

DESCARTES' THEORY OF EXPLANATION IN NATURAL PHILOSOPHY. THE CASE OF REFRACTION.

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1. DESCARTES' THEORY OF EXPLANATION

Descartes' theory of explanation is expounded in three of his works, the *Discourse*, *Principles of Philosophy* and the *Regulae*.¹ His aim in the *Discourse* is to find undoubtable knowledge to serve as a basis for his natural philosophy. He applies the method of systematic doubt to arrive at a firm foundation for subsequent knowledge by uncovering any beliefs which have no absolute certainty. With this method he arrives at the certainty of self-consciousness. This certainty allows him to iteratively reconstruct the objects of his consciousness, progressing from the self to God and finally to the external world. Descartes' proof for the existence of God in the third Meditation is an application of his method. The existence of God is a primary truth with absolute certainty, derived through a strict demonstration. Everything derived from primary truths by causal reasoning can also be considered absolute certainty. Descartes describes his method in the *Discourse*:

First I tried to discover the general principles, or first causes, of all that is or can be in the world, without taking for this purpose considering anything but God alone, its Creator, and without deriving these principles from anything but certain seeds of truths which are natural in our souls. After that, I examined what were the first and most ordinary effects that we could infer from these causes. And it seems to me that I thereby discovered the skies, the stars, an earth, and even on the earth water, air, fire, minerals, and some other things, which are the most commonest and simplest of all, and thus the easiest to understand.²

Descartes' methodology of deducing the laws of nature from his metaphysics is applied extensively in his *Principles of Philosophy*. The primary cause of all motion in the world is God Himself. Once he sets things in motion they will continue to move in the same way and for the

¹ For the original text and correspondence, I have used R. DESCARTES, *Oeuvres complètes*, ed. Ch. ADAM and P. TANNERY (=AT; Paris, 1897-1910, 12 vols.). The English translations are from P. J. OLSAMP, *Discourse on Method, Optics, Geometry and Meteorology* (Indianapolis, 1965), M.S. MAHONEY, *The Principles of Philosophy. On Motion* (Princeton, 1995), and J. COTTINGHAM, R. STOOHOFF, D. MURDOCH (eds.) *The Philosophical Writings of Descartes* (=CSM, Cambridge, 1985, 3 vols.). Some parts of *Le Monde* not in CSM have been translated by M.S. MAHONEY, <http://www.princeton.edu/~hos/mike/texts/cartes/world/worldfr.htm>.

² *Discourse*, p. 52.

same reason, unless some other motion interferes with it. This primary cause of motion allows Descartes to derive his *first law of nature*:

Indeed, from the same immutability of God can be known certain rules or laws of nature, which are the secondary and particular causes of the diverse motions that we perceive in individual bodies. The first of these is that any object, insofar as it is simple and undivided, remains, in and of itself, always in the same state and is never changed, unless by external cause.³

But one can proceed further along these lines. As the motions initiated by God are conserved in matter, it also follows that moving parts of matter will proceed along straight lines, which leads us to *the second law of nature*:

The second law of nature is that any part of matter, considered apart, never tends to continue to be moved along any oblique lines, but only along straight lines, even if many are often forced to deflect due to the collision of others, and, as has been said shortly before, in any motion a circle is somehow made from all the matter moved at the same time. The cause of this rule is the same as that of the one preceding, namely the immutability and simplicity of the operation by which God conserves motion in matter.⁴

In his *Principles of Philosophy* Descartes continues to derive additional laws, proceeds in proving these laws and formulates seven rules for determining the direction of colliding bodies in motion.

Given such explicit method and set of rules, one may feel disappointed looking for applications of the method in Descartes' natural philosophy. In his explanations of refraction in the *Optics*, of hydrostatics in *Le Monde* or on the rainbow in *Meteorology*, one fails to find any direct application of his rules, and in the case of the *Optics* and *Le Monde* not even a reference to his method. Why is it that Descartes, whose *method* is so very fundamental to his philosophy, resorts to other sorts of explanation when explaining natural phenomena? The key to the answer, according to Peter Dear and contrary to common perception of seventeenth century science, is the limited importance attributed by Descartes to experience.

As long as experience remained ungrounded in reason, it also remained indeterminate; reason might always change its verdicts. But even if experience were definitive, it could never, as Descartes saw, provide causal explanations: appearances and causal principles constitute logically distinct categories.⁵

When asked for his opinion by Mersenne, Descartes severely doubted Galileo's rules on falling bodies. Although supported by experience they

³ *Principles of Philosophy. On Motion*, Part II, § 37, translation by MAHONEY.

⁴ *ibid.*, § 39.

⁵ DEAR, *Discipline and Experience: the mathematical way in the scientific revolution* (Chicago, 1995), p. 136.

lacked a causal explanation of gravity to convince Descartes. His scepticism is easily understood from the point of his methodology. As he points out in the *Regulae*, ‘Philosophers, of course, recognize that cause and effect are correlatives, but in the present case, if we want to know what the effect is, we must know the cause first, and not *vice versa*.’⁶ Descartes was most sceptical in his appreciation of empirical data for which he had an explanation derived from primary causes. His rules of impact of colliding bodies, cited above, were manifestly contradicting experience. To his death he refused to accept any empirical evidence against his laws of motion. Descartes’ theory of explanation is therefore important in understanding his demonstration of refraction in the *Optics*.⁷ In this paper I argue that most of the critique on the *Optics* stems from a lack of understanding the Cartesian theory of explanation.

2. A BRIEF INTRODUCTION ON REFRACTION

Richard Feynman’s lecture on refraction illustrates how an all too simple model of science prevails in the scientific community. Citing Ptolemy’s refraction tables and introducing Snell’s law he observes:

This, then, is one of the important steps in the development of physics; first we observe an effect, then we measure it and list it in a table; then we find a rule by which one thing can be connected with another.⁸

In his account on Fermat’s principle of least time Feynman takes the opportunity to complete science’s path to glory:

Now, in the further development of science, we want more than just a formula. First we have an observation, then we have numbers that measure, then we have a law which summarizes all the numbers. But the real glory of science is that we can find a way of thinking such that the law is evident.⁹

Although there is a common understanding by serious scholars that Descartes discovered the sine law of refraction independently from Snell, he is not granted a single citation by Feynman, as he is denied the discovery of the law in most of the Anglo-Saxon literature on the subject.¹⁰ The discovery of the sine law of refraction by Descartes has

⁶ *Regulae*, Rule VI, AT, X, p. 383, CSM, I, p. 22,

⁷ I am very much indebted to the first chapter of Sabra’s PhD thesis of 1955, reproduced in A.I. SABRA, *Theories of light: From Descartes to Newton* (New York, 1967), though I disagree with Sabra’s treatment of Descartes’ explanation of refraction which depends on the velocity of light, a concept without meaning for Descartes.

⁸ FEYNMAN, *Lectures on Physics* (Redwood City, 1962-1963, 3 vols.), vol. I, p. 26-2.

⁹ FEYNMAN, *Lectures on Physics*, I, p. 26-3. Out of all possible paths between two points, light takes the path which requires the least time.

¹⁰ Authors who take Descartes’ discovery seriously include: D.C. LINDBERG, *Theories of Vision from al-Kindi to Kepler* (Chicago, 1985), SABRA, *Theories of light*; A.M. SMITH, ‘Descartes’ theory of light and refraction: a discourse on method’ *Transactions of the American Philosophical Society* (1987), 77, pt. 3, p. 1-92; M. S. MAHONEY, *The Mathematical Career of Pierre de Fermat (1601-1665)* (Princeton, 1994), and S. GAUKROGER, *Descartes. An intellectual biography* (Oxford, 1995).

been a source of controversy since its very first publication in the *Optics* of 1637. At best, present-day scholars describe Descartes' demonstration of the law as 'quite inadequate' and 'an implausible route to discovery'.¹¹ Others have ridiculed Descartes' arguments for the apparent inconsistencies and arbitrariness and dismiss the proof all together.¹² Many authors, starting with Isaac Vossius in 1662, even see in Descartes' proof an attempt to cover up plagiarism.¹³ Many reference works, especially those published outside continental Europe, do not grant Descartes the honor of the discovery of the sine law.¹⁴ Why is this demonstration of refraction so hard to accept? Or, if one does not choose to accept the demonstration of Descartes as a path of discovery, why is the proof not taken seriously as a justification of sine law? Part of the answer lies in the lack of understanding of Descartes' theory of explanation. As will be explained, Descartes had specific reasons to adopt the manner of explanation as used in the *Optics*. For the other part of the answer we can point to the mismatching figures, strange analogies and apparent inconsistencies of the proof. In this paper we will take Descartes' demonstration seriously and try to show that there are no inconsistencies and that the line of reasoning by Descartes is a valid justification of the discovered law. In order to achieve this, we will place Descartes' rationale within a framework with three important viewpoints: 1) The demonstration of the law of refraction should be situated within Descartes' specific theory of explanation for natural philosophy, 2) the discovery of the sine law is not an instance of revolutionary science but at the contrary, has to be seen as a culmination of a centuries-old tradition in optics, and 3) a rejection of the validity of Descartes' proof is founded in a misunderstanding of his analogies and a confusion of terms between competing research traditions. In order to understand Descartes' explanation of refraction, we have to go back as far as Ptolemy's *Optics*.¹⁵ Someone who is not familiar with

¹¹ GAUKROGER, *Descartes. An intellectual biography*, p. 140.

¹² 'As a demonstration, [Descartes'] argument was preposterous. Its foundation consisted of assumptions which were arbitrary and contradictory'. R. WESTFALL, *The construction of Modern Science* (New York, 1971), p. 53. Ernst Mach called Descartes' explanation 'unintelligible and unscientific'. MACH, *Die Prinzipien der physikalischen Optik: Historisch und erkenntnispsychologisch entwickelt* (Leipzig, 1921), Trans. J. S. ANDERSON and A. F. A. YOUNG, *The Principles of Physical Optics* (London, 1921).

¹³ VOSSIUS, *Responsio ad objecta Job. De Bruyn* (1663), p. 32; *De Lucis natura et proprietate* (Amsterdam, 1662), p. 36, cited in G. MILHAUD, *Descartes Savant* (Paris, 1921), p. 105. Milhaud shows evidence that completely ruins the hypothesis of plagiarism.

¹⁴ For example, S. DUKE-ELDER and D. ABRAMS, *System of Ophthalmology, Vol 5: Ophthalmic Optics and refraction* (Kimpton, 1970), giving a fairly complete history of refraction theory state: 'The first work on optics to utilize Snell's law was *La Dioptrique*', implying that Descartes only gives a justification for the law as discovered by Snell. See also MACH, *The Principles of Physical Optics*: 'The attempt of Descartes to prove this law *theoretically* is a terrible attempt of the *pedantic* method of demonstration', p. 32, and 'After actually reading these discussions in Chapter II of Descartes' *Dioptrics*, it will scarcely be assumed, even apart from the statements of Huygens, that Descartes discovered the law of refraction', p. 33.

¹⁵ G. GOVI, *L'Optica di Cl. Tolomeo da Eugenio* (Turin, 1885), plate VIII. See also A. LEJEUNE Albert (1956) *L'Optique de Claude Ptolémée dans la version latine d'après l'arabe de l'émir Eugène de Sicile* (Louvain,

the history of optics, will be surprised with the level of sophistication of Ptolemy's study of refraction. Book V on refraction not only contains detailed measurements of refraction in different media, it also contains tens of figures dealing with refraction in objects with flat, convex and concave surfaces and the explanation of several phenomena of refraction such as the apparent change in size of refracted images and atmospheric refraction. But even Ptolemy in the second century AD could rely on a body of knowledge that was founded centuries before him by Euclid (*Optics*, ca. 300 B.C.) and Heron of Alexandria (*Catoptrics*, ca. A.D. 60). The figures used by Descartes in the *Optics* and most of today's textbooks are very alike the drawings from Ptolemy.¹⁶ The original third-century drawing shows us several tacit laws known by Ptolemy and important for the explanation of refraction:

1. The rectilinear propagation of the visual ray, which is a prerequisite for the geometrical treatment of vision and hence Descartes' proof of the sine law.¹⁷
2. Visual rays reflect and refract in a single plane, e.g. the surface of the piece of paper on which the figure is printed.
3. Refraction is caused by a change of media with different [optical] density.
4. When traveling from a rare to a more dense medium, rays are refracted towards the normal.
5. For given media and an angle of incidence the angle of refraction can be predicted.

Ptolemy provided a scientific explanation of refraction that satisfies the requirements of deductive-nomological (DN) explanation to a high degree.¹⁸ However, he failed to give a quantitative law. Instead of quantifying, Ptolemy gave some vague inequalities and ratios between angles of incidence and corresponding angles of refraction. One could also argue that 'change in medium travelled by the visual ray' is not a sufficient causal explanation of refraction. What makes light rays change their direction upon entering a medium with a different optical density? In the ten centuries following Ptolemy the most important contribution to the answer of this question came from the Arab scholar Abū Alī Al-Hasan Ibn Al-Hasan Ibn al-Haytham, more commonly referred to as Alhazan. Alhazan's *Optics* or *Kitāb al-manāẓir* in Arabic (translated in Latin early thirteenth century as *De aspectibus*) had a profound influence on

1956), figure 81, p 231. English translation by A.M. SMITH, 'Ptolemy's theory of visual perception: An English Translation of the Optics with Introduction and Commentary', *Transactions of the American Philosophical Society* (1996), 86, Pt. 2.

¹⁶ LEJEUNE, *L'Optique de Claude Ptolémée*, figure 81, p. 231.

¹⁷ Euclid's first postulate in the *Optics*: visual rays proceed in a straight line indefinitely.

¹⁸ C. G. HEMPEL, *Aspects of Scientific Explanation and other Essays in the Philosophy of Science* (New York, 1965), p. 337 a.n.: The explanandum is a logical consequence of 1) the set of sentences describing the facts and 2) the general laws on which the explanation rests.

medieval Europe and on seventeenth century natural philosophers.¹⁹ Alhazen was the first to decompose reflected and refracted light rays into normal (vertical) and tangential (horizontal) components, which was an important step in geometrical optics. Alhazen's theory of light explains refraction in terms of resistance and speeds that depend on the optical media. Light rays that enter the second medium, perpendicular to the surface, are not refracted and the perpendicular is therefore the path of the least resistance. Light rays passing obliquely from a rare to a dense medium are, because of this reason, refracted towards the normal. In 1604 Kepler's *Optics* were published.²⁰ This landmark in the history of optics provides the most complete account of the perspectivist's theory of refraction. Kepler was specifically interested in the exact measure of atmospheric refraction in order to correct astronomical observations. Starting from the right hypothesis, he came short in arriving at the sine law.²¹ By the beginning of the seventeenth century it had become clear that what was to be found, was a quantitative law describing some proportionality between the angles of incidence and refraction as a constant ratio. In a matter of decades at least four scientists discovered such law independently from each other. Thomas Harriot discovered the sine law probably before 1602, but made the serious error of perishing before publishing.²² Harriot set up his own experiments to measure refraction and evidence of his knowledge of the sine law was communicated by a young collaborator, called Walter Warner, to John Pell who dutifully took notes about the set up and the reasoning.²³ Wilibrord Snell, also called Snellius, determined the cosec ratio of the refractive index perhaps as early as 1621. Like Harriot he failed to publish his findings. Evidence of his discovery survived through the annotations in his Risner edition of Alhazen's *Optics*.²⁴ Descartes discovered the sine law probably between 1626 and 1628 and was the first to offer a complete systematic justification in his *Optics* published in 1637.²⁵ Fermat then provided a different derivation of the same law, as we shall see below.

¹⁹ The edition used by most of the seventeenth century natural philosophers, including Descartes, was F. RISNER (ed.), *Opticae thesaurus Alhazeni Arabis libri septem* (Basel, 1572). The first three books of Alhazen's *Optics* have been translated into English: A.I. SABRA, *The Optics of ibn al-Haytham* (London, 1989, 2 vols.).

²⁰ J. KEPLER, *Ad Vitellionem paralipomena, quibus astronomiae pars optica traditur* (Frankfurt, 1604). English translation by W. DONAHUE, *Kepler's Optics* (Santa Fe, 2000). Page numbers refer to the original edition.

²¹ For an analysis of Kepler's model see A. HEEFFER, "Kepler's near discovery of the sine law: A qualitative computational model", in C. DELRIEUX and J. LEGRIS (eds.), *Computer Modeling of Scientific Reasoning* (Bahia Blanca, Argentina, 2003), p. 93-102.

²² R.S. FISHMAN, "Perish, Then Publish. Thomas Harriot and the Sine Law of Refraction", *Archives of Ophthalmology* (Mar 2000), 118, p. 405-9.

²³ Pell's manuscript with the notes still survives in the British Museum. Birch MSS 4407, F. 183a.

²⁴ J.H. VOLLGRAFF, *Risneri opticae cum annotationibus Willibrordi Snelli* (Ghent, 1918).

²⁵ The first written reference is in Beekman's Journal of 1628, describing the sine law under the heading *Angulus refractionis a Des Cartes exploratus*. C. DE WAARD, *Journal tenu par Isaac Beekman de 1604 à 1634* (den Haag, 1939 – 1953, 4 vols), vol. III, p. 98.

By way of provisional conclusion we can state that Descartes' proof of the sine law of refraction was the culmination of an understanding of a physical phenomenon within the perspectivists tradition. The discovery of the sine law is not a discovery of refraction but a quantification of a since long well described, observed and measured physical process.

3. DESCARTES' THEORY OF EXPLANATION FOR PHYSICAL PHENOMENA

Descartes had two options when writing the second chapter of the *Optics*. He could deduce the sine law from primary truths as an application of his method explained above. Or he could proceed in a different way, more suited for explaining physical phenomena: starting from hypotheses that are founded on empirical grounds, and then deducing the causes. He favored the second option for reasons he explained in the *Discourse*:

I have called them hypotheses only in order for you to know that I think I can deduce them from those first truths which I have explained above, but that I expressly wished not to do so, because of certain minds who imagine that they can comprehend in a day what has taken another person twenty years to think out, ... and I wish to prevent them from taking the opportunity to build some extravagant philosophy upon what they will believe to be my principles, and then attributing the blame on me.²⁶

Descartes' decision not to reveal his 'first truths' can easily be understood when one knows that he was writing these words shortly after he had learned of what had happened to Galileo in Italy. His choice for the alternative method of explanation was inspired by his reluctance to let his first principles be abused for 'extravagant philosophy'. Descartes had to resort to the reasoning from empirical observation of the effects to the causes of these effects. As we have stressed before, Descartes displayed a healthy dose of scepticism towards reasoning from experiences. Fortunately he was convinced that the hypotheses he started from in the *Optics* were physical principles that also could be derived from his primary truths. In fact, in the *Discourse* he apologizes for having proceeded in the reverse way:

And if some of the things of which I have spoken at the beginning of the *Optics* and the *Meteorology* are displeasing at first, because I call them hypotheses and do not seem to worry about proving them, have the patience to read all of it with care and that you will find that you are satisfied with them.²⁷

But still, Descartes was having doubts about this alternative form of explanation. In a letter to Mersenne, the year after the *Discourse* appeared

²⁶ *Discourse*, p. 61.

²⁷ *Discourse*, p. 60.

in print, he expressed an uneasiness about physical explanations that do not derive their truth through deduction from metaphysics. Demonstrating physical principles on the base of sheer observation and experience is asking for the impossible:

You ask whether I consider what I have written about refraction to be a demonstration; and I believe it is, at least in so far as it possible to give a demonstration in this manner without having previously demonstrated the principles of Physics by Metaphysics (which I hope to do one day, but which hitherto has not been done), and in so far any question of Mechanics, or of Optics, or of Astronomy, or any other matter that is not purely Geometrical or Arithmetical, has ever been demonstrated. But to require of me Geometrical demonstrations in a matter which depends on Physics is to demand that I achieve impossible things.²⁸

According to Sabra, this quotation is crucial in understanding Descartes' theory of explanation.²⁹ Descartes expresses explicitly his doubts about the possibility of exact demonstration in physics.

The way Descartes explains physical phenomena is as follows: First we gather information about the external world by observation and contrived experiments. We then formulate hypotheses about these observations. The choice of hypothetical constructions is limited not only by observation, but also by our first principles. The hypotheses do not have to be deducible from our first principles, but at least should not contradict them. The hypotheses function as the *explanandum* of the causal explanation we are constructing. The causes for the *explanandum* can be derived by deduction if the effects are sufficiently precise and specific. For the difficulty with this method is that several causes could explain one and the same effect, as Descartes admits in the *Discourse*:

But I must also confess that the power of nature is so ample and so vast, and these principles so simple and so general, that I almost never notice any particular effect such that I do not see right away that it can be derived from these principles in many different ways; and that my greatest difficulty is usually to discover in which of these ways the effect is derived. And to do that I know no other expedient than again to search for certain experiments which are such that their result is not the same when we explain the effect by one hypothesis, as when we explain it by another.³⁰

What we want to find is that particular cause that is necessary and sufficient to explain the effect. There is no other way out this difficulty than to make our experiments sufficiently precise and have them to contain as many empirical data as is necessary to deduce the causes from

²⁸ Descartes to Mersenne, 27 May 1638, in AT, II, p. 141-2.

²⁹ In his acclaimed study on seventeenth century optics he writes: 'No interpretation of Descartes' theory of explanation would be satisfactory without taken this passage, and the problems with it raises into full consideration', SABRA, *Theories of light*, p. 23.

³⁰ *Discourse*, p. 52.

the effects. By increasing the relevance of our observations we limit the number of explanations that are possible for our hypothesis. Thus, on the one hand we have an *a priori* method for deriving laws about nature from first principles, on the other hand we have an *a posteriori* method for deducing the causes from observed effects (see figure 1). What if both are possible for the same explanation, as Descartes thinks is possible for refraction? Morin argues in a letter to Descartes in 1638 that this amounts to circularity.³¹ Ever since Aristotle asymmetry of explanation is required.³² Descartes agrees with the need of asymmetry but denies committing the fallacy of circularity:

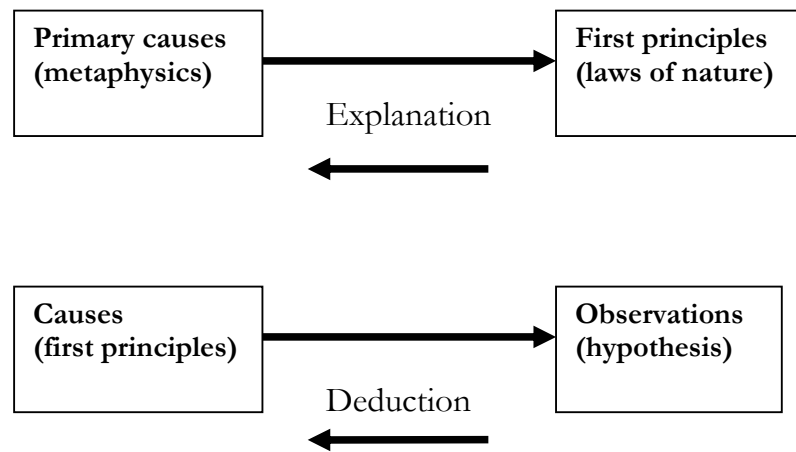


Figure 1: Descartes' a priori and a posteriori method of explanation

For it seems to me that the arguments there follow from each other in such a way that, just as the last ones are proven by the first, which are their causes, so those are reciprocally proven by the last, which are their effects. And you must not think that I commit in this the fallacy that the logicians call arguing in a circle; for as experiment makes of these effects very certain, the causes from which I derive them do not serve so much to prove these effects as to explain them; but, on the contrary, the principles or causes are proven by their effects.³³

Descartes distinguishes between proving and explaining.³⁴ An explanation shows how an effect is necessitated by its cause. In the *Optics*

³¹ 'Et s'il est vrai que prouver des effets par une cause posée, puis prouver cette mesme cause par les mesmes effets, ne soit pas un cercle logique, Aristotle l'a mal entendu, et on peut dire qu'il ne s'en peut faire aucun'. Morin to Descartes, 22 Feb 1638, in AT I, p. 538.

³² D.H. RUBEN, *Explaining Explanation* (London, 1993), p. 101 and also chapters VI and VII.

³³ *Discourse*, p. 60-61.

³⁴ 'Vous dites aussi que *prouver des effets par une cause, puis prouver cette cause par les mêmes effets, est un cercle logique*; ce que j'avoue: mais je n'avoue pas pour cela que c'en soit un d'expliquer des effets par une cause, puis de la prouver par eux; car il y a grande différence entre prouver et expliquer'. Descartes to Morin, 13 July 1638, in AT II, p. 197-198.

however, he *proves* that the only possible cause of refraction is the principle of transmission of action of light. Ironically, some years later, Fermat provided a derivation of the sine law based on a first principle in optics, namely the minimal path in time. He therefore achieved what Descartes would consider a superior explanation: deducing the sine law from primary truths or demonstrating the effect from its cause.

3. CONTINUITY WITH THE PERSPECTIVIST'S TRADITION

Some, like Richard Feynman cited above, hold the image of the scientific revolution in the seventeenth century that broke away with all traditions and philosophies of the past. In this view, a discovery of the sine law would be seen as a victory of the scientific method that combined new mathematical techniques with careful observation and experimentation to result in the discovery of new laws about nature. Of course, we do not want to challenge here important contributions to the philosophy of science such as the concepts of paradigm and incommensurability, but we want to stress specifically that the discovery of the sine law can and must be understood within a tradition of inquiry and experimentation dating back to ancient Greece. On the other hand we don't want to subscribe to the continuity thesis or popular social reconstructivist texts.³⁵ There is an obvious middle way in which an explanation pays respect to scientific traditions and accounts for newly discovered methods as summarized by Peter Dear:

That there could have been a genuine novelty in seventeenth-century developments, but a novelty that possessed wholesale continuity with what went before, has been a rather trivial proposition that has fallen between the cracks of explicit discussion.³⁶

If the discovery of the sine law is not a revolution in science, but has to be understood within a centuries-old tradition, how do we account for the circumstances of the discovery? Why did several people discover the sine law independently in the first decades of the seventeenth century and not in the centuries before? All observational data from Ptolemy, Alhazen and Witelo were available and relatively easy to reproduce. Why was the sine law beyond reach for Kepler who studied refraction very intensely and was in command of the mathematical language to formulate the law of refraction? We believe there were three main conceptual steps, prerequisite in arriving at a quantitative law of refraction. It took several centuries to complete the three. The first step was the principle of geometrical optics in which the laws of geometry are applied to light rays. Euclid and Heron provided the foundations in their works. The second conceptual breakthrough was the decomposition of

³⁵ 'There was no such thing as the Scientific Revolution, and this is the book about it', S. SHAPIN, *The scientific revolution* (Chicago, 1996), p. 1.

³⁶ DEAR, *Discipline and Experience*, p. 2.

motion into horizontal (tangential) and perpendicular (normal) components. As there is no direct relationship between the angles of incidence and refraction, reasoning about the orthogonal components of motion is an essential step. The seventeenth-century natural philosophers could rely on a practice that was initiated by Alhazen and continued within the perspectivist's tradition. The third conceptual step is a principle of conservation. As we shall see in detail below, there are several possible ways that lead to the sine law but they all presuppose that some quantitative aspect is conserved through refraction. The most elegant one is Fermat's principle of the path of least action that is valid independently of the density of the media. For the perspectivists and Descartes it was the more arbitrary choice of one the orthogonal components of motion that remains constant during refraction. Bacon and Peckham had some notion of conservation and came very close to a quantitative determination of the direction of the refracted rays. They probably lacked the skills of analytic art to verbose their insights in a mathematical way. It is significant that Thomas Harriot, Willebrord Snell, René Descartes and Pierre de Fermat, who have discovered the sine law independently from each other, were all excellent mathematicians. The conceptual barriers in arriving at a quantitative law of refraction were taken by the thirteenth century.³⁷ It took the skills of seventeenth century *geometers* to finally arrive at the sine law. We will now look into detail how these conceptual steps are applied in Descartes' proof of the sine law.

4. PREMISES AND FIRST PRINCIPLES

In this section we will investigate the first principles used by Descartes to prove the law of refraction. We distinguish six principles and we will show how they are used within the justification of the sine law. We will demonstrate the interrelation of each of the principles or premises within the perspectivist's tradition and establish how they agree or conflict with Fermat's proof of 1662. All premises are summarized in table 1. Descartes primarily uses analogies as a mode of explanation in the *Optics*. We will not fall into the trap of taking these analogies all too literally as many commentators have done before. The use of analogies is a direct consequence of Descartes' choice for reversing the method of explanation by deducing the causes from their effects. The mechanism of analogy in explanation is explicitly described in Rule Eight of the *Regulae*.³⁸ The analogies are mainly used for pointing out the mechanical properties of light and their choice is evidently founded in the

³⁷ The idea of conceptual barriers for explaining scientific discoveries is from H. MARGOLIS, *Paradigms & Barrier: How Habits of Mind Govern Scientific Beliefs* (Chicago, 1993).

³⁸ 'If [the philosopher] is unable to discern at once what the nature of light's action is, in accordance with Rule Seven he will make an enumeration of all the other natural powers, in the hope that a knowledge of some other natural power will help him to understand this one, if only by way of analogy', *Regulae*, in AT, X, p. 395; translation from CSM, p. 29.

perspectivist’s tradition, back to Alhazen. In present-day terms, the ‘determination’ can best be interpreted as a force vector.

	a. Perspectivists	b. Descartes	c. Fermat
1. Speed of light	Temporal (Alhazen, Wittelo, Bacon)	Instantaneous (also Kepler, Pecham) $v = \infty$	Finite and constant in homogenous media
2. Resistance to light	Proportional to optical density (Alhazen, Wittelo, Bacon)	Inverse proportional to optical density (also Hooke and Newton)	Proportional to optical density (also Huygens and Leibniz)
3. Cause of refraction	Change of density	Change of density	Change of density
4. Decomposition of motion in orthogonal components	(Alhazen, Witelo, Kepler)	$F^2 = F_t^2 + F_n^2$	$F^2 = F_t^2 + F_n^2$
5. Conservation during refraction	Vertical component is strongest (Alhazen, Bacon and Pecham)	Horizontal component of ‘determination’ $f_t \sin i = f_r \sin r$	Shortest path in time $\min\left(\frac{d_1}{v_1} + \frac{d_2}{v_2}\right)$
6. Direction of the refracted ray	Proportional to relative densities (Pecham)	Horizontal component proportional to ease of passage	Shortest path in time

Table 1: Premises of three competing theories on refraction

The speed of light is crucial in contemporary explanations of the laws of refraction. The refractive index of a transparent medium is defined as the speed of light in that medium relative to the speed of light in vacuum. However, in medieval and seventeenth-century discussions on refraction its importance should not be overstated. From Alhazen to Descartes, arguments about the speed of light are closely intertwined with a more general enquiry into visual perception.

1.a. Perspectivists: Speed of light is finite

Lindberg shows how the perspectivists argued against the unambiguous claim of Aristotle in *De Anima* that light is propagated instantaneously. According to Alhazen, it is only because the high speed of light that we have the illusion that light rays take no time to propagate:

Therefore, light passes from the air outside the aperture to the air within the aperture only in time; but that time is completely hidden from sense because of the swiftness of the reception of the forms of light by the air.³⁹

Roger Bacon, in his *Opus Majus*, did not agree with Alhazen but comes to the same conclusion through a different line of reasoning. Witelo argues for the instantaneous propagation of light through mathematical reasoning, but only in situations where light is unimpeded. When traveling through media, the speed of light becomes inverse proportional

³⁹ ALHAZEN, *De aspectibus*, cited and translated by LINDBERG, ‘Medieval Latin theories of the speed of light’, in R. TATON, ed., *Roemer et la Vitesse de la Lumière* (Paris, 1978) p. 45-72, p. 52.

with the density of the medium. John Peckham, and later Kepler and Descartes did not follow Alhazen but accepted the authority of Aristotle.

1.b. Descartes: Instantaneous propagation of light

In chapter 14 of *Le Monde*, Descartes gives twelve properties of light, the third being the instantaneous propagation of light rays. The principle of the instantaneous action of light was central to Descartes' general theory, so much so that he writes to Beeckman: 'it is so certain that if it could be proven false, I am ready to confess that I know nothing at all of philosophy'.⁴⁰ Some authors point out the apparent inconsistency between the physical properties of light in *Le Monde* and the justification of refraction in the *Optics*. However, we will see that Descartes' careful formulation of refraction in the *Optics* does in no way compromise his theory of light as explained in *Le Monde*. Descartes' analogy of the blind man's stick is precisely chosen because of the supposed property of instantaneous action of light:

This will prevent you from finding it strange at first that this light can extend its rays in an instant from the sun to us; for you know that the action with which we move one of the ends of a stick must thus be transmitted in an instant to the other end, and that it would have to go from the earth to the heavens in the same manner.⁴¹

On the other hand, the analogy is very well suited to point out the problem of instantaneous propagation of light in Descartes' corpuscular theory: all bodies have some stiffness and the lack of complete rigidity makes instantaneous transmission of motion impossible. Huygens, who accepted Descartes' idea of a medium that mechanically transports light, would criticize the idea of complete rigidity and introduced elasticity as a property of the traversed medium. This successful idea led him to the wave theory.

When Descartes writes about *vitesse*, he always refers to the speed of movement within the mechanical analogies he uses, such as the speed of a ball. Descartes *never* talks about the 'speed of light' in the *Optics* for the simple reason that in his view light does not move but extends itself within an instant. This is important in the discussion about inconsistencies.

⁴⁰ 'Dixi nuper, cum una essemus, lumen in instanti non quidem moveri, ut scribis, sed (quod pro eodem habes) a corpore luminoso ad oculum pervenire, addidique etiam – hoc mihi esse tam certum, ut si falsitatis argui posset, nil me prorsus scire in Philosophia confiteri paratus sim', Descartes to Beeckman, 22 Aug 1634, in AT I, p 307-8.

⁴¹ *Discourse*, p. 67.

1.c. Fermat: speed of light is finite and constant in homogeneous media

Fermat did not venture on natural philosophy. His mathematical brilliancy, however, would have allowed him to effectively apply mathematical laws and techniques to the study of nature. In his excellent biography on Fermat, Michael Mahony states that Fermat's lack of interest in mathematical physics is the simple reason for Fermat's very limited output on the subject:

Simply put, he did not believe in it. He was either too poor or too good a philosopher to be swept up by the movement to mathematize nature. He saw no *a priori* reason to assume that natural phenomena obeyed exact mathematical laws of behaviour, and he saw a good deal of a posteriori evidence to suggest that they did not.⁴²

As an example, Fermat explained that the octave is determined by the double ratio, but he conjectured to Mersenne that it may also be that the human ear cannot distinguish between that precise interval and imprecise ones close to it.⁴³ In 1657, Fermat was given a copy of a book by Marin Cureau de la Chambre, called *Light*. In this book Cureau applied the principle of the shortest path to derive the law of reflection. Of course, naively applying this principle to refraction would imply that light rays continue in a straight line through more dense media. Fermat went further to define a principle of least action, accounting for the difference in resistance to light in different media. This principle, of course, implied a finite speed of light, depending on the density of the medium through which it travelled. Although Fermat was reluctant to bluntly oppose Descartes and Cureau on the infinite velocity of light it is clear that he believed in a finite and constant velocity of light in homogenous media.⁴⁴ In the principle of least action, he found the wording that avoided controversies over physical nature of light.

2.a. Perspectivists: resistance to light is proportional to optical density

Alhazen, Wittelo and Bacon all agreed that transparent bodies resist the movement of light with varying degree, depending on their transparency. More importantly, the degree of density determines the resistance to

⁴² MAHONEY, *The Mathematical Career of Pierre de Fermat*, p. 370.

⁴³ 'De sorte, par mon sentiment et par ces raisons, j'estime qu'il serait bien malaisé de trouver une proportion différente de la double qui fit l'octave plus exactement que celle-là. Je vous avoue bien qu'il y en a infinies qui effectivement feront accords différents et lesquels néanmoins la différence ne sera pas comprises par l'ouïe la plus délicate qui puisse être; et de là on pourrait conclure que peut-être la vraie octave consiste pas précisément en la proportion double.' Fermat to Mersenne, 26 April 1636, in C. DE WAARD (ed.), *Correspondance du Père Marin Mersenne, religieux minime* (Paris, 1932-1986, 16 vol), VI, p. 51-2.

⁴⁴ See his letter to Cureau of 1 jan 1662, in V. COUSIN, *Oeuvres de Descartes* (1824-1826, Paris), VI, p. 485-507.

light and thus the degree of deviation from the direct path. This is unambiguously read in Roger Bacon:

For the denser the second body, the more the refracted species deviates from the direct path, on account of the resistance of the denser medium, for the density resists the ray.⁴⁵

2.b. Descartes: resistance to light is inverse proportional to optical density

The idea that light penetrates rarer bodies more easily than denser ones is older than the perspectivists and dates back to Aristotle. However, Descartes departs from this widely accepted view and states:

the harder and firmer are the small particles of a transparent body, the more easily do they allow the light to pass.⁴⁶

Why did Descartes, who so closely follows the perspectivist's analysis of refraction, take a completely opposite stand on the resistance to light? Some have suggested that the principle of inverse proportionality is an arbitrary and awkward premise, necessary as an artificially constructed step in Descartes' proof.⁴⁷ However, there is nothing artificial about the fact that light travels easier through denser bodies. For Descartes, it is a direct consequence of his general philosophy on the mechanical properties of the corpuscular nature of matter. As explained in *Le Monde*, light 'is transmitted from one place to another by means of several bodies that touch one another and that continuously fill all the space between the places'.⁴⁸ Therefore, the harder the bodies, the easier the passage. It is noteworthy that Newton later adopted the same principle of inverse proportionality, although he had reservations about the mechanical interpretation that he exchanged for a dynamical interpretation of his own.⁴⁹ Historians of science who criticized Descartes' demonstration of the sine law failed to interpret Descartes within his own philosophy. Since the second half of the nineteenth century it was established that light travels slower in denser media. However, in the seventeenth century Descartes' viewpoint was perfectly reasonable. Indeed, within his mechanical philosophy of nature, it was a

⁴⁵ Bacon, *De multiplicatione specierum*. 'Nam quanto corpus secundum est densius, tanto minus recedit fractio ab incesso rectu, propter resistentiam medii densioris, densitas enim resistit radio', in D.C. LINDBERG, *Roger Bacon's Philosophy of nature: A Critical Edition and English Translation of De Multiplicatione specierum and De speculis comburentibus* (Oxford, 1983), p. 107-109.

⁴⁶ *Discourse*, p. 82.

⁴⁷ Descartes deftly solved the problem [of determining the direction of refraction] by the facile (and dubiously mechanical) proposal that the denser the body, the less its resistance to the passage of light'. D.C. LINDBERG, 'The Cause of Refraction in Medieval Optics', *The British Journal for The History of Science* (1968), 4, p. 23-38, p. 35.

⁴⁸ *Le Monde*, chap 14, properties of light 9-10, translation by MAHONEY.

⁴⁹ SABRA, *Theories of light*, p. 301, show how Descartes' principle gains greater plausibility in Newton's account of refraction.

logical consequence that light passes more easily through dense bodies. When Descartes uses the term ‘resistance to light’ he by no means implies any relation between resistance and speed of light. As we have argued before, ‘speed’ is no meaningful property in Descartes’ mechanical interpretation of light. Still, some present-day historians mistakenly interpret these premises in terms of speed. Smith concludes that the analogy of a projectile under water ‘casts doubt in turn on Descartes’ claim that, since light inclines towards the normal when passing from rarer to denser medium, it must be gaining, rather than losing speed’⁵⁰ Sabra notes ‘that Descartes never actually took that step; he never used the word ‘speed’ in connection with light’.⁵¹ Then continues to base his complete analysis of Descartes’ proof on the assumption incorrectly attributed to Descartes ‘that the velocity of light is characteristic of the medium it is traversing’.⁵² This mistaken assumption casts serious doubts on this otherwise excellent analysis of Descartes’ demonstration of the sine law. In fact, in Kuhnian terminology, ‘resistance to light’ allows communication between two competing research programs: the term reconciles two opposing views. For Beeckman, Fermat and later Huygens and Leibniz, resistance to light is reciprocal to the speed of light. In Descartes’ mechanistic worldview, resistance to light refers to the degree of transmission of action of light. But both viewpoints did agree that refraction was caused by the differences in resistance to light of the media traversed.

2.c. Fermat: resistance to light is proportional to optical density

Although Fermat believed that light travels slower in media with higher density, this is not a necessary premise for his proof. The principle of least action together with the observation that refracted rays bend towards the perpendicular when entering media with higher density, is sufficient for his proof. Actually, the proportionality of the speed of light to the optical density is a consequence of his derivation of the sine law. Sabra reconstructed the complete algebraic derivation that Fermat left out of his proof and finally leads to $\frac{\sin i}{\sin r} = \frac{v_i}{v_r}$.⁵³ From this it can be deduced immediately that the speed of light in the refracted (more dense) media is less than in the media of incidence.

⁵⁰ SMITH, ‘Descartes’s theory of light’, p. 31.

⁵¹ SABRA, *Theories of light*, p. 113.

⁵² Ibid. p 114.

⁵³ Ibid. p. 146-147. Fermat’s proof is included at the end of his letter to Cureau of 1 Jan 1662 and titled “Analyse pour les réfractions”, in COUSIN, *Oeuvres de Descartes*, p. 495-507.

3. Refraction is caused by change of optical density

The change of optical density as a cause for refraction is known since antiquity and Ptolemy formulated the principle in book V of his *Optics*. Not only does the change in density cause refraction, the density of the media also determines the amount of refraction:

this sort of bending of the visual ray does not occur [the same way] in all liquids and rare media; what happens, instead, is that in each one of these [media] the amount of deflection is determined solely by the way in which the medium allows penetration.⁵⁴

Only when the when light enters perpendicularly to the surface, the rays will continue unrefracted.

light will be found to extend rectilinearly in a transparent body as long as that body is of uniform transparency. If however, light meets the first body through which it extended, it will not pass into it along the straight lines in which it formerly extended unless those straight lines are perpendicular to the surface of the second transparent body.⁵⁵

This premise is present in all three accounts of refraction by the perspectivists, Descartes and Fermat.

4. Motion can be decomposed in orthogonal components

As stated above, the orthogonal decomposition of motion was the second important step in arriving at a quantitative law of refraction. The importance of this instrument of decomposition of motion or force cannot be understated. For evident reasons the sine law cannot be stated without decomposition. It is not surprising to find it used by Fermat or Descartes in their derivation of the sine law, as both are considered to be the originators of analytic geometry. But it is surprising to see it so explicitly formulated by Alhazen:

Now if the motion within an object is oblique with respect to the surface of the object, it is composed of a motion in the direction of the perpendicular passage ... and of a motion along the perpendicular to this [first] perpendicular. Therefore when light is moved through a dense transparent substance along an oblique line, its passage through that transparent substance takes place by means of a motion composed of the two aforesaid motions.⁵⁶

The importance of the tool of decomposition is illustrated by the fact that, except for Bacon, all theorists that have written on refraction after Alhazen, have used it. Also Kepler writes that:

⁵⁴ SMITH, 'Ptolemy's theory of visual perception', p. 229.

⁵⁵ SABRA, *The Optics of Ibn al-Haytham*, p. 68.

⁵⁶ *Ibid.*, p. 28.

The motion of light striking obliquely is composed of a motion perpendicular and a motion parallel to the surface of the dense medium.⁵⁷

In the *Optics* Descartes gives an explicit account of the principle of decomposition as a necessary step in his proof of the sine law:

It must be noted that the determination to move towards a certain direction, as well as movement and any sort of quantity generally, can be divided among all the parts of which we can image that it is composed.⁵⁸

5. The conservation principle

Some characteristic of the ray of incidence is preserved during refraction. As we have decomposed the original motion into components it becomes easier now to talk about what is conserved of the original motion. It is not possible that both the normal and tangential component of the motion rest unchanged because there would be no refraction. Hence there are only three possibilities: 1) the vertical component is conserved, 2) the horizontal component is conserved or 3) something else is conserved which determines both components as different from the original ones. Strangely enough, each of these possibilities has led to a theory on refraction.

5.a. Perspectivists: the vertical component is strongest

Although Alhazen was the first to decompose the propagation of light into components he does not specify what happens to both components during refraction.⁵⁹ However, using analogies he demonstrates that the vertical component is stronger than the horizontal one. A first analogy is that of a falling stone. The impact of the stone will be maximal if it is not diverted from its perpendicular course. Even when it is deviated from its course it will return along its original path because of the strength of perpendicular approach. A second analogy is that of an iron ball or sword. When an object is hit or cut with an iron ball or sword, its penetration will be greatest when the blow is perpendicular to the surface of the object. When the blow is applied obliquely, much of its effect will be lost. It is the vertical component that determines the effect of the blow. Bacon, Wittelo and Peckham have repeated these analogies. But this is as far as it goes. Smith reads in Bacon a rule of vertical conservation, but this is rather contradicted by Bacon himself.⁶⁰ First of all, Bacon

⁵⁷ DONAHUE, *Kepler's Optics*, p. 85.

⁵⁸ *Discourse*, p. 76.

⁵⁹ Although some authors contribute the conservation of the vertical component to Alhazen, Sabra states that nothing is said about this in the Arabian text; SABRA, *The Optics of ibn al-Haytham*, p. 97.

⁶⁰ Citing BACON 'when a species passes from a subtler into a coarser substance, it maintains its ease of traversal in the second substance, so that its passage through the two substances is, insofar as possible, proportional and uniform', Smith concludes: 'Therefore, if the act is to remain natural,

does not use Alhazen's terminology of components but speaks of species generated by an agent, and secondly, for Bacon both species are affected by refraction: 'both perpendicular and oblique species are somewhat impeded [by refraction] but oblique species more so'.⁶¹ Vertical species for Bacon are different from oblique ones and thus behave different in dense media. But there is no reason to assume a principle of conservation here. In fact, I have not read any explanation in perspectivist's texts that supports a principle of vertical conservation.

5.b. Descartes: the horizontal component is conserved

The principle of conservation of the horizontal component is arguably the most problematic one in Descartes' proof of the law of refraction. Fermat had problems with the use of analogies and, of course, with the inverse proportionality of density and resistance to light, but his main objection to Descartes' proof in 1637 lays in the arbitrariness of this premise. For the next twenty years Fermat had doubts about the correctness of the sine law. Granting Descartes some principle of conservation for the sake of the argument, he could not accept his choice for the horizontal component. Not that Fermat believed the principle of horizontal conservation to be wrong, but in his view Descartes failed to give any good reason for accepting this premise.⁶² Let us see how Descartes formulated the principle of conservation:

of the two parts of which we can imagine this determination to be composed, only the one ... to tend from high to low can be changed in any manner ... and the one that was causing it to tend toward the right must always remain the same as it was, because in no way does the cloth [the surface] oppose its going in this direction⁶³

One important point here: the principle of conservation of the horizontal component refers to the determination of the movement. What does Descartes mean with 'determination': direction, speed, motion or force? Descartes was most explicit about his choice for the term 'determination' in a letter to Mydorge, some time after the dispute with Fermat.⁶⁴ Determination is the tendency to move and is independent of speed.

power must be conserved throughout along the vertical or normal'. SMITH, 'Descartes's theory of light', p. 55-56.

⁶¹ LINDBERG, *Roger Bacon's Philosophy of nature*, p. 112.

⁶² 'Pourquoi donc notre sceptique sera-t-il obligé d'accorder gratuitement et sans preuve que le mouvement qui se fait vers le droite dans sa figure de la page 18 avance également vers ledit côté droit, après qu'il a changé de milieu? Ce n'est pas que cette proposition ne puisse être vraie, mais elle ne l'est qu'au cas que la conclusion que M. Descartes en tire soit véritable, c'est-à-dire que la raison ou proportion pour mesurer les réfractions, ait été par lui légitimement et véritablement assignée', Fermat to Clerselier, 10 March 1657, in COUSIN, *Oeuvres de Descartes*, VI, p. 419.

⁶³ *Discourse*, p. 78.

⁶⁴ 'Je remarque d'abord, il veut que j'aie supposé telle différence entre la détermination à se mouvoir çà ou là et la vitesse qu'elles ne se trouvent pas ensemble, ni ne puissent être diminuées par une même

5.c. Fermat: the shortest path in time is conserved

Fermat found in the book of de la Chambre the principle of the shortest path applied to the law of reflection, a principle dating back to Heron of Alexandria. Of course, the shortest path cannot be applied to refraction. The refracted ray would thus continue its path of incidence. But it brought Fermat to the idea of a more general principle of economy in nature. Although he used the term *Naturam per vias brevioris operari*, he evidently referred to the easiest path and not the shortest one. As shown in figure 2, the path of least action can be calculated using some imaginary point O that differs from D by some small quantity ϵ using Fermat's method of maxima and minima.⁶⁵ The resulting path is the one in which light meets the least resistance.

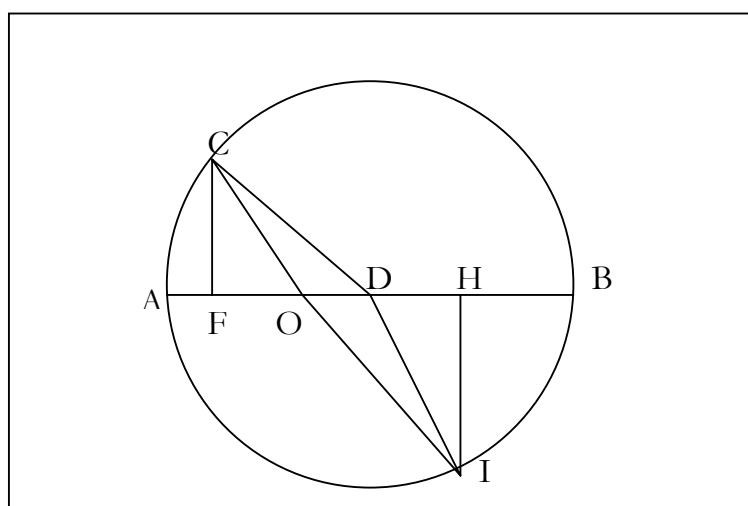


Figure 2: Fermat's principle

Direction of refraction is proportional to relative densities

For a quantitative law on refraction it was crucial to link the direction of the refracted ray with the relative densities of the media traversed. This relation is clearly present in Kepler's analysis cited above, but was already present in the perspectivist's tradition as the following quotations show. Roger Bacon explicitly speaks of proportions:

cause, savoir par la toile CBE; ce qui est contre mon sens et contre la vérité, vu même que cette détermination ne peut être sans quelque vitesse, bien qu'une même vitesse puisse avoir diverses déterminations, et une même détermination être jointe à diverses vitesses.' (italics by Descartes), Descartes to Mydorge, 24 feb 1638, in COUSIN, *Oeuvres de Descartes*, VI, p. 404-405.

⁶⁵ Figure 47 from plate III in COUSIN, *Oeuvres de Descartes*, V. The diagram belongs to *Analysis ad Refractiones*, an appendix to the letter of Fermat to de la Chambre of 1 Jan 1662, in COUSIN p. 485-507.

when a species passes from a subtler into a coarser substance, it maintains its ease of traversal in the second substance, so that its passage through the two substances is, insofar as possible, proportional and uniform.⁶⁶

As does Pecham:

since there is variation in the ease of traversing media, it will necessarily be found that the direction of the ray in the second medium ... is proportioned to the direction in the first medium as the resistance of the second medium to the resistance of the first.⁶⁷

Horizontal component is proportional to the ease of passage

Now we finally arrive at Descartes' principle, which allows a direct derivation of the sine law. Given that the horizontal component of the 'light force' is conserved, it will be proportional to the relative densities. If light enters a denser medium where there is less resistance by some factor n , the horizontal component will be smaller by this factor n . Unfortunately this principle is formulated somewhat awkwardly:

[if one] augments the force of its movement by for instance one-third, so that afterwards it can make as much headway in two moments as it previously made in three [... then] there is a third less distance between FE and HB than between HB and AC.⁶⁸

Literary taken, Descartes could be misunderstood in comparing four thirds of the distance with three halves. Mistakes like this contributed to the misunderstandings and the critique on the *Optics*. Another such example is the carelessness with which Descartes submitted his figures to the printer.

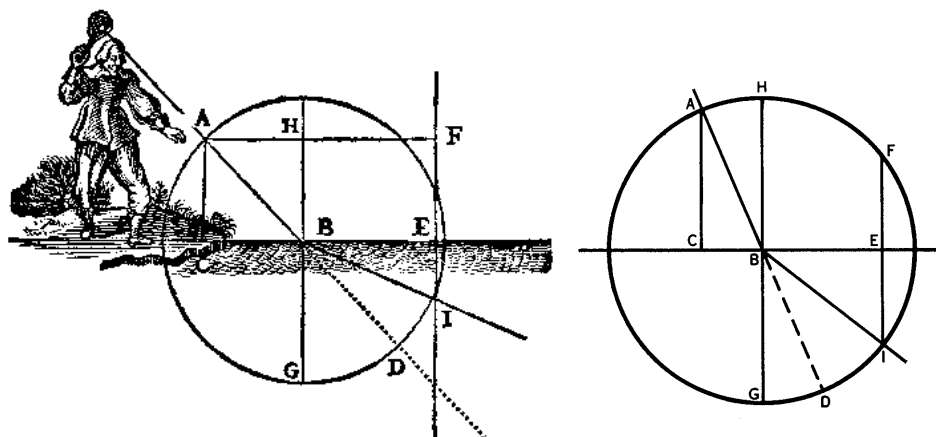


Figure 3: from the *Optics*. The right figure shows the correct proportions.

⁶⁶ LINDBERG, *Roger Bacon's Philosophy of nature*, p. 116.

⁶⁷ PECHAM, *Perspectiva Communis*, prop. I-15, quoted and translated by LINDBERG, 'The Cause of refraction', p. 33.

⁶⁸ *Discourse*, p. 80.

Descartes intends that the distance $|HF|$ is twice that of $|AH|$, but the printer ‘misunderstood’ the instructions. In contrast with talented draftsmen like Harriot and Newton, Descartes was very poor in drawing and was well aware of it. While getting help from Mersenne and Van Schooten, the final woodcuts in the *Discourse* contained several smaller errors. Hobbes took some pleasure in pointing out to Mersenne the misrepresentation of ratios in figure 17 of the *Optics*.

5. CONCLUSION

Kepler asked whether the ratio of densities between the two media in which refraction occurs, could be measured by the ratios of lines in a geometrical analysis. Descartes gives the answer to this question as a conclusion of his demonstration of the sine law:

Only it is necessary to take care that the inclination be measured by the quantity of the straight lines, such as CB or AH , and EB or IG , and the like, compared to each other; not by that of angles like ABH or GBI , nor even less by those, like DBI , which we call angles of refraction. For the ratio or proportion between these angles varies with the different inclinations of the rays: whereas that between the lines AH and IG , or others such, remains the same in all refraction caused by the same bodies.⁶⁹

Descartes considered the observed ratio of the sines a hypothesis. In his explanation of refraction he deduce the first principles that are causing this natural phenomenon: the mechanical properties of light and the conservation of the horizontal component of determination of light. Although the method of the *Discourse* prescribes to find the laws of nature from primary causes in metaphysics, this is not always possible in natural philosophy, as is the case for the refraction law. Therefore Descartes proceeded from observed effect to first principles. In fact, this is most likely the chronology that is the basis for his discovery of the sine law. By studying Kepler’s *Optics*, Descartes succeeded in answering the question to which geometrical proportion the ratio of optical densities corresponds, the crucial question which Kepler failed to answer. Descartes most likely found that the secants FH and FN in Kepler’s drawing fitted up the required proportion. However, this was not something that could be derived deductively, so it had to be considered as a hypothesis. Descartes found out that the ratio of the secants did not vary during contrived observations. So he had to look for the principles that caused this to happen. A reformulation of the proportion of secants as the ratios of sines between the angles of incidence and refraction was more convenient as this corresponded with one of the components of the action of light. The explanation of refraction in the *Optics* is therefore

⁶⁹ *Discourse*, p. 80.

a logical consequence of the chronology of the discovery and the theory of explanation by Descartes.

Fermat on the other hand, and without intend, proceeded in the Cartesian way advocated in the *Discourse*: he deduced the sine law from the first principle of economy of nature: light proceeds along the path of the shortest time. Although Descartes would have disagreed with this principle, as it contradicted most of his theory in *le Monde*, he would have admired Fermat's dexterous deduction of the laws of nature from general principles.