Structured Proofs and Adaptive Logic

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A central feature of adaptive logics is that for any provisional derivation of p based on $\Gamma \vdash_{LLL} Dab(C_1, \ldots, C_n) \vee p$, it requires the normal behaviour of all C_i with respect to all premises in Γ, and not merely with respect to that subset $\gamma \subseteq \Gamma$ from which p was initially derived (as reflected in a proof by the line numbers from which it is effectively derived). As long as Γ is considered as a theory, being a set of statements structured by at most a (total) preference ordering, this is a sensible approach towards provisional deductive steps.

The latter is, however, not always the case. Within several contexts it is, for instance, preferable to consider a set of premises as a structured set (e.g. a poset). Hereby we mean that a set of premises can be seen as a number of subsets that are not fully related, but which possibly share some of their premises. A few examples are : (i) discussions with an explicit structure - i.e. a (transitive) *reply-to* relation, (ii) alternative evolutions, (iii) mutually incompatible extensions of a theory.

In such a case, the strategy for provisional derivations within the adaptive logic framework should be modified in this way: $\Gamma \vdash_{AL} p$ iff $\gamma \vdash_{LLL} Dab(C_1, \ldots, C_n) \vee p$ holds, $\gamma \subseteq \Gamma$ whose elements are related, and the conditions on which p was derived behave normally with respect to γ', the largest extension of γ with respect to the internal structure of Γ. The resulting consequence relation is weaker as its reach is only local, but it is also more versatile and yields a more fine-grained approach by validating more - but only locally valid - consequences.

One way of modelling this is to consider a structured set of premises as a labelled graph with premises as labelled nodes, and relations as (directed) labelled edges. Using modal semantics, such structures can be described in a straightforward way, and notions as 'related premises' and 'largest extensions of subsets of premises' can easily be formalised. When it comes to the formulation of the corresponding dynamic proof-formats, we have to deal with an additional problem, viz. the integration of relational structures within proofs. In this paper a labelled deductive system that solves the problem at the level of proofs is considered.

As a concrete example we focus on the case of discussions with an explicit structure, and show that this approach allows for the distinction of 'lines of argument' and 'local agreements' within a discussion. Furthermore we argue that the labelled approach, although yielding exactly the same consequences as a series of distinct proofs for every $\gamma \subseteq \Gamma$, is a useful and expressive extension of the regular dynamic proof-formats.

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