

Alleged Assassins

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Outline

- 1 The Problem of Modal Modification
- 2 Solutions in terms of Procedural Semantics
- 3 Transparent Intensional Logic
- 4 Modal Constructive Type Theory
- 5 Conclusions

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a is an alleged assassin
?

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- what is the logical structure of the premise?
- what follows as conclusion?

Property Modification

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- Let M be a modifier and F a property. Then (MF) is the result of the procedure of applying the function M to the argument F .
- A full semantic theory of modification must include the following variants:
 - ▶ **Subsective**: $(M'F)a \therefore Fa$
 - ▶ **Privative**: $(M''F)a \therefore \neg Fa$
 - ▶ **Intersective**: $(M'''F)a \therefore M^*a \wedge Fa$
 - ▶ **Modal**: M'''' oscillates between subsection and privation

3 Negative Characterizations of M''''

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- Fails to validate either of $Fa, \neg Fa$ as conclusion.

Task

A *positive* characterization of modal modification.

A solution to privative modification

- [Primiero and Jespersen, 2010] offers two analyses of *privative modification* using two variants of *procedural semantics*:

Realism: Tichý's Transparent Intensional Logic

Idealism: Martin-Löf's Constructive Type Theory

A solution to privative modification

- [Primiero and Jespersen, 2010] offers two analyses of *privative modification* using two variants of *procedural semantics*:
 - Realism: Tichý's Transparent Intensional Logic
 - Idealism: Martin-Löf's Constructive Type Theory
- Common basic idea is to analyze *modal modification* in terms of *possibility/contingency*:
 - TIL: alethic
 - CTT: epistemic

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The Common Core

- 1 a notion of *construction*
- 2 a *functional* language
- 3 a *typed* universe
- 4 an *interpreted* syntax

What Distinguishes TIL from CTT

	TIL	CTT
Semantics	model-theoretic	proof-theoretic
Modifier	property to property	set to set

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Basic and Functional Types

- *Ground Types*: o, l, τ, ω
- *Property*: $(ol)_{\tau\omega}$
- *Property modifier*: $((ol)_{\tau\omega}(ol)_{\tau\omega})$
- *Proposition*: $o_{\tau\omega}$
- *Propositional modifier*: $(o_{\tau\omega}o_{\tau\omega})$

Sentential Meaning

“a is an alleged assassin”

$\lambda w \lambda t \text{ [[} \textit{Alleged Assassin} \textit{]}_{wt} a \text{]}$

The speech act of allegation

$$\frac{\lambda w \lambda t [Alleges_{wt} b \lambda w' \lambda t' [F_{w't'} a]]}{\lambda w \lambda t [\exists x [\exists P [Alleges_{wt} x P]]]} \text{ EG}$$

“*b* alleges that *a* is an *F*”
“somebody alleges something”

Introduction rule for *Alleged*

$$\lambda f[[\textit{Alleged } f]_{wt} a] = \lambda f[\exists x[\textit{Alleges}_{wt} x \lambda w \lambda t[f_{wt} a]]]$$

“being a property that *a* is alleged to have equals
being a property that somebody alleges *a* to have”

Elimination Rule for *Alleged*

$$\frac{[[\textit{Alleged Assassin}]_{wt} a]}{\exists w'[\exists t'[\textit{Assassin}_{w't'} a]] \wedge \exists w''[\exists t''\neg[\textit{Assassin}_{w''t''} a]]}$$

Introduction rule for *Allegedly*

$$\lambda P[\textit{Allegedly } P] = \lambda P[\lambda w \lambda t [\exists x [\textit{Alleges}_{wt} x P]]]$$

“being an alleged proposition equals
being a proposition that somebody alleges”

Elimination rule for *Allegedly*

$$\frac{[Allegedly P]_{wt}}{\exists w'[\exists t[P_{w't'}]] \wedge \exists w''[\exists t''[\neg P_{w''t''}]]}$$

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Two initial comments

- 1 Given the judgemental structure of formulas in CTT, we can model only the propositional modifier:
 - ▶ from 'a is an **alleged** assassin' to '**Allegedly**, a is an assassin'
- 2 The standard constructive syntax does not allow to deal with the contingency required by modal modifiers:
 - ▶ an extended language is required

Definition (Alphabet)

The syntax is defined by the following alphabet:

$\mathcal{K} : \{type, type_{inf}\}$ (verifiers, possibly terminating processes)

$Types := A \mid \perp \mid A \wedge B \mid A \vee B \mid A \rightarrow B \mid A \supset B.$

$Terms := x_i \mid a_i \mid (a_i, b_j) \mid (x_i(b_j)) \mid a_i(b_j).$

$Contexts := \Gamma_i \mid \Delta_i \mid \Box_i \Gamma \mid \Diamond_i \Gamma$

$Judgements := \Delta_i; \Gamma_i \vdash A \text{ type} \mid \Box_i(A \text{ true}) \mid \Diamond_i(A \text{ true}) \mid$

$\circ_{i,j} \Gamma \vdash \circ_{i,j}(A \text{ true}).$

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$\circ_{i,j} \Gamma \vdash \circ_{i,j}(A \text{ true}).$

Modal Modification Rule: Introduction

Allegedly [a is an assassin]

$$\frac{\textit{Assassin type}[\Gamma] \quad \textit{Property}_i \textit{ type}_{inf} \in \Gamma \quad \textit{Alleged}(x)[x : \textit{Assassin}]}{\Box\Gamma, \Diamond(\textit{Property}_i) \vdash a : \textit{Assassin}[x_i/p_i : \textit{Property}_i]}$$

Modal Modification Rule: Elimination I

It is proven that [a is an assassin]

$$\frac{\Box\Gamma, \Diamond(\text{Property}_i) \vdash a : \text{Assassin}[x_i/p_i : \text{Property}_i] \quad p_i : \text{Property}_i}{\Box(\Gamma, p_i : \text{Property}_i) \vdash a : \text{Assassin}}$$

$$\frac{A \text{ type}_{inf} \quad x : A \vdash B \text{ type}_{inf} \quad a : A}{(x(b))(a) = b[a/x] : B \text{ type}[a/x]} \beta\text{-conversion}$$

Modal Modification Rule: Elimination II

The allegation that [a is an assassin] is false.

$$\frac{\Box \Gamma, x_i : \text{Property}_i \vdash a : \text{Assassin}[x_i / p_i : \text{Property}_i] \quad p_i : \text{Property}_i \rightarrow \perp}{a : \text{Assassin} \rightarrow \perp}$$

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Summary of this presentation

- 1 Oscillation between subsection and privation
- 2 Alethic vs. Epistemic Possibility
- 3 For TIL, if $(M''''F)a$ is true, then at some pair w_t (empirical parameters) Fa is true and at another such pair Fa is false
- 4 For CTT, if $(M''''F)a$ true is an admissible judgment to make, then conditions for Fa true are known to be satisfiable, but not all are asserted as verified

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