A Disjunction is Exclusive Until Proven Otherwise
Introducing the Adaptive Logics Approach to Gricean Pragmatics

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Abstract

In Gricean pragmatics, *generalized conversational implicatures* (**GCI**) are the pragmatic rules that allow the hearer to derive the intended meaning of the sentences uttered by the speaker. Moreover, in contradistinction to particularized conversational implicatures, **GCI** only depend on what is said, and not on the linguistic context.

One of the main characteristics of **GCI** consists in their being defeasible. In other words, when new utterances are made, some of the earlier consequences obtained by application of **GCI** might have to be withdrawn. Because of this characteristic, Levinson argued that **GCI** should be formally modeled as *non–monotonic inference rules*. In this paper, I will argue that all attempts so far failed to do so in a satisfactory way. Moreover, I will contend that by relying on the *adaptive logics approach*, **GCI** can be captured satisfactorily as non–monotonic inference rules. I will do so by showing how the *or*–implicature, one of the best–known of the **GCI**, can be captured by adaptive means.

Keywords

Gricean pragmatics, scalar implicatures, presumptive meanings, defaults in semantics and pragmatics, adaptive logics

1 Introducing Gricean Pragmatics

H.P. Grice (1989, p. 26) states that participants in a conversation should make their contributions in accordance with what is required by the accepted purpose or direction of the talk exchange in which they are engaged (the *cooperative principle*). Moreover, the infamous *Gricean maxims* (of quantity, quality, relation, and manner) are the result of Grice's attempt to flesh out the main characteristics of communicative acts governed by the cooperative principle:[1]

(Maxim of Quantity) Make your contribution as informative as is required (for the current purposes of the exchange). Do not make your contribution more informative than is required.

(Maxim of Quality) Try to make your contribution one that is true.

- Do not say what you believe is false.
- Do not say for which you lack adequate evidence.

(Maxim of Relation) Be relevant.

(Maxim of Manner) Be perspicuous.

- Avoid obscurity of expression.
- Avoid ambiguity.
- Be brief (avoid unnecessary prolixity).
- Be orderly.

At face value, a lot of utterances made in conversation seem to run counter to the cooperative principle, for example in the case of irony, analogy,... These utterances are usually not intended to be uncooperative (or in conflict with Grice's maxims), but are made under the assumption that the hearer is able to derive the intended meaning of the utterance by presupposing the utterance to be cooperative. In most cases, the hearer is indeed capable to do so. This shows that the Gricean maxims should not be interpreted as actual guidelines directed at speakers, but are better understood as "presumptions about utterances, presumptions that we as listeners rely on *and* as speakers exploit" (Bach, 2006, p. 24, my italics).

Conversational Implicatures. In order to get at the actual meaning of an utterance made in conversation, a hearer frequently has to derive some statements that do not follow logically from that utterance, but only pragmatically (i.e. in a way that reconciles the utterance with the cooperative principle). These derivations are governed by pragmatic rules, called *conversational implicatures*. In this paper, I will focus on a particular kind of conversational implicatures, viz those that solely depend on what is actually said, i.e. the linguistic content of the utterance, and that do not depend on the extra–linguistic context of the utterance, such as a.o. the shared background knowledge of the participants in a conversation. In other words, I will focus on *generalized conversational implicatures* (**GCI**).

[1]The maxims below are taken from Grice (1989, pp. 26–27).

Formally Characterizing GCI. The fact that **GCI** only depend on what is linguistically given, is not the only distinctive property of **GCI**. Another is that they are defeasible (see e.g. Levinson, 2000). Hence, when new utterances are made, some of the consequences derived by means of **GCI** might have to be withdrawn. Because of this last characteristic, Levinson (2000) argued that **GCI** should be modeled formally as non–monotonic inference rules. Although he didn't attempt to do this himself, others did — for an overview, see e.g. Jasczolt (2008), Levinson (2000, pp. 45–54), Thomason (1997), and Wainer (2007, pp. 212–213).

Most attempts to capture **GCI** formally do so by means of non–monotonic logics that are based on *default logic* (Reiter, 1987), or *circumscription* (McCarthy, 1987). The most elaborated approach of this kind (at least to my knowledge) is the one presented by Wainer (2007). My main objection to these approaches is that they are usually restricted to the semantic plane, which means that a proof theoretic characterization is missing. As a consequence, it can hardly be claimed that these approaches truly capture **GCI** as non–monotonic *inference rules*. They at most do so partially, viz on the metatheoretic level. Hence, these approaches deviate too much from what actually happens when people apply **GCI** in conversation in order to capture the dynamic process involved in an adequate way. Even when an algorithm is provided enabling one to decide whether or not a formula is a **GCI**–consequence of a set of utterances, these algorithms don't resemble human reasoning at all.

Other formal approaches that intend to capture **GCI** suffer from the same problems as those above. The most common one is the so–called *substitutional approach* — see e.g. Gazdar (1979) and Horsten (2005). This approach proceeds by substituting those (parts of) statements that trigger **GCI** by expressions that explicitly express the full, intended meaning of that statement. If defeasibility is accounted for at all, it is done extra–logically. As indicated before, this doesn't resemble human conversation at all. Consequently, it is more suitable not to interpret this approach as a way to model the way **GCI** are applied in conversation, but as a means to determine the communicative content of an utterance.

Aim of this Paper. In this paper, I will show that the use of **GCI** in actual conversation can be captured satisfactorily by relying on the *adaptive logics approach* (see Batens, 2001, 2007; Batens et al., 2007). For, adaptive logics (**AL**) are a kind of dynamic, non–monotonic logics that can not only be characterized semantically, but also proof theoretically. Hence, it is possible to devise adaptive logics in which **GCI** are treated as truly non–monotonic *inference rules* (let's call these logics $\mathbf{AL}^{\mathbf{gci}}$). More specifically, in the $\mathbf{AL}^{\mathbf{gci}}$–proof theories, the consequences of applying **GCI** are only accepted for as long as there is no reason to reject them (for example, for as long as the speaker in the conversation hasn't asserted a sentence that contradicts those consequences). In the end, $\mathbf{AL}^{\mathbf{gci}}$ only retain the unproblematic applications of **GCI**, while rejecting the problematic ones. As such, $\mathbf{AL}^{\mathbf{gci}}$ nicely capture the way **GCI** are used in conversation.

To keep things simple, I will only show how the adaptive logics approach can be applied to capture one of the best–known **GCI**, viz the *or*–implicature. The latter states that if a disjunction is asserted in a conversation, this should be interpreted as the assertion of an exclusive disjunction. Hence, the assertion of a disjunction implicates (i.e. implies pragmatically) that one of the disjuncts is true, but that not both are.

($or -$ **implicature**) The assertion $A\_\texttt{or}\_B$ implicates that $\texttt{not}\_(A\_\texttt{and}\_B)$.

In spite of the imposed restriction, I will show that the proposed approach can easily be generalized to other **GCI**.

Besides simplicity, there is another reason to limit the attention to the *or*–implicature in particular. For, all attempts to capture the *or*–implicature as a non–monotonic inference rule by means of a non–monotonic logic face a problem with respect to the interpretation of the logical connectives — a problem exposed by Horsten (2005). Although this problem is related to the *or*–implicature, a general formal characterization of **GCI** is only possible in case this problem has been solved. I will attempt to do so in section 2. Afterwards, in section 3, I will show how the adaptive logics approach can explicate the application of the *or*–implicature in conversation.

2 A Problem for Classical Gricean Pragmatics

Horsten (2005) pointed to a problem for all formal approaches that wish to capture the *or*–implicature as a non–monotonic inference rule. The problem is related to the interpretation of the logical connectives (and of the disjunction in particular). Grice (1989) states that these should be interpreted classically. As most neo–Griceans have taken this for granted, all formal approaches have tried to implement the *or*–implicature (as well as all other **GCI**) in the context of *classical logic* (**CL**). However, Horsten showed that the *or*–implicature cannot be implemented as a genuine non–monotonic inference rule in the context of (propositional) **CL**. He did so by considering two (rather) standard non–monotonic derivability relations, which he called respectively *weak* derivability and *strong* derivability. Both are based on the *consistent extensions* of a particular premise set Γ, which are obtained by consistently extending the **CL**–consequence set of Γ by applications of the *or*–implicature (here interpreted as $A \vee B$ implies $\neg(A \wedge B)$). A formula A is now considered a weak consequence of Γ iff A follows from at least one consistent extension of Γ, and a strong consequence iff A follows from all consistent extensions of Γ. By relying on the **CL**–validity of the inference rule *addition* (**ADD**: $A \vdash A \vee B$), Horsten showed that all formulas that are compatible with the premise set Γ are weak consequences of Γ, while no formula that is not a **CL**–consequence of Γ is a strong consequence of Γ. In particular, this is a consequence of the fact that the *or*–implicature can be applied to all disjunctions that are a **CL**–consequence of Γ, even to those that were generated by means of **ADD**. As any formula will be a disjunct of some of these disjunctions, there is a consistent extension of Γ for each formula compatible with Γ. On the other hand, no formula is true in all consistent extensions, for there is always a consistent extension in which its negation is true — I will not spell out the specifics of the argument, for these can be found in Horsten (2005, pp. 113–117). Hence, in the former case there are too much consequences obtained by means of the *or*–implicature, while in the latter case there are too few. Of course, this doesn't need to be considered as an insurmountable problem, for different ways to restrict the **GCI**–consequences remain possible. But, as Horsten himself rightly points out:

> One can of course try to eliminate the undesirable consequences by imposing further restrictions on applications of $C\vee$ [the *or*–implicature]. But this would make the nonmonotonic rule rather complicated, compared to the apparent simplicity of the rule as it is used in daily communication. (Horsten, 2005, p. 117)

Henceforth, I will call the above problem the *implementation*–problem of the *or*–implicature.

Because of the implementation–problem, two possible lines of reasoning remain open. The first is to side with Grice, and to maintain that the logical connectives are to be interpreted classically. This obviously implies giving up the attempt to formally characterize the *or*–implicature in a way that adequately explicates its use (and that of the other **GCI**) in communication. This is the option taken by Horsten (2005). As a consequence, he turned to a substitutional approach in order to capture the *or*–implicature formally (see section 1 for my objections to this kind of approach). On the other hand, the second possibility is to state that Grice must be wrong, and that the logical connectives aren't to be interpreted classically. Obviously, the question remains how they should be interpreted then. Only Verhoeven & Horsten (2005) tried to answer this question. They state that the connectives are best interpreted according to the logic **RAD**. The latter is a peculiar logic that was originally devised to explicate *relevantly assertable disjunctions*, disjunctions that do not contain redundant disjuncts (see Verhoeven, 2007). By limiting the applications of the *or*–implicature to relevantly assertable disjunctions, Verhoeven & Horsten are able to avoid the problem generated by the inference rule **ADD**. For, a disjunction obtained by means of **ADD** obviously contains a redundant disjunct, and so, is not relevantly assertable. Nonetheless, the **RAD**–approach of Verhoeven & Horsten does not solve the implementation–problem satisfactorily, for the implementation in the context of **RAD** doesn't capture the full inferential strength of the *or*–implicature. For example, the formula $\neg B$ doesn't follow from the formulas $A \vee B$ and A (a problem mentioned by Verhoeven & Horsten themselves). The reason for this odd kind of behavior is obvious: in view of A, the disjunction $A \vee B$ is not relevantly assertable. Hence, the *or*–implicature cannot be applied to it, preventing the pragmatic derivation of $\neg B$ from A and $A \vee B$. Moreover, the **RAD**–approach is also philosophically flawed, for it confuses the viewpoint of the speaker in a conversation with the viewpoint of the hearer. As the logic **RAD** was devised to distinguish the relevantly from the irrelevantly *assertable* disjunctions, it obviously captures part of the reasoning process of the speaker. The *or*–implicature though, is typically an inference step that is associated with the hearer, for it is a step made to reconcile the assertions of the speaker with the cooperative principle. As a consequence, the logic **RAD** cannot be considered the right logic to capture the context of implementation of the *or*–implicature.

I agree with Verhoeven & Horsten that the logical connectives should not be interpreted classically (and that Grice must be wrong on this point). As a consequence, in this paper, the *or*–implicature will also be implemented in the context of a non–classical logic, viz the logic $\mathbf{CL^u}$ (see section 2.1). However, in contradistinction to the **RAD**–approach, the one based on the logic $\mathbf{CL^u}$ solves the implementation–problem in a satisfactory way (see section 2.2), for not only does the implementation in the context of $\mathbf{CL^u}$ capture the full inferential strength of the *or*–implicature, it even does so in a way that is philosophically apt (by focussing on the viewpoint of the hearer).

2.1 The Logic $\mathbf{CL^u}$

The logic $\mathbf{CL^u}$ is obtained by adding additional connectives to the logic **CL**. These new connectives are particularly weak, for the logic $\mathbf{CL^u}$ allows for *gaps* with respect to all of them. As a consequence, the logic $\mathbf{CL^u}$ is a logic related to the gap– and glut–logics characterized in Batens (1999).

Language Schema. The logic $\mathbf{CL^u}$ is based on the language $\mathcal{L}^\mathbf{u}$ that is obtained by extending the classical propositional language $\mathcal{L}$ with the non–classical connectives $\overset{\texttt{not}}{\neg}, \overset{\texttt{and}}{\wedge}, \overset{\texttt{or}}{\vee}, \overset{\texttt{imp}}{\supset}, \overset{\texttt{iff}}{\equiv}$ (henceforth, the *gap–connectives*). Only the negations, conjunctions and disjunctions of the language $\mathcal{L}^\mathbf{u}$ will be taken as primitive, the others are defined in the standard way (see table 1). Also the set of well–formed formulas $\mathcal{W}^\mathbf{u}$ is defined in the usual way.

Language	Letters	Connectives	Defined Connectives	Set of Formulas
$\mathcal{L}$	$\mathcal{S}$	$\neg, \wedge, \vee$	$\supset, \equiv$	$\mathcal{W}$
$\mathcal{L}^\mathbf{u}$	$\mathcal{S}$	$\neg, \wedge, \vee, \overset{\texttt{not}}{\neg}, \overset{\texttt{and}}{\wedge}, \overset{\texttt{or}}{\vee}$	$\supset, \equiv, \overset{\texttt{imp}}{\supset}, \overset{\texttt{iff}}{\equiv}$	$\mathcal{W}^\mathbf{u}$

Table 1: Language Schema of $\mathbf{CL^u}$

Proof Theory. Syntactically, the logic $\mathbf{CL^u}$ is fully characterized by means of the axiom system of **CL**, extended by the axiom schemas presented in table 2. The latter clearly show in which sense the gap–connectives are weaker than the classical ones: while most of the *analyzing* classical inference rules are valid for the gap–connectives, none of the *constructive* inference rules are. Obviously, the analyzing

$\mathrm{A}\overset{\texttt{not}}{\neg}$	For $A \in \mathcal{S}$, $\overset{\texttt{not}}{\neg}A \supset \neg A$	$\mathrm{A}\overset{\texttt{not}}{\neg}\overset{\texttt{not}}{\neg}$	$\overset{\texttt{not}}{\neg}\overset{\texttt{not}}{\neg}A \supset A$
$\mathrm{A}\overset{\texttt{and}}{\wedge}$	$(A\overset{\texttt{and}}{\wedge}B) \supset (A \wedge B)$	$\mathrm{A}\overset{\texttt{not}}{\neg}\overset{\texttt{and}}{\wedge}$	$\overset{\texttt{not}}{\neg}(A\overset{\texttt{and}}{\wedge}B) \supset (\overset{\texttt{not}}{\neg}A \vee \overset{\texttt{not}}{\neg}B)$
$\mathrm{A}\overset{\texttt{or}}{\vee}$	$(A\overset{\texttt{or}}{\vee}B) \supset (A \vee B)$	$\mathrm{A}\overset{\texttt{not}}{\neg}\overset{\texttt{or}}{\vee}$	$\overset{\texttt{not}}{\neg}(A\overset{\texttt{or}}{\vee}B) \supset (\overset{\texttt{not}}{\neg}A \wedge \overset{\texttt{not}}{\neg}B)$

Table 2: Additional axiom schemas of $\mathbf{CL^u}$

inference rules are those that allow to derive less complex formulas from more complex ones (e.g. *simplification*, *disjunctive syllogism*,...), while the constructive inference rules are those that allow to derive more complex formulas from less complex ones (e.g. *conjunction*, *addition*,...). Remark that, in view of the implementation–problem, it is indicative that the inference rule **ADD** is not valid for the gap–disjunction (more on this in section 2.2 below):

Fact 1 *For* $A, B \in \mathcal{W}^{\mathbf{u}}$, $A \nvdash_{\mathbf{CL^u}} A\overset{\texttt{or}}{\vee}B$.

Semantics. Let $\mathcal{S}, \mathcal{W}^{\mathbf{d}} \subset \mathcal{W}^{\mathbf{u}}$ be respectively the set of sentential letters and the set of well–formed formulas of which the main connectives are gap–connectives.

Definition 1 $\mathcal{W}^{\mathbf{d}} = \{\overset{\texttt{not}}{\neg}A \mid A \in \mathcal{S}\} \cup \{\overset{\texttt{not}}{\neg}\overset{\texttt{not}}{\neg}A \mid A \in \mathcal{W}^{\mathbf{u}}\} \cup \{A\overset{\texttt{and}}{\wedge}B \mid A, B \in \mathcal{W}^{\mathbf{u}}\} \cup \{\overset{\texttt{not}}{\neg}(A\overset{\texttt{and}}{\wedge}B) \mid A, B \in \mathcal{W}^{\mathbf{u}}\} \cup \{A\overset{\texttt{or}}{\vee}B \mid A, B \in \mathcal{W}^{\mathbf{u}}\} \cup \{\overset{\texttt{not}}{\neg}(A\overset{\texttt{or}}{\vee}B) \mid A, B \in \mathcal{W}^{\mathbf{u}}\}$.

Moreover, let $\mathcal{W}^{\mathbf{m}}, \mathcal{W}^{\mathbf{n}} \subset \mathcal{W}^{\mathbf{u}}$ be sets of well–formed formulas of which the main connectives are a specific combination of both classical and gap–connectives.

Definition 2 $\mathcal{W}^{\mathbf{m}} = \{\overset{\texttt{not}}{\neg}\neg A \mid A \in \mathcal{W}^{\mathbf{u}}\} \cup \{\overset{\texttt{not}}{\neg}(A \wedge B) \mid A, B \in \mathcal{W}^{\mathbf{u}}\} \cup \{\overset{\texttt{not}}{\neg}(A \vee B) \mid A, B \in \mathcal{W}^{\mathbf{u}}\}$.

Definition 3 $\mathcal{W}^{\mathbf{n}} = \{\neg\overset{\texttt{not}}{\neg}A \mid A \in \mathcal{W}^{\mathbf{u}}\} \cup \{\neg(A\overset{\texttt{and}}{\wedge}B) \mid A, B \in \mathcal{W}^{\mathbf{u}}\} \cup \{\neg(A\overset{\texttt{or}}{\vee}B) \mid A, B \in \mathcal{W}^{\mathbf{u}}\}$.

A $\mathbf{CL^u}$-model M for the language $\mathcal{L}^{\mathbf{u}}$ is an assignment function v, characterized as follows:

C1.1 $v : \mathcal{S} \mapsto \{0, 1\}$
C1.2 $v : \mathcal{W}^{\mathbf{d}} \mapsto \{0, 1\}$
C1.3 $v : \mathcal{W}^{\mathbf{m}} \mapsto \{0, 1\}$

The assignment function v of the model M is extended to a valuation function $v_M : \mathcal{W}^{\mathbf{u}} \mapsto \{0, 1\}$ by means of the following semantic postulates:

C2.1a For $A \in \mathcal{S}$, $v_M(A) = 1$ iff $v(A) = 1$.
C2.1b For $A \in \mathcal{W}^{\mathbf{m}}$, $v_M(A) = 1$ iff $v(A) = 1$.
C2.2a For $A \in \mathcal{S}$, $v_M(\neg A) = 1$ iff $v_M(A) = 0$.
C2.2b For $A \in \mathcal{S}$, $v_M(\overset{\texttt{not}}{\neg}A) = 1$ iff $v_M(A) = 0$ *and* $v(\overset{\texttt{not}}{\neg}A) = 1$.

C2.2c For $\neg A \in \mathcal{W}^{\mathbf{n}}$, $v_M(\neg A) = 1$ iff $v_M(A) = 0$.
C2.3a $v_M(A \wedge B) = 1$ iff $v_M(A) = 1$ and $v_M(B) = 1$.
C2.3b $v_M(A \overset{\text{and}}{\wedge} B) = 1$ iff $v_M(A) = 1$ and $v_M(B) = 1$ *and* $v(A \overset{\text{and}}{\wedge} B) = 1$.
C2.4a $v_M(A \vee B) = 1$ iff $v_M(A) = 1$ or $v_M(B) = 1$.
C2.4b $v_M(A \overset{\text{or}}{\vee} B) = 1$ iff $v_M(A) = 1$ or $v_M(B) = 1$ *and* $v(A \overset{\text{or}}{\vee} B) = 1$.
C2.5a $v_M(\neg\neg A) = 1$ iff $v_M(A) = 1$.
C2.5b $v_M(\overset{\text{not}}{\neg}\overset{\text{not}}{\neg} A) = 1$ iff $v_M(A) = 1$ *and* $v(\overset{\text{not}}{\neg}\overset{\text{not}}{\neg} A) = 1$.
C2.6a $v_M(\neg(A \wedge B)) = 1$ iff $v_M(\neg A) = 1$ or $v_M(\neg B) = 1$.
C2.6b $v_M(\overset{\text{not}}{\neg}(A \overset{\text{and}}{\wedge} B)) = 1$ iff $v_M(\overset{\text{not}}{\neg} A) = 1$ or $v_M(\overset{\text{not}}{\neg} B) = 1$ *and* $v(\overset{\text{not}}{\neg}(A \overset{\text{and}}{\wedge} B)) = 1$.
C2.7a $v_M(\neg(A \vee B)) = 1$ iff $v_M(\neg A) = 1$ and $v_M(\neg B) = 1$.
C2.7b $v_M(\overset{\text{not}}{\neg}(A \overset{\text{or}}{\vee} B)) = 1$ iff $v_M(\overset{\text{not}}{\neg} A) = 1$ and $v_M(\overset{\text{not}}{\neg} B) = 1$ *and* $v(\overset{\text{not}}{\neg}(A \overset{\text{or}}{\vee} B)) = 1$.

A well–formed formula A of the language $\mathcal{L}^{\mathbf{u}}$ is *verified by a model M* in case $v_M(A) = 1$. Also, a model M is a *model of a premise set* Γ in case M verifies all elements of Γ. Finally, validity and semantic consequence are defined as follows:

Definition 4 (Validity) $\vDash_{\mathbf{CL^u}} A$ *iff A is verified by all* **CL$^{\mathbf{u}}$**-*models.*

Definition 5 (Semantic Consequence) $\Gamma \vDash_{\mathbf{CL^u}} A$ *iff A is verified by all* **CL$^{\mathbf{u}}$**-*models of* Γ.

In view of section 2.2 below, some remarks are necessary. First of all, it is important to notice that a **CL$^{\mathbf{u}}$**–assignment function assigns a truth value not only to sentential letters, but also to the elements of the sets $\mathcal{W}^{\mathbf{d}}$ and $\mathcal{W}^{\mathbf{m}}$. Secondly, a **CL$^{\mathbf{u}}$**–valuation function doesn't assign a truth value to all elements of $\mathcal{W}^{\mathbf{d}}$ and $\mathcal{W}^{\mathbf{m}}$ in a purely inductive way, for the valuation function also refers to the truth value given by the assignment function it is based on. Nonetheless, the truth conditions of elements of $\mathcal{W}^{\mathbf{d}}$ (not those of elements of $\mathcal{W}^{\mathbf{m}}$!) are such that, if all elements of $\mathcal{W}^{\mathbf{d}}$ are assigned truth by the assignment function v, then all those elements $\mathcal{W}^{\mathbf{d}}$ for which the classical truth conditions are satisfied, will be true in the model M based on v. As a consequence, the same is obviously true for all well–formed formulas that only contain gap–connectives (henceforth, *gap–formulas*), as is expressed by lemma 1 below.

Notational Convention 1 *Where* $A \in \mathcal{W}$, $A^{\mathbf{u}}$ *is used to refer to the gap–formula obtained by replacing all classical connectives in A by the corresponding gap–connectives.*

Lemma 1 *If v is the assignment function of the* **CL$^{\mathbf{u}}$**–*model M and for all* $B \in \mathcal{W}^{\mathbf{d}}$: $v(B) = 1$*, then for all* $A \in \mathcal{W}$: $v_M(A) = 1$ *iff* $v_M(A^{\mathbf{u}}) = 1$.

Proof. Suppose that for all $B \in \mathcal{W}^{\mathbf{d}}$: $v(B) = 1$, with v the assignment function of the model M. This means that the truth conditions of A and $A^{\mathbf{u}}$ are the same in M (because of the semantic characterization of **CL$^{\mathbf{u}}$**). As a consequence, $v_M(A) = v_M(A^{\mathbf{u}})$, from which follows that $v_M(A) = 1$ iff $v_M(A^{\mathbf{u}}) = 1$. □

Soundness and Completeness. The soundness and completeness proofs below are inspired by those presented in Batens & De Clercq (2004). First, consider soundness.

Theorem 1 (Soundness) *If* $\Gamma \vdash_{\mathbf{CL^u}} A$ *then* $\Gamma \vDash_{\mathbf{CL^u}} A$.

Proof. Soundness is proven in the standard way, viz by proving the semantic validity of all the $\mathbf{CL^u}$–axioms and –rules. However, as all cases are completely straightforward, this is left to the reader. □

Before proving completeness, remark that the *deduction theorem* is obviously provable for the logic $\mathbf{CL^u}$, as is *compactness* with respect to derivability — both proofs are completely standard. Keeping this in mind, the completeness theorem is now provable as well.

Theorem 2 (Completeness) *If* $\Gamma \vDash_{\mathbf{CL^u}} A$ *then* $\Gamma \vdash_{\mathbf{CL^u}} A$.

Proof. Suppose $\Gamma \nvdash_{\mathbf{CL^u}} A$. Consider a sequence $B_1, B_2, ...$ that contains all well–formed formulas of the language $\mathcal{L}^{\mathbf{u}}$. We then define:

$$
\begin{aligned}
\Delta_0 &= Cn_{\mathbf{CL^u}}(\Gamma) \;\; (= \text{the } \mathbf{CL^u}\text{–consequence set of } \Gamma) \\
\Delta_{i+1} &= Cn_{\mathbf{CL^u}}(\Delta_i \cup \{B_{i+1}\}) \text{ if } A \notin Cn_{\mathbf{CL^u}}(\Delta_i \cup \{B_{i+1}\}) \text{, and} \\
\Delta_{i+1} &= \Delta_i \text{ otherwise.} \\
\Delta &= \Delta_0 \cup \Delta_1 \cup \ldots
\end{aligned}
$$

Each of the following is provable:

(i) $\Gamma \subseteq \Delta$ (by the construction of Δ).
(ii) $A \notin \Delta$ (by the construction of Δ).
(iii) Δ is deductively closed (by the construction of Δ).
(iv) Δ is non–trivial (as $A \notin \Delta$).
(v) Δ is prime, i.e. if $C \vee D \in \Delta$, then $C \in \Delta$ or $D \in \Delta$.

Suppose that (1) $C \vee D \in \Delta$, but that (2) $C \notin \Delta$ and $D \notin \Delta$. From (2) follows that there must be an m and n such that $\Delta_m \cup \{C\} \vdash_{\mathbf{CL^u}} A$ and $\Delta_n \cup \{D\} \vdash_{\mathbf{CL^u}} A$ (by the construction of Δ). From these follow that $\Delta_m \vdash_{\mathbf{CL^u}} C \supset A$ and $\Delta_n \vdash_{\mathbf{CL^u}} D \supset A$ (by the deduction theorem of $\mathbf{CL^u}$). But, this also means that $\Delta \vdash_{\mathbf{CL^u}} C \supset A$ and $\Delta \vdash_{\mathbf{CL^u}} D \supset A$ (by the construction of Δ, together with the syntactic compactness of $\mathbf{CL^u}$). From this, together with (1), follows that $A \in \Delta$ (by the deductive closure of Δ). Contradiction.

It is now possible to define a $\mathbf{CL^u}$–model M from Δ. First of all, the assignment function v of M is defined as follows:

P1.1 For $A \in \mathcal{S}$, $v(A) = 1$ iff $A \in \Delta$.

P1.2 For $A \in \mathcal{W}^{\mathbf{m}}$, $v(A) = 1$ iff $A \in \Delta$.
P1.3 For $A \in \mathcal{W}^{\mathbf{d}}$, $v(A) = 1$ iff $A \in \Delta$.

Next, for all well–formed formulas C of the language $\mathcal{L}^{\mathbf{u}}$, it can be proven that $v_M(C) = 1$ iff $C \in \Delta$. This is done by a straightforward induction on the complexity of well–formed formulas. First, consider the base cases. In view of C2.1*a*–1*b*, P1.1–2 warrant that, if $C \in \mathcal{S}$ or $C \in \mathcal{W}^{\mathbf{m}}$, then $v_M(C) = 1$ iff $C \in \Delta$ — as the proof is completely straightforward, it is left to the reader. Also, for $C \in \mathcal{S}$, it can easily be shown that $v_M(\neg C) = 1$ iff $\neg C \in \Delta$, and that $v_M(\overset{\text{not}}{\neg} C) = 1$ iff $\overset{\text{not}}{\neg} C \in \Delta$. Consider the proofs below.

$v_M(\neg C) = 1$ iff $v_M(C) = 0$ (by C2.2a).
iff $v(C) = 0$ (by C2.1a).
iff $C \notin \Delta$ (by P1.1).
iff $\neg C \in \Delta$ ($\Downarrow$ As Δ is deductively closed, $C \vee \neg C \in \Delta$. Hence, as Δ is prime and as $C \notin \Delta$, $\neg C \in \Delta$; $\Uparrow$ Because Δ is not trivial, which wouldn't be the case if both $C \in \Delta$ and $\neg C \in \Delta$).

$v_M(\overset{\text{not}}{\neg} C) = 1$ iff $v_M(C) = 0$ *and* $v(\overset{\text{not}}{\neg} C) = 1$ (by C2.2b).
iff $v(C) = 0$ (by C2.1) *and* $\overset{\text{not}}{\neg} C \in \Delta$ (by P1.3).
iff $C \notin \Delta$ (by P1.1) *and* $\overset{\text{not}}{\neg} C \in \Delta$.
iff $\overset{\text{not}}{\neg} C \in \Delta$ ($\Downarrow$ Immediately; $\Uparrow$ Because Δ is not trivial, which would not be the case if both $C \in \Delta$ and $\overset{\text{not}}{\neg} C \in \Delta$).

Next, consider the induction cases. However, as all of these are proven analogously, I will only discuss one case in detail, the one for formulas of the form $\overset{\text{not}}{\neg}(A \overset{\text{and}}{\wedge} B)$. The remaining cases are left to the reader.

$v_M(\overset{\text{not}}{\neg}(A \overset{\text{and}}{\wedge} B)) = 1$ iff $v_M(\overset{\text{not}}{\neg} A) = 1$ or $v_M(\overset{\text{not}}{\neg} B) = 1$ *and* $v(\overset{\text{not}}{\neg}(A \overset{\text{and}}{\wedge} B)) = 1$ (by C2.6b).
iff $\overset{\text{not}}{\neg} A \in \Delta$ or $\overset{\text{not}}{\neg} B \in \Delta$ (by the induction hypothesis) *and* $\overset{\text{not}}{\neg}(A \overset{\text{and}}{\wedge} B) \in \Delta$ (by P1.3).
iff $\overset{\text{not}}{\neg}(A \overset{\text{and}}{\wedge} B) \in \Delta$ ($\Downarrow$ Immediately; $\Uparrow$ As $\overset{\text{not}}{\neg}(A \overset{\text{and}}{\wedge} B) \in \Delta$ and as Δ is deductively closed, $\overset{\text{not}}{\neg} A \vee \overset{\text{not}}{\neg} B \in \Delta$. Hence, as Δ is prime, $\overset{\text{not}}{\neg} A \in \Delta$ or $\overset{\text{not}}{\neg} B \in \Delta$).

The induction proof shows that for all $C \in \mathcal{W}^{\mathbf{u}}$, $v_M(C) = 1$ iff $C \in \Delta$. In view of (i) and (ii) above, this also means that for all $B \in \Gamma$, $v_M(B) = 1$, and that $v_M(A) = 0$. As a consequence, $\Gamma \nvDash_{\mathbf{CL^u}} A$ (by definition 5). □

2.2 Distinguishing *What Has Been Asserted* from *What Has Been Derived*

In conversation, the hearer obviously applies the *or*–implicature only to disjunctions that are part of the sentences that *have been asserted* by the speaker. Hence, the hearer doesn't apply the *or*–implicature to disjunctions that *have merely been derived* by herself from what the speaker said. For example, if the disjunction A_or_(B_or_C) has been asserted by the speaker, the hearer will obviously apply the *or*–implicature to this disjunction. Moreover, if the sentence not_A has been asserted as well, the hearer will also apply the *or*–implicature to the sentence B_or_C. Although the latter has been derived from the assertions explicitly made by the speaker, it nonetheless constitutes an essential part of the message the speaker intended to transfer. Otherwise, it wouldn't have occurred inside an assertion made by the speaker. Hence, this kind of derivable sentences will be called *implicit assertions*. On the other hand, the hearer will not apply the *or*–implicature to the disjunction (A_or_(B_or_C))_or_D, for this disjunction does not constitute an essential part of the message intended to be transferred (quite the contrary, for it is considered uncooperative to assert a disjunction known to contain a redundant disjunct), but has *merely* been derived by the hearer from the assertions made by the speaker.

Whenever the logical connectives are interpreted classically (as Grice would have it), it is impossible to distinguish between sentences that have been asserted by the speaker (explicitly or implicitly) and sentences that have merely been derived by the hearer from the sentences asserted by the speaker. Consequently, formal approaches based on the classical interpretation of the logical connectives apply the *or*–implicature to all disjunctions that are derivable from the assertions made by the speaker. However, as Horsten showed, this yields the implementation–problem (as explained at the beginning of section 2). Hence, to overcome the implementation–problem, it is necessary to distinguish between what has been asserted (explicitly or implicitly) and what has been derived. By taking the logic $\mathbf{CL^u}$ as the context of implementation of the *or*–implicature, it is possible to make this kind of distinction: gap–formulas are taken to correspond to (explicit and implicit) assertions, while classical formulas (i.e. formulas stated in the classical language $\mathcal{L}$) are taken to correspond to the conclusions a hearer has derived from those assertions. As a consequence, by taking $\mathbf{CL^u}$ as the context of implementation, the application of the *or*–implicature can be restricted to formulas that express asserted disjunctions, viz gap–disjunctions. Formally, this means that the *or*–implicature is interpreted as follows:

$$A \overset{\text{or}}{\vee} B \text{ pragmatically implies } \neg(A \wedge B).$$

This interpretation of the *or*–implicature avoids the implementation–problem because the inference rule **ADD** is not valid for gap–disjunctions (as has been shown in the previous section). Hence, it is impossible to derive gap–disjunctions with redundant disjuncts, so that all applications of the *or*–implicature will be relevant applications. Moreover, the *or*–implicature can always be applied to a gap–disjunction, even if also one of its disjuncts can be derived. Consequently, in contradistinction to the **RAD**–approach (discussed at the beginning of section 2), the logic $\mathbf{CL^u}$ doesn't unnecessarily reduce the inferential strength of the *or*–implicature.

In what follows premise sets are restricted such as to contain only gap–formulas. Consequently, the formal implementation of the *or*–implicature in the context of the logic $\mathbf{CL}^{\mathbf{u}}$ really captures the viewpoint of the hearer in a conversation (and doesn't confuse this with the viewpoint of the speaker). For, as a premise set only contains gap–formulas, it clearly corresponds to the explicit assertions made by the speaker (and heard by the hearer). Moreover, because of the weakness of the gap–connectives, only those gap–disjunctions are derivable from the premises that correspond to disjunctions that are an essential part of the message the speaker intended to transfer (i.e. disjunctions that correspond to implicit assertions). All other consequences, viz those containing classical connectives, correspond to the sentences the hearer has merely derived from the assertions made by the speaker.

Interpretation of the Logical Connectives. Despite the fact that the logical connectives that occur in assertions are not interpreted classically when the logic $\mathbf{CL}^{\mathbf{u}}$ is taken as the context of implementation of the *or*–implicature, the situation is not that different from the case when **CL** is taken as the context of implementation. For, in the context of $\mathbf{CL}^{\mathbf{u}}$, the hearer is able to derive all and only those sentences, from the assertions made by the speaker, that he would also be able to derive if these assertions were interpreted classically. Formally, this is expressed by theorem 3 below that states the following: if only gap–formulas are allowed to enter a premise set (which is the case here), then the classical $\mathbf{CL}^{\mathbf{u}}$–consequences of a premise set (i.e. those $\mathbf{CL}^{\mathbf{u}}$–consequences that only contain classical connectives) are the **CL**–consequences that can be derived from the classical counterpart of that premise set.[2]

Definition 6 $\Gamma^{\mathbf{u}} = \{A^{\mathbf{u}} \mid A \in \Gamma, \textit{ with } \Gamma \subset \mathcal{W}\}$.

Theorem 3 *For* $\Gamma \subset \mathcal{W}$ *and* $A \in \mathcal{W}$, $\Gamma^{\mathbf{u}} \vdash_{\mathbf{CL}^{\mathbf{u}}} A$ *iff* $\Gamma \vdash_{\mathbf{CL}} A$.

Proof. $\Leftarrow$ Suppose $\Gamma^{\mathbf{u}} \vdash_{\mathbf{CL}^{\mathbf{u}}} A$ and $\Gamma \nvdash_{\mathbf{CL}} A$. Hence, there is a **CL**–model M such that for all $B \in \Gamma$, $v_M(B) = 1$ and $v_M(A) = 0$. From M (based on the assignment function v), a $\mathbf{CL}^{\mathbf{u}}$–model M' (based on the assignment function v') is defined in the following way:

AF1 For $A \in \mathcal{S}$, $v'(A) = v(A)$.
AF2 For $A \in \mathcal{W}^{\mathbf{d}}$, $v'(A) = 1$.
AF3 For $A \in \mathcal{W}^{\mathbf{m}}$, $v'(A) = 1$.

As a consequence, for all $B \in \Gamma$, $v_{M'}(B) = 1$ and $v_{M'}(A) = 0$ (by AF1, and because A only contains classical connectives). Moreover, for all $B \in \Gamma$, $v_{M'}(B^{\mathbf{u}}) = 1$ (by AF2 and theorem 1). As a consequence, $\Gamma^{\mathbf{u}} \nvDash_{\mathbf{CL}^{\mathbf{u}}} A$ (by definition 5). Contradiction.

$\Rightarrow$ Suppose $\Gamma \vdash_{\mathbf{CL}} A$. For all $B \in \Gamma$, $\Gamma^{\mathbf{u}} \vdash_{\mathbf{CL}^{\mathbf{u}}} B$ (because of the semantic characterization of $\mathbf{CL}^{\mathbf{u}}$, if the truth conditions for $B^{\mathbf{u}}$ are satisfied, then the truth conditions for B are satisfied as well). In view of the supposition, this immediately gives

[2] Put differently, it is possible to define the **CL**–consequence relation by means of the logic $\mathbf{CL}^{\mathbf{u}}$.

us $\Gamma^{\mathbf{u}} \vdash_{\mathbf{CL^u}} A$. □

In conclusion, the logical connectives are interpreted only slightly non–classical in the context of $\mathbf{CL^u}$, just in so far as to make a clear distinction between what the speaker has actually said and what the hearer has derived from this. Nonetheless, this approach is clearly preferable to the classical one, for it is completely in accordance with the situation in actual conversation.

3 The Adaptive Logics Approach

In this section, I will show how the adaptive logics approach can be applied to formally capture the use of the *or*–implicature in actual conversation. This will be done by characterizing the adaptive logic $\mathbf{CL}^{\texttt{or}}$. The latter implements the *or*–implicature as a truly non–monotonic inference rule in the context of the logic $\mathbf{CL^u}$. As a consequence, the implementation–problem doesn't occur. Moreover, the logic $\mathbf{CL}^{\texttt{or}}$ can be characterized proof theoretically, so that the adaptive logics approach proves to capture the *or*–implicature more realistically compared to other formal approaches.

As the logic $\mathbf{CL}^{\texttt{or}}$ is a standard adaptive logic, I will first characterize the standard format of adaptive logics (see section 3.1). Afterwards, I will characterize the logic $\mathbf{CL}^{\texttt{or}}$ in some more detail (see section 3.2).

3.1 The Standard Format

All standard adaptive logics (**AL**) have a uniform characterization. This characterization is called the *standard format* of adaptive logics and was presented most thoroughly in Batens (2007) and Batens et al. (2007). The main advantage of the standard format is that all **AL** that are characterized accordingly are provided with a standard semantics and proof theory. Moreover, a lot of metatheoretic properties follow immediately (most importantly, soundness and completeness). Below, I will give a very general and intuitive characterization of the standard format, leaving aside the semantics and the proof theory.

General Characterization. All standard adaptive logics are fully characterized by means of the following three elements: a *lower limit logic* (**LLL**), a *set of abnormalities* Ω (a set of formulas characterized by a logical form $\texttt{F}$), and an *adaptive strategy*.

The **LLL** is the stable part of an adaptive logic. Proof theoretically, this means that all **LLL**–inference rules are monotonic inference rules: all consequences derived from a premise set by means of those inference rules cannot be withdrawn anymore. In other words, all **LLL**–consequences of a premise set are also **AL**–consequences of that premise set. Obviously, the **LLL** of the adaptive $\mathbf{CL}^{\texttt{or}}$ will be the logic constituting the context in which the *or*–implicature is implemented.

An adaptive logic typically enables to derive more consequences from a premise set than its **LLL**. In general, the supplementary **AL**–consequences are obtained by interpreting as much abnormalities (elements of Ω) as possible as false. In fact, this comes down to the following: when a formula $A \vee Dab(\Delta)$, with $Dab(\Delta)$ a finite disjunction of abnormalities (also called a *Dab*–formula), is **LLL**–derivable from a premise set, all elements of Δ will be interpreted as false for as long as there is no reason not to do so. The formula A is then considered a *conditional* **AL**–consequence of that premise set, unless or until it turns out that there are reasons to consider some elements of Δ to be true (in which case A cannot safely be considered as true anymore). Proof theoretically, this means that **AL** allow applications of certain inference rules in a defeasible way. Obviously, the defeasible inference rule validated by the adaptive logic $\mathbf{CL}^{\circ\mathtt{r}}$, will correspond to the *or*–implicature.

Whether a conditional **AL**–consequence of a premise set is also a *final* **AL**–consequence, depends on the *Dab*–formulas that are **LLL**–derivable from that premise set (these are also called the *Dab*–consequences of that premise set). Obviously, not all abnormalities occurring in such a *Dab*–formula can be interpreted as false (otherwise the *Dab*–formula itself cannot possibly be true). Hence, some of the **AL**–consequences that were derived by presupposing those abnormalities to be false, will have to be rejected. Which of these that will be, is determined by the adaptive strategy of an adaptive logic. As such, the adaptive strategy can be regarded as a kind of guideline to cope effectively with the *Dab*–consequences of a premise set. In any case, the lesser *Dab*–consequences a premise set has, the more **AL**–consequences follow from that premise set. This is why adaptive logicians sometimes say that **AL** adapt themselves to the premise sets they are applied to.

Dynamic Behavior. Because of the conditional status of some of the **AL**–consequences, **AL** display an external as well as an internal dynamics. The external dynamics corresponds to non–monotonicity: if the premise set is extended, some conditionally derived **AL**–consequences of the premise set may not be derivable anymore. The internal dynamics is a proof theoretic feature: growing insights in the premises, obtained by deriving new consequences from the premises (in casu *Dab*–consequences), may lead to the withdrawal of earlier reached conclusions, or to the rehabilitation of earlier withdrawn conclusions. Obviously, the dynamic behavior of **AL** resembles the dynamics of **GCI** (see section 1). As such, **AL** seem particularly well–suited to explicate the pragmatic processes triggered in conversation.

3.2 The Adaptive Logic $\mathbf{CL}^{\circ\mathtt{r}}$

The logic $\mathbf{CL}^{\mathbf{u}}$ provides the context in which the *or*–implicature is implemented by the adaptive logic $\mathbf{CL}^{\circ\mathtt{r}}$. Hence, the **LLL** of the latter is the logic $\mathbf{CL}^{\mathbf{u}}$. The set of abnormalities is the set $\Omega^{\mathtt{or}}$, which is defined as follows:[3]

Definition 7 $\Omega^{\mathtt{or}} = \{(A \vee B)^{\mathbf{u}} \wedge (A \wedge B) \mid A, B \in \mathcal{W}\}$

Finally, the adaptive strategy is the *normal selections* strategy.

In view of the standard format, this general characterization clearly shows in which way the *or*–implicature is captured by the logic $\mathbf{CL}^{\mathtt{or}}$. For, as long as an abnormality $(A \vee B)^{\mathbf{u}} \wedge (A \wedge B)$ can be considered as false, the formula $\neg((A \vee B)^{\mathbf{u}} \wedge (A \wedge B))$ will be considered as true. From this follows that the formula $\neg((A \vee B)^{\mathbf{u}}) \vee \neg(A \wedge B)$ will be considered as true as well. Hence, in case the formula $(A \vee B)^{\mathbf{u}}$ is true unconditionally (i.e. $\mathbf{CL}^{\mathbf{u}}$–derivable from the premise set), the formula $\neg(A \wedge B)$ can be considered as true (for it is derivable by means of the inference rule disjunctive syllogism). Obviously, this corresponds to the *or*–implicature! However, in case it turns out that the formula $A \wedge B$ is also true unconditionally (i.e. is $\mathbf{CL}^{\mathbf{u}}$–derivable from the premise set), the abnormality $(A \vee B)^{\mathbf{u}} \wedge (A \wedge B)$ cannot be considered as false anymore, which results in the rejection of the consequences obtained by presupposing so. As a consequence, the *or*–implicature at hand will be rejected.

Semantics. The semantics of the logic $\mathbf{CL}^{\mathtt{or}}$ is a so–called *preferential semantics*. Hence, the $\mathbf{CL}^{\mathtt{or}}$–consequences of a premise set Γ are determined by reference to sets of preferred $\mathbf{CL}^{\mathbf{u}}$–models (ergo, **LLL**–models) of Γ. These sets are called the *selected sets* of $\mathbf{CL}^{\mathbf{u}}$–models of Γ. More specifically, the $\mathbf{CL}^{\mathtt{or}}$–consequences of Γ are those formulas that are verified by all models of at least one such a selected set.

Definition 8 $\Gamma \vDash_{\mathbf{CL}^{\mathtt{or}}} A$ *iff* A *is verified by all elements of a selected set of* $\mathbf{CL}^{\mathbf{u}}$*–models of* Γ.

Whether a particular $\mathbf{CL}^{\mathbf{u}}$–model M of Γ will make it to a selected set Σ, depends on its *abnormal part*, i.e. the set of abnormalities it verifies.

Definition 9 *Where* M *is a* $\mathbf{CL}^{\mathbf{u}}$*–model, the abnormal part of* M *is the set* $Ab(M) = \{A \in \Omega \mid A \text{ is verified by } M\}$.

Moreover, it also depends on the adaptive strategy of the logic $\mathbf{CL}^{\mathtt{or}}$, for the strategy determines the actual selection criterion. As the adaptive strategy of the logic $\mathbf{CL}^{\mathtt{or}}$ is the normal selections strategy, a selected set Σ is defined by means of a two–step procedure. Firstly, the *minimally abnormal models* of a premise set Γ are defined.

Definition 10 *A* $\mathbf{CL}^{\mathbf{u}}$*–model* M *of* Γ *is* minimally abnormal *iff there is no* $\mathbf{CL}^{\mathbf{u}}$*–model* M' *of* Γ *such that* $Ab(M') \subset Ab(M)$.

[3]Remember that $A^{\mathbf{u}}$ is used to refer to the gap–formula obtained by replacing all classical connectives in A by the corresponding gap–connectives (see notational convention 1).

Secondly, all minimally abnormal models that verify the same abnormalities, are grouped together into distinct sets. These sets are the selected sets of $\mathbf{CL^u}$–models of the premise set Γ.

Definition 11 $\Phi(\Gamma) = \{Ab(M) \mid M$ *is a minimally abnormal model of* $\Gamma\}$.

Definition 12 *A set* Σ *of* $\mathbf{CL^u}$*–models of* Γ *is a* selected set *iff for some* $\phi \in \Phi(\Gamma)$*,* $\Sigma = \{M \mid M$ *verifies all elements of* Γ *and* $Ab(M) = \phi\}$.

Proof Theory. As the logic $\mathbf{CL}^{\mathtt{or}}$ is a standard adaptive logic, its proof theory has some characteristic features that are shared by all **AL**. First of all, a $\mathbf{CL}^{\mathtt{or}}$–proof is a succession of stages, each consisting of a sequence of lines. Adding a line to a proof means to move on to the next stage of the proof. Secondly, the lines of a $\mathbf{CL}^{\mathtt{or}}$–proof consist of four elements (instead of the usual three): a line number, a formula, a justification, and an adaptive condition. The latter is a finite subset of $\Omega^{\mathtt{or}}$ (the set of abnormalities). Thirdly, as long as all elements of the adaptive condition of a line *i* can be considered as false, the formula on line *i* is considered as derivable from the premise set. In order to indicate that not all elements of the adaptive condition of line *i* can be considered as false anymore, line *i* is marked (formally, this is done by placing the symbol ✓ next to the adaptive condition). Obviously, when a line is marked, the formula on that line is not considered as derivable anymore. Finally, the markings of a $\mathbf{CL}^{\mathtt{or}}$–proof are dynamic. At some stage of the proof, a line might be marked (resp. unmarked), while at a later stage, it might become unmarked (resp. marked) again. Obviously, the dynamics of the $\mathbf{CL}^{\mathtt{or}}$–proof theory correspond to the dynamics involved in the use of the *or*–implicature.

Now, consider the $\mathbf{CL}^{\mathtt{or}}$–proof theory in particular. It consists of both *deduction rules* and a *marking criterion*. The deduction rules determine how new lines may be added to a proof, while the marking criterion determines at every stage of the proof which lines have to be marked. The deduction rules are listed in shorthand notation, with

$$A \qquad \Delta$$

expressing that A occurs in the proof on the condition Δ.

PREM If $A \in \Gamma$:

$$\begin{array}{ll} \ldots & \ldots \\ \hline A & \emptyset \end{array}$$

RU If $A_1, \ldots, A_n \vdash_{\mathbf{CL^u}} B$:

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \end{array}$$

RC If $A_1, \ldots, A_n \vdash_{\mathbf{CL^u}} B \vee Dab(\Theta)$

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \cup \Theta \end{array}$$

Notice that the deduction rules are fully determined by the logic $\mathbf{CL^u}$ (the **LLL** of the logic $\mathbf{CL}^{\circ\mathrm{r}}$) and by the set of abnormalities $\Omega^{\circ\mathrm{r}}$. Hence, the deduction rules are independent of the adaptive strategy of the logic $\mathbf{CL}^{\circ\mathrm{r}}$. This is not the case for the marking criterion, for the latter strongly depends on the adaptive strategy, in casu the normal selections strategy.[4] To determine whether or not a line has to be marked at a certain stage of a $\mathbf{CL}^{\circ\mathrm{r}}$–proof, the normal selections strategy refers to the *Dab–consequences* of the premise set that have been derived at that stage of the proof.

Definition 13 *$Dab(\Delta)$ is a Dab–consequence of a premise set Γ at stage s of the proof iff $Dab(\Delta)$ is derived at stage s on the condition $\emptyset$.*

More specifically, the normal selections strategy lays down that a line i with condition Δ has to be marked at stage s in case $Dab(\Delta)$ is a *Dab*–consequence of the premise set at stage s.

Definition 14 (Marking for Normal Selections) *Line i is marked at stage s of the proof iff, where Δ is its condition, $Dab(\Delta)$ is a Dab–consequence of Γ at stage s.*

To conclude, a formula A is derivable from a premise set Γ iff A has been derived as the second element of an unmarked line in a proof from Γ. However, because of the dynamic nature of the proofs, this definition of derivability is stage–dependent. For, markings may change at every stage, so that for every new stage, it has to be reconsidered whether or not a formula is derivable from the premise set. Also a stable notion of derivability can be defined. It is called *final derivability*, which refers to the fact that for some formulas, derivability can only be decided at the final stage of a proof.

Definition 15 *A is finally derived from Γ on a line i of a proof at stage s iff (i) A is the second element of line i, (ii) line i is not marked at stage s, and (iii) every*

[4] In general, the proof theories of **AL** that are based on the same **LLL** and the same set of abnormalities, but on different adaptive strategies, only differ with relation to the marking criterion (see e.g. Batens, 2007).

extension of the proof in which line i is marked may be further extended in such a way that line i is unmarked.

Because of its stability, the notion of final derivability is used to define the $\mathbf{CL}^{\mathtt{or}}$–consequence relation.

Definition 16 $\Gamma \vdash_{\mathbf{CL}^{\mathtt{or}}} A$ *iff A is finally derived on a line of a proof from* Γ.

Soundness and Completeness. As the logic $\mathbf{CL}^{\mathtt{or}}$ is characterized according to the standard format, soundness and completeness follow immediately, as was shown in Batens et al. (2007).

Definition 17 $\Gamma \vdash_{\mathbf{CL}^{\mathtt{or}}} A$ *iff* $\Gamma \vDash_{\mathbf{CL}^{\mathtt{or}}} A$.

Example. In order to illustrate the $\mathbf{CL}^{\mathtt{or}}$–proof theory, consider the proof below which is based on the premise set $\Gamma = \{p \overset{\mathtt{or}}{\vee} q, r \overset{\mathtt{or}}{\vee} p, \overset{\mathtt{not}}{\neg} r, p, q\}$.

1	$p \overset{\mathtt{or}}{\vee} q$	—;PREM	$\emptyset$
2	$r \overset{\mathtt{or}}{\vee} p$	—;PREM	$\emptyset$
3	$\overset{\mathtt{not}}{\neg} r$	—;PREM	$\emptyset$
4	$\neg(p \wedge q)$	1;RC	$\{(p \overset{\mathtt{or}}{\vee} q) \wedge (p \wedge q)\}$
5	$\neg(r \wedge p)$	2;RC	$\{(r \overset{\mathtt{or}}{\vee} p) \wedge (r \wedge p)\}$
6	p	2,3;RU	$\emptyset$
7	$\neg q$	4,6;RU	$\{(p \overset{\mathtt{or}}{\vee} q) \wedge (p \wedge q)\}$

At stage 7 of the proof, no *Dab*–formulas have been derived yet. Hence, no markings occur, so that all formulas that have been derived conditionally are considered as $\mathbf{CL}^{\mathtt{or}}$–derivable. However, this changes in case the proof is extended as follows:

...	...	...	...	
4	$\neg(p \wedge q)$	1;RC	$\{(p \overset{\mathtt{or}}{\vee} q) \wedge (p \wedge q)\}$	✓
5	$\neg(r \wedge p)$	2;RC	$\{(r \overset{\mathtt{or}}{\vee} p) \wedge (r \wedge p)\}$	
6	p	2,3;RU	$\emptyset$	
7	$\neg q$	4,6;RU	$\{(p \overset{\mathtt{or}}{\vee} q) \wedge (p \wedge q)\}$	✓
8	q	—;PREM	$\emptyset$	
9	$(p \overset{\mathtt{or}}{\vee} q) \wedge (p \wedge q)$	1,6,8;RU	$\emptyset$	

At stage 9 of the proof, a *Dab*–formula has been derived on line 9. As a consequence, lines 4 and 7 are marked, which means that the formulas on those lines are not considered as $\mathbf{CL}^{\mathtt{or}}$–derivable anymore. It is easy to check that the proof cannot be extended in such a way that also line 5 is marked. Hence, the formula on line 5 is finally derivable from Γ.

The Dual of the *Or*–Implicature. The $\overline{and}$–implicature is the dual of the *or*–impli-cature. This implicature states that if it is asserted that a conjunction is false, then this should be interpreted as the assertion that not both conjuncts are true, but that only one of these is. Formally, this comes down to the following:

($\overline{and}$ – **implicature**) The assertion $\texttt{not\_}(A\texttt{\_and\_}B)$ implicates that $A\texttt{\_or\_}B$.

To show that the adaptive logics approach can handle more **GCI** than only the *or*–implicature, I will briefly characterize the logic $\mathbf{CL}^{\texttt{or}/\overline{\texttt{and}}}$. The latter extends the logic $\mathbf{CL}^{\texttt{or}}$ in such a way that also the $\overline{and}$–implicature is captured. More **GCI** are captured by extending the logic $\mathbf{CL}^{\texttt{or}/\overline{\texttt{and}}}$ even further. As all these extensions are obtained analogously, the adaptive logics approach to **GCI** proves to be quite general.

In fact, the logic $\mathbf{CL}^{\texttt{or}/\overline{\texttt{and}}}$ is characterized equivalently to the logic $\mathbf{CL}^{\texttt{or}}$. Only the set of abnormalities $\Omega^{\texttt{or}/\overline{\texttt{and}}}$ of the former differs from the set of abnormalities $\Omega^{\texttt{or}}$ of the latter. More specifically, $\Omega^{\texttt{or}/\overline{\texttt{and}}} = \Omega^{\texttt{or}} \cup \Omega^{\overline{\texttt{and}}}$, where $\Omega^{\overline{\texttt{and}}}$ is defined as follows:

Definition 18 $\Omega^{\overline{\texttt{and}}} = \{(\neg(A \wedge B))^{\mathbf{u}} \wedge (\neg A \wedge \neg B) \mid A, B \in \mathcal{W}\}$

The set $\Omega^{\texttt{or}/\overline{\texttt{and}}}$ is a simple extension of the set $\Omega^{\texttt{or}}$. As this is the only difference between the logics $\mathbf{CL}^{\texttt{or}/\overline{\texttt{and}}}$ and $\mathbf{CL}^{\texttt{or}}$, both the semantics and proof theory of $\mathbf{CL}^{\texttt{or}/\overline{\texttt{and}}}$ are completely analogous to the semantics and proof theory of $\mathbf{CL}^{\texttt{or}}$. Hence, there is no need to characterize these here. Nonetheless, to illustrate the logic $\mathbf{CL}^{\texttt{or}/\overline{\texttt{and}}}$, consider the proof below, based on the premise set $\{\overset{\texttt{not}}{\neg}(p \overset{\texttt{and}}{\wedge} q), r \overset{\texttt{or}}{\vee} \overset{\texttt{not}}{\neg} p, \overset{\texttt{not}}{\neg} r, \overset{\texttt{not}}{\neg} q\}$.

1	$\overset{\texttt{not}}{\neg}(p \overset{\texttt{and}}{\wedge} q)$	—;PREM	$\emptyset$	
2	$r \overset{\texttt{or}}{\vee} \overset{\texttt{not}}{\neg} p$	—;PREM	$\emptyset$	
3	$\overset{\texttt{not}}{\neg} r$	—;PREM	$\emptyset$	
4	$p \vee q$	1;RC	$\{\overset{\texttt{not}}{\neg}(p \overset{\texttt{and}}{\wedge} q) \wedge (\neg p \wedge \neg q)\}$	✓
5	$\neg(r \wedge \neg p)$	2;RC	$\{(r \overset{\texttt{or}}{\vee} \overset{\texttt{not}}{\neg} p) \wedge (r \wedge \neg p)\}$	
6	$\neg p$	2,3;RU	$\emptyset$	
7	q	4,6;RU	$\{\overset{\texttt{not}}{\neg}(p \overset{\texttt{and}}{\wedge} q) \wedge (\neg p \wedge \neg q)\}$	✓
8	$\overset{\texttt{not}}{\neg} q$	—;PREM	$\emptyset$	
9	$\overset{\texttt{not}}{\neg}(p \overset{\texttt{and}}{\wedge} q) \wedge (\neg p \wedge \neg q)$	1,6,8;RU	$\emptyset$	

4 Conclusion

By relying on the adaptive logics approach, I have shown how **GCI** can be modeled formally as non–monotonic inference rules. I have done so by focussing on the *or*–implicature. First of all, I have argued that the *or*–implicature is best implemented in the context of the logic $\mathbf{CL}^{\mathbf{u}}$ (instead of in the context of **CL**), as this non–classical logic avoids the implementation–problem in a realistic way, by differentiating between the (explicit and implicit) assertions made by a speaker and the consequences derived from those assertions by the hearer. Afterwards, I have presented the adaptive logic $\mathbf{CL}^{\circ\mathrm{r}}$ that actually captures the *or*–implicature in a formal way. As $\mathbf{CL}^{\circ\mathrm{r}}$ does so in the context of the logic $\mathbf{CL}^{\mathbf{u}}$, the implementation–problem is avoided. Moreover, as the $\mathbf{CL}^{\circ\mathrm{r}}$–proof theory nicely explicates the dynamic use of the *or*–implicature in actual conversation, the *or*–implication is captured more realistically compared to other formal approaches. Finally, I have also shown how the logic $\mathbf{CL}^{\circ\mathrm{r}}$ can be extended to capture other **GCI** as well. This illustrates the generality of the adaptive logics approach.

Further Research. Despite the fact that the adaptive logics approach to **GCI** is fairly general, there are nonetheless some lines of further research. First of all, I have taken premise sets to correspond with the assertions made by the speaker (and heard by the hearer) in a conversation. However, not only assertions may compel a hearer to reject some of the consequences obtained by applying **GCI**, the background knowledge shared by speaker and hearer may do so as well. Hence, the approach should be extended to include this kind of background knowledge as well. Secondly, I have only considered **GCI**. There are also other kinds of implicatures, viz particularized conversational implicatures. It is still an open question whether the approach can be extended to include this kind of conversational implicatures as well.

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