b)$ is the weakest property which, if it is actual in some state, guarantees that b will be an actual property of the system under the condition that we obtain a positive response from performing the ideal first kind measurement associated to property a.

In other words, our analysis explicitly supports the claim made in [2] that the action of the Sasaki hook assigns causes and has a fundamental dynamic nature. As such we link this interpretation to what we have called "dynamic operational quantum logic" (see for instance [1, 8]) and clarify the point already made in [2]: the Sasaki hook should not be thought of as a static implication but rather as inducing a labeled dynamic implication that assigns causes.

References

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