The Standard Format for Adaptive Logics as a Step towards Universal Logic

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1 Introductory Remarks

adaptive logics interpret a premise set “as normally as possible” with respect to some standard of normality
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  - external dynamics: non-monotonicity
  - internal dynamics: revise conclusions as insights in premises grow
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  absence of positive test for derivability (at predicative level)
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technical reason for dynamics:
   absence of positive test for derivability (at predicative level)

• many reasoning patterns explicated by an adaptive logic
• number of known inference relations characterized by an adaptive logic
many (not all) adaptive logics seem to have a common structure
some can be given this structure under a translation

the structure is central for
  proof theory, semantics, soundness and completeness,
  proofs of further properties, computational aspects, ...

whence the plan:
• describe the structure: the SF (standard format)
• define the proof theory and semantics from the SF
• prove as many properties as possible by relying on the SF only

the results are provisional (as everything):
• not all adaptive logics have been phrased in SF
• a more general characterization may be possible
  (with sets of properties depending on specifications)
2 The Standard Format

- lower limit logic

- set of abnormalities $\Omega$

- strategy
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- *lower limit logic*
  
  standard (monotonic, compact, . . . ) logic

- *set of abnormalities* $\Omega$
  
  characterized by a (possibly restricted) logical form

- *strategy*
  
  Reliability, Minimal Abnormality, Simple strategy, . . .
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  standard (monotonic, compact, ...) logic

- **set of abnormalities Ω**
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upper limit logic:
\[ \text{ULL} = \text{LLL} + \text{axiom/rule that trivializes abnormalities} \]
semantically: the \text{LLL}-models that verify no abnormality
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Upper limit logic:
\[
\text{ULL} = \text{LLL} + \text{axiom/rule that trivializes abnormalities}
\]
semantically: the $\text{LLL}$-models that verify no abnormality

General idea behind adaptive logics:
\[
Cn_{\text{AL}}(\Gamma) : Cn_{\text{LLL}}(\Gamma) + \text{what follows if as many members of } \Omega \text{ are false as the premises permit}
\]
Example: the inconsistency-adaptive $\text{ACLuN}^m$

- **lower limit logic**: $\text{CLuN}$

- **set of abnormalities**: $\Omega = \{ \exists (A \land \sim A) \mid A \in \mathcal{F} \}$

- **strategy**: Minimal Abnormality

**upper limit logic:**

$$\text{CL} = \text{CLuN} + (A \land \sim A) \supset B$$

semantically: the $\text{CLuN}$-models that verify no inconsistency

**corrective** adaptive logic (if $\text{CL}$ is the standard)
Example: logic of inductive generalization: $\textbf{IL}^m$

- *lower limit logic*: $\textbf{CL}$

- *set of abnormalities*: $\Omega = \{\exists A \land \exists \neg A \mid A \in F^o\}$

- *strategy*: Minimal Abnormality

**upper limit logic:**

$\textbf{UCL} = \textbf{CL} + \exists \alpha A(\alpha) \supset \forall \alpha A(\alpha)$

semantically: the uniform $\textbf{CL}$-models ($v(\pi^r) \in \{\emptyset, D^{(r)}\}$)

*ampliative* adaptive logic (if $\textbf{CL}$ is the standard)
Example: Strong Consequence Relation (Rescher–Manor)

consider $\text{ACLuN}^m$ with classical negation ($\neg$) occurring in the language

let $W^\not\sim$ be the closed formulas that do not contain $\sim$

the theorems in $W^\not\sim$ are those of $\text{CL}$ (with $\neg$ the standard negation)

let $\Gamma^\sim\neg = \{\sim\neg A \mid A \in \Gamma\}$

where $\Gamma \cup \{A\} \subseteq W^\not\sim$: $\Gamma \vdash_{\text{Strong}} A$ iff $\Gamma^\sim\neg \models_{\text{ACLuN}^m} A$

corrective consequence relation characterized by an adaptive logic (under a translation)
Conventions

- to simplify the metatheoretic proofs, we add (where necessary) all logical symbols of CL to the LLL
  - harmless
  - these symbols need not occur in the premises or conclusion
  - notation: \( \neg, \sqsubseteq, \sqcap, \sqcup, (\sqcap \alpha), (\sqcup \alpha) \), and \( \equiv \)

so LLL contains CL (in one sense, even if it may be weaker in another)
Conventions

- to simplify the metatheoretic proofs, we add (where necessary) all logical symbols of $\text{CL}$ to the $\text{LLL}$
  - harmless
  - these symbols need not occur in the premises or conclusion
  - notation: $\neg$, $\sqsubseteq$, $\sqcap$, $\sqcup$, $(\sqcap \alpha)$, $(\sqcup \alpha)$, and $\equiv$

so $\text{LLL}$ contains $\text{CL}$ (in one sense, even if it may be weaker in another)

- $\text{Dab}$-formula: classical disjunction of the members of a finite $\Delta \subset \Omega$
  notation: $\text{Dab}(\Delta)$
3 Proofs

- rules of inference  (determined by $\text{LLL}$ and $\Omega$)

- a marking definition  (determined by $\Omega$ and the strategy)
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Dynamics of the proofs controlled by attaching conditions (finite subsets of $\Omega$) to derived formulas
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Line of annotated proof: number, formula, justification, condition
3 Proofs

- rules of inference  (determined by \text{LLL} and \Omega)

- a marking definition  (determined by \Omega and the strategy)

Dynamics of the proofs controlled by attaching \textit{conditions} (finite subsets of \Omega) to derived formulas

Line of annotated proof: number, formula, justification, \textit{condition}

The \textit{rules} govern the conditions
3  Proofs

- rules of inference  (determined by LLL and Ω)

- a marking definition  (determined by Ω and the strategy)

Dynamics of the proofs controlled by attaching conditions (finite subsets of Ω) to derived formulas

Line of annotated proof: number, formula, justification, condition

The rules govern the conditions

Marking definition: determines for every line i at every stage s of a proof whether i is IN or OUT in view of \{ the condition of i, the Dab-formulas derived \}
Rules of inference  
(depend on LLL and Ω, not on the strategy)

**PREM**  If $A \in \Gamma$: 

\[
\begin{array}{c}
\text{A} \\
\text{B}
\end{array} 
\hline
\text{A} \quad \emptyset
\]

**RU**  If $A_1, \ldots, A_n \vdash_{LLL} B$: 

\[
\begin{array}{c}
A_1 \quad \Delta_1 \\
\ldots \quad \ldots \\
A_n \quad \Delta_n
\end{array} 
\hline
B \quad \Delta_1 \cup \ldots \cup \Delta_n
\]

**RC**  If $A_1, \ldots, A_n \vdash_{LLL} B \sqcup Dab(\Theta)$: 

\[
\begin{array}{c}
A_1 \quad \Delta_1 \\
\ldots \quad \ldots \\
A_n \quad \Delta_n
\end{array} 
\hline
B \quad \Delta_1 \cup \ldots \cup \Delta_n \cup \Theta
\]
Rules of inference (depend on LLL and Ω, not on the strategy)

PREM  If \( A \in \Gamma \):
\[
\begin{array}{c}
A \\
\hline
\emptyset
\end{array}
\]

RU  If \( A_1, \ldots, A_n \vdash_{\text{LLL}} B \):
\[
\begin{array}{c}
A_1 \quad \Delta_1 \\
\ldots \quad \ldots \\
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B \quad \Delta_1 \cup \ldots \cup \Delta_n
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\hline
B \quad \Delta_1 \cup \ldots \cup \Delta_n \cup \Theta
\end{array}
\]

for example:
\[
\begin{align*}
p, \ p \supset q & \vdash_{\text{CLuN}} q \\
\neg p, \ p \lor q & \vdash_{\text{CLuN}} q \lor (p \land \neg p)
\end{align*}
\]
Marking definitions

proceed in terms of the minimal $Dab$-formulas that are derived at the stage of the proof

$Dab(\Delta)$ is a minimal $Dab$-formula at stage $s$:  
$Dab(\Delta)$ derived on line with condition $\emptyset$
no $Dab(\Delta')$ with $\Delta' \subset \Delta$ derived on line with condition $\emptyset$
Marking Definition for Reliability

where $Dab(\Delta_1), \ldots, Dab(\Delta_n)$ are the minimal $Dab$-formulas derived on condition $\emptyset$ at stage $s$,

$$U_s(\Gamma) = \Delta_1 \cup \ldots \cup \Delta_n$$

Definition

where $\Delta$ is the condition of line $i$,

line $i$ is marked at stage $s$ iff $\Delta \cap U_s(\Gamma) \neq \emptyset$
Marking Definition for Minimal Abnormality

where $Dab(\Delta_1), \ldots, Dab(\Delta_n)$ are the minimal $Dab$-formulas derived on condition $\emptyset$ at stage $s$,

- $\Phi^o_s(\Gamma)$: set of all sets that contain one member of each $\Delta_i$
- $\Phi^*_s(\Gamma)$: contains, for any $\varphi \in \Phi^o_s(\Gamma)$, $Cn_{LLL}(\varphi) \cap \Omega$
- $\Phi_s(\Gamma)$: $\varphi \in \Phi^*_s(\Gamma)$ that are not proper supersets of a $\varphi' \in \Phi^*_s(\Gamma)$

minimal sets of abnormalities that should be true in order for all $Dab$-formulas derived at stage $s$ to be true

Definition

where $A$ is the formula and $\Delta$ is the condition of line $i$,

line $i$ is marked at stage $s$ iff,

(i) there is no $\varphi \in \Phi_s(\Gamma)$ such that $\varphi \cap \Delta = \emptyset$, or

(ii) for some $\varphi \in \Phi_s(\Gamma)$, there is no line at which $A$ is derived on a condition $\Theta$ for which $\varphi \cap \Theta = \emptyset$
Marking Definition for the Simple strategy

Definition
where $\Delta$ is the condition of line $i$,
line $i$ is marked at stage $s$ iff some $A \in \Delta$ is derived on condition $\emptyset$

only suitable iff, for all $\Gamma$,
$$\Gamma \vdash_{LLL} Dab(\Delta) \quad \text{iff} \quad \text{for some } A \in \Delta, \Gamma \vdash_{LLL} A.$$ 

in other words: if $Dab(\Delta)$ is derived on condition $\emptyset$,
then, for some $A \in \Delta$, $A$ is derivable on condition $\emptyset$

in this case, Reliability and Minimal Abnormality both coincide with the Simple Strategy
Derivability at a stage vs. **final derivability**

idea: \( A \) derived on an unmarked line \( i \) and the proof is **stable** with respect to \( i \)

stability concerns a specific line

**Definition**

\( A \) is *finally derived* from \( \Gamma \) at line \( i \) of a proof at stage \( s \) iff

(i) \( A \) is the second element of line \( i \),
(ii) line \( i \) is unmarked at stage \( s \), and
(iii) any extension of the proof may be further extended in such a way that line \( i \) is unmarked.

**Definition**

\( \Gamma \vdash_{AL} A \) (\( A \) is *finally AL-derivable* from \( \Gamma \)) iff \( A \) is finally derived at a line of a proof from \( \Gamma \).
Two remarks:

even at the predicative level, there are criteria for final derivability

• **ULL** extends **LLL** by validating some further rules
• **AL** extends **LLL** by validating some **applications** of those **ULL**-rules
Extremely simple propositional example for $\text{ACLuN}^r$ (and $\text{ACLuN}^m$)

<table>
<thead>
<tr>
<th></th>
<th>Formula</th>
<th>Premise</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$(p \land q) \land t$</td>
<td>PREM</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>2</td>
<td>$\neg p \lor r$</td>
<td>PREM</td>
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</tr>
<tr>
<td>3</td>
<td>$\neg q \lor s$</td>
<td>PREM</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>4</td>
<td>$\neg p \lor \neg q$</td>
<td>PREM</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>5</td>
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<tr>
<td>6</td>
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<td>1, 2; RC</td>
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<th>Set</th>
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</tr>
<tr>
<td>7</td>
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<td>$s$</td>
<td>1, 3; RC</td>
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<td>(s)</td>
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<td>((p \land \neg p) \lor (q \land \neg q))</td>
<td>1, 4; RU</td>
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<td>1, 2; RC ({p \land \neg p})</td>
<td>(\checkmark)</td>
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<td>((p \land \neg p) \lor (q \land \neg q))</td>
<td>1, 4; RU (\emptyset)</td>
<td></td>
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<tr>
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<td>(p \land \neg p)</td>
<td>1, 5; RU (\emptyset)</td>
<td></td>
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Extremely simple propositional example for ACLuN$^r$ (and ACLuN$^m$)

1. $$(p \land q) \land t$$  
   PREM  $\emptyset$
2. $$\sim p \lor r$$  
   PREM  $\emptyset$
3. $$\sim q \lor s$$  
   PREM  $\emptyset$
4. $$\sim p \lor \sim q$$  
   PREM  $\emptyset$
5. $$t \supset \sim p$$  
   PREM  $\emptyset$
6. $$r$$  
   1, 2; RC  $$\{p \land \sim p\}$$  $\checkmark$
7. $$s$$  
   1, 3; RC  $$\{q \land \sim q\}$$
8. $$(p \land \sim p) \lor (q \land \sim q)$$  
   1, 4; RU  $\emptyset$
9. $$p \land \sim p$$  
   1, 5; RU  $\emptyset$

nothing interesting happens when the proof is continued

no mark will be removed or added
4 Semantics

\( Dab(\Delta) \) is a minimal \( Dab \)-consequence of \( \Gamma \):

\[
\Gamma \models_{LLL} Dab(\Delta) \text{ and, for all } \Delta' \subset \Delta, \Gamma \not\models_{LLL} Dab(\Delta')
\]
4 Semantics

$Dab(\Delta)$ is a minimal $Dab$-consequence of $\Gamma$:

$\Gamma \models_{LLL} Dab(\Delta)$ and, for all $\Delta' \subset \Delta$, $\Gamma \not\models_{LLL} Dab(\Delta')$

where $Dab(\Delta_1)$, $Dab(\Delta_2)$, ... are the minimal $Dab$-consequences of $\Gamma$, $U(\Gamma) = \Delta_1 \cup \Delta_2 \cup ...$
4 Semantics

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$U(\Gamma) = \Delta_1 \cup \Delta_2 \cup \ldots$

where $M$ is a LLL-model: $Ab(M) = \{A \in \Omega \mid M \models A\}$
4 Semantics

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$$U(\Gamma) = \Delta_1 \cup \Delta_2 \cup \ldots$$

where $M$ is a $LLL$-model: $Ab(M) = \{A \in \Omega \mid M \models A\}$

the $AL$-semantics selects some $LLL$-models of $\Gamma$ as $AL$-models of $\Gamma$

the selection depends on $\Omega$ and on the strategy
Reliability

a LLL-model $M$ of $\Gamma$ is reliable iff $Ab(M) \subseteq U(\Gamma)$

$\Gamma \models_{AL^r} A$ iff all reliable models of $\Gamma$ verify $A$
Reliability

a LLL-model $M$ of $\Gamma$ is reliable iff $Ab(M) \subseteq U(\Gamma)$

$\Gamma \models_{ALr} A$ iff all reliable models of $\Gamma$ verify $A$

Minimal Abnormality

a LLL-model $M$ of $\Gamma$ is minimally abnormal iff

there is no LLL-model $M'$ of $\Gamma$ for which $Ab(M') \subset Ab(M)$

$\Gamma \models_{ALm} A$ iff all minimally abnormal models of $\Gamma$ verify $A$
Reliability

A LLL-model $M$ of $\Gamma$ is **reliable** iff $\text{Ab}(M) \subseteq U(\Gamma)$

$\Gamma \models_{AL^r} A$ iff all reliable models of $\Gamma$ verify $A$

Minimal Abnormality

A LLL-model $M$ of $\Gamma$ is **minimally abnormal**

iff

there is no LLL-model $M'$ of $\Gamma$ for which $\text{Ab}(M') \subset \text{Ab}(M)$

$\Gamma \models_{AL^m} A$ iff all minimally abnormal models of $\Gamma$ verify $A$

**Simple strategy**: either of the above if the Simple strategy is suitable
Abnormal $\Gamma$

Normal $\Gamma$
flip-flop (if $\Omega$ not suitably restricted or because of strategy)
flip-flop (if $\Omega$ not suitably restricted or because of strategy)

there are no $\mathbf{AL}$-models, but only $\mathbf{AL}$-models of some $\Gamma$
5 Some Metatheory

5.1 Preliminaries

5.2 On the ULL

5.3 Strong Reassurance

5.4 Soundness and Completeness

5.5 Some Further properties
5.1 Preliminaries

LLL is reflexive, transitive, monotonic, compact, contains CL (see before) and has a characteristic semantics

\(\Omega\): all formulas of a (possibly restricted) logical form \(F\)

provisos:
- if \(A\) has the form \(F\), then \(A \vdash_{LLL} Dab(\Delta)\) for some (finite) \(\Delta \in \Omega\)
- every \(A \in \Omega\) is falsified by a LLL-model

the provisos are only required for obtaining a standard ULL
in a standard way, not for the rest of the metatheory

strategy: we shall consider only Reliability and Minimal Abnormality
(the Simple strategy reduces to these where it is sensible)
5.2 On the ULL

Definition \( \Gamma \vdash_{\text{ULL}} A \) iff \( \Gamma \cup \Omega^- \vdash_{\text{LLL}} A \)

viz. ULL: exactly as LLL, except that it trivializes abnormalities
5.2 On the ULL

Definition \( \Gamma \vdash_{\text{ULL}} A \iff \Gamma \cup \Omega^\neg \vdash_{\text{LLL}} A \)

viz. ULL: exactly as LLL, except that it trivializes abnormalities

Theorem 1
Where \( \Omega \) is characterized by the logical form \( F + a \) (possibly empty) restriction, ULL is LLL + the axiom schema \( \neg F \).

Proof.
(1) LLL + \( \neg F \) contains ULL: obvious

(2) ULL contains LLL + \( \neg F \):
   suppose: \( B \) has the form \( F \)
   there is a finite \( \Delta \in \Omega \) such that \( B \vdash_{\text{LLL}} Dab(\Delta) \)
   for every \( C \in \Delta, \vdash_{\text{ULL}} \neg C \)
   so \( \vdash_{\text{ULL}} \neg Dab(\Delta) \) and also \( \vdash_{\text{ULL}} \neg B \)
\textbf{ULL} = \textbf{LLL} + \neg \textbf{F}

\textbf{ULL}-semantics: the \textbf{LLL}-models that verify no member of \( \Omega \)
ULL = LLL + ¬F

ULL-semantics: the LLL-models that verify no member of Ω

**Theorem 2**

LLL + the axiom schema ¬F is sound and complete w.r.t. the ULL-semantics.

Obvious in view of the proof of Theorem 1.
Theorem 3

\( \Gamma \vdash_{ULL} A \) iff there is a finite \( \Delta \subseteq \Omega \) such that \( \Gamma \vdash_{LLL} A \cup Dab(\Delta) \).

(Derivability Adjustment Theorem)

Proof.
The following six statements are equivalent:

1. \( \Gamma \vdash_{ULL} A \)
2. \( \Gamma \cup \Omega^- \vdash_{LLL} A \) \hspace{1cm} (Def. ULL)
3. \( \Gamma' \cup \Delta^- \vdash_{LLL} A \) for a finite \( \Gamma' \subseteq \Gamma \) and a finite \( \Delta \subseteq \Omega \) \hspace{1cm} (LLL compact)
4. \( \Gamma' \vdash_{LLL} A \cup Dab(\Delta) \) for those \( \Gamma' \) and \( \Delta \) \hspace{1cm} (LLL contains CL)
5. \( \Gamma \vdash_{LLL} A \cup Dab(\Delta) \) for a finite \( \Delta \subseteq \Omega \) \hspace{1cm} (LLL monotonic)

‘motor’ for the adaptive logic: one tries to get as close to ULL as possible by considering \( Dab(\Delta) \) as false whenever \( \Gamma \) permits so
obvious:

**Theorem 4**

ULL contains CL

ULL is reflexive, transitive, monotonic, and uniform

ULL is compact
5.3 Strong Reassurance

Stopperedness, Smoothness

if a model of the premisses is not selected, this is justified by the fact that a selected model of the premisses is less abnormal
5.3 Strong Reassurance

Stopperedness, Smoothness

if a model of the premisses is not selected, this is justified by the fact that a selected model of the premisses is less abnormal

\[ \mathcal{M}_{\Gamma}^{LLL} \text{: the } LLL\text{-models of } \Gamma \]
\[ \mathcal{M}_{\Gamma}^{m} \text{: the } AL^m\text{-models of } \Gamma \]
\[ \mathcal{M}_{\Gamma}^{r} \text{: the } AL^r\text{-models of } \Gamma \]
Theorem 5

If \( M \in \mathcal{M}_{\Gamma}^{LLL} - \mathcal{M}_{\Gamma}^{m} \), then there is a \( M' \in \mathcal{M}_{\Gamma}^{m} \) such that \( Ab(M') \subset Ab(M) \). (Strong Reassurance for Minimal Abnormality.)
Theorem 5
If $M \in \mathcal{M}_\Gamma^{LLL} - \mathcal{M}_\Gamma^m$, then there is a $M' \in \mathcal{M}_\Gamma^m$ such that $Ab(M') \subset Ab(M)$. (Strong Reassurance for Minimal Abnormality.)

Proof. (holds vacuously if $\mathcal{M}_\Gamma^m = \mathcal{M}_\Gamma^{LLL}$)
**Theorem 5**

If $M \in \mathcal{M}_\Gamma^{LLL} - \mathcal{M}_\Gamma^m$, then there is a $M' \in \mathcal{M}_\Gamma^m$ such that $Ab(M') \subset Ab(M)$. (Strong Reassurance for Minimal Abnormality.)

Proof. (holds vacuously if $\mathcal{M}_\Gamma^m = \mathcal{M}_\Gamma^{LLL}$)

Consider $M \in \mathcal{M}_\Gamma^{LLL} - \mathcal{M}_\Gamma^m$; $D_1, D_2, \ldots$ list of all members of $\Omega$

$\Delta_0 = \emptyset$

$\Delta_{i+1} = \Delta_i \cup \{\neg D_{i+1}\}$

if $Ab(M') \subseteq Ab(M)$ for some $M'$ of $\Gamma \cup \Delta_i \cup \{\neg D_{i+1}\}$, otherwise

$\Delta_{i+1} = \Delta_i$

$\Delta = \Delta_0 \cup \Delta_1 \cup \Delta_2 \cup \ldots$

$\Gamma \cup \Delta$ has LLL-models (compactness of LLL + construction)
Theorem 5

If $M \in \mathcal{M}_\Gamma^{\text{LLL}} - \mathcal{M}_\Gamma^m$, then there is a $M' \in \mathcal{M}_\Gamma^m$ such that $\text{Ab}(M') \subset \text{Ab}(M)$. (Strong Reassurance for Minimal Abnormality.)

Proof. (holds vacuously if $\mathcal{M}_\Gamma^m = \mathcal{M}_\Gamma^{\text{LLL}}$)

Consider $M \in \mathcal{M}_\Gamma^{\text{LLL}} - \mathcal{M}_\Gamma^m$. $\Gamma \cup \Delta$ has LLL-models

Step 1. If $M'$ is a model of $\Gamma \cup \Delta$, then $\text{Ab}(M') \subset \text{Ab}(M)$.

Suppose that there is a $D_j \in \Omega$ such that $D_j \in \text{Ab}(M') - \text{Ab}(M)$. Let $M''$ be a model of $\Gamma \cup \Delta_{j-1}$ for which $\text{Ab}(M'') \subseteq \text{Ab}(M)$. As $D_j \notin \text{Ab}(M), D_j \notin \text{Ab}(M'')$. Hence $M''$ is a model of $\Gamma \cup \Delta_{j-1} \cup \{\neg D_j\}$ and $\text{Ab}(M'') \subseteq \text{Ab}(M)$. So $\neg D_j \in \Delta_j \subseteq \Delta$. As $M'$ is a model of $\Gamma \cup \Delta$, $D_j \notin \text{Ab}(M')$. But this contradicts the supposition.

$\Rightarrow$
Theorem 5
If $M \in \mathcal{M}_\Gamma^{\text{LLL}} - \mathcal{M}_\Gamma^m$, then there is a $M' \in \mathcal{M}_\Gamma^m$ such that $\text{Ab}(M') \subset \text{Ab}(M)$. (Strong Reassurance for Minimal Abnormality.)

Proof. (holds vacuously if $\mathcal{M}_\Gamma^m = \mathcal{M}_\Gamma^{\text{LLL}}$)
Consider $M \in \mathcal{M}_\Gamma^{\text{LLL}} - \mathcal{M}_\Gamma^m$. $\Gamma \cup \Delta$ has LLL-models

Step 1. If $M'$ is a model of $\Gamma \cup \Delta$, then $\text{Ab}(M') \subset \text{Ab}(M)$.

Step 2. Every model of $\Gamma \cup \Delta$ is a minimal abnormal model of $\Gamma$.
Suppose that $M'$ is a model of $\Gamma \cup \Delta$, but is not a minimal abnormal model of $\Gamma$. Hence [...] there is a model $M''$ of $\Gamma$ for which $\text{Ab}(M'') \subset \text{Ab}(M')$.

It follows that $M''$ is a model of $\Gamma \cup \Delta$. If it were not, then, as $M''$ is a model of $\Gamma$, there is a $\neg D_j \in \Delta$ such that $M'$ verifies $\neg D_j$ and $M''$ falsifies $\neg D_j$. But then $M'$ falsifies $D_j$ and $M''$ verifies $D_j$, which is impossible in view of $\text{Ab}(M'') \subset \text{Ab}(M')$.

Consider any $D_j \in \text{Ab}(M') - \text{Ab}(M'') \neq \emptyset$. As $M''$ is a model of $\Gamma \cup \Delta_{j-1}$ that falsifies $D_j$, it is a model of $\Gamma \cup \Delta_{j-1} \cup \{\neg D_j\}$. As $\text{Ab}(M'') \subset \text{Ab}(M')$ and $\text{Ab}(M') \subset \text{Ab}(M)$, $\text{Ab}(M'') \subset \text{Ab}(M)$. It follows that $\Delta_j = \Delta_{j-1} \cup \{\neg D_j\}$ and hence that $\neg D_j \in \Delta$. But then $D_j \notin \text{Ab}(M')$. Hence, $\text{Ab}(M'') = \text{Ab}(M')$. So the supposition leads to a contradiction.
Theorem 5
If $M \in \mathcal{M}_\Gamma^{LLL} - \mathcal{M}_\Gamma^m$, then there is a $M' \in \mathcal{M}_\Gamma^m$ such that $Ab(M') \subset Ab(M)$. (Strong Reassurance for Minimal Abnormality.)

Proof. (holds vacuously if $\mathcal{M}_\Gamma^m = \mathcal{M}_\Gamma^{LLL}$)
Consider $M \in \mathcal{M}_\Gamma^{LLL} - \mathcal{M}_\Gamma^m$. $\Gamma \cup \Delta$ has LLL-models

Step 1. If $M'$ is a model of $\Gamma \cup \Delta$, then $Ab(M') \subset Ab(M)$.

Step 2. Every model of $\Gamma \cup \Delta$ is a minimal abnormal model of $\Gamma$. 

Lemma
\[ M^m_\Gamma \subseteq M^r_\Gamma \]  (all Minimal Abnormal models are Reliable models)

Theorem 6
If \( M \in M^\text{LLL}_\Gamma - M^r_\Gamma \), then there is a \( M' \in M^r_\Gamma \) such that \( Ab(M') \subset Ab(M) \). (Strong Reassurance for Reliability.)
5.4 Soundness and Completeness (for Reliability)

Lemma
There is a \( {\mathcal{A}} \)L-proof from \( \Gamma \) in which \( A \) is derived on the condition \( \Delta \) iff \( \Gamma \vDash_{LLL} A \cup \text{Dab}(\Delta) \).

Proof.
\( \Rightarrow \) By an obvious induction on the length of the proof.
\( \Leftarrow \) In view of the compactness of \( LLL \), there is a \( LLL \)-proof of \( A \cup \text{Dab}(\Delta) \) from \( \Gamma \).

So there is a \( {\mathcal{A}} \)L-proof from \( \Gamma \), obtained by applications of PREM and RU, in which \( A \cup \text{Dab}(\Delta) \) is derived on the condition \( \emptyset \).

By applying RC to the last step, one obtains a proof from \( \Gamma \) in which \( A \) is derived on the condition \( \Delta \).
Theorem 7

$\Gamma \vdash_{\text{AL}} A$ iff $\Gamma \vdash_{\text{LLL}} A \sqcup Dab(\Delta)$ and $\Delta \cap U(\Gamma) = \emptyset$ for a finite $\Delta \subset \Omega$.

Proof.
Both directions obvious in view of previous Lemma and the definition of $\Gamma \vdash_{\text{AL}} A$. 

Theorem 7
\( \Gamma \vdash_{AL^r} A \) iff \( \Gamma \vdash_{LLL} A \cup Dab(\Delta) \) and \( \Delta \cap U(\Gamma) = \emptyset \) for a finite \( \Delta \subset \Omega \).

Theorem 8
\( \Gamma \vDash_{AL^r} A \) iff \( \Gamma \vDash_{LLL} A \cup Dab(\Delta) \) and \( \Delta \cap U(\Gamma) = \emptyset \) for a finite \( \Delta \subset \Omega \).

Proof.
\( \Rightarrow \)
all models in \( M^r_\Gamma \) verify \( A \)
so \( \Gamma \cup (\Omega - U(\Gamma))^- \vdash_{LLL} A \)
so \( \Gamma' \cup \Delta^- \vdash_{LLL} A \) for finite \( \Gamma' \subset \Gamma \) and \( \Delta \subset \Omega \) compact
so \( \Gamma' \vdash_{LLL} A \cup Dab(\Delta) \)
so \( \Gamma \vdash_{LLL} A \cup Dab(\Delta) \)
monotonic
\( \Leftarrow \)
suppose there are \( LLL \)-models of \( \Gamma \) and they all verify \( A \cup Dab(\Delta) \)
so there are \( AL^r \)-models of \( \Gamma \) Strong Reassurance
all \( AL^r \)-models of \( \Gamma \) falsify \( Dab(\Delta) \)
so all \( AL^r \)-models of \( \Gamma \) verify \( A \)
Theorem 7
\[ \Gamma \vdash_{\text{AL}} A \iff \Gamma \vdash_{\text{LLL}} A \cup Dab(\Delta) \text{ and } \Delta \cap U(\Gamma) = \emptyset \text{ for a finite } \Delta \subset \Omega. \]

Theorem 8
\[ \Gamma \models_{\text{AL}} A \iff \Gamma \models_{\text{LLL}} A \cup Dab(\Delta) \text{ and } \Delta \cap U(\Gamma) = \emptyset \text{ for a finite } \Delta \subset \Omega. \]

Lemma
\[ \Gamma \vdash_{\text{LLL}} A \cup Dab(\Delta) \iff \Gamma \models_{\text{LLL}} A \cup Dab(\Delta). \]

Proof. By the soundness and completeness of LLL.
Theorem 7
\( \Gamma \vdash_{AL^r} A \text{ iff } \Gamma \vdash_{LLL} A \cup Dab(\Delta) \text{ and } \Delta \cap U(\Gamma) = \emptyset \) for a finite \( \Delta \subset \Omega \).

Theorem 8
\( \Gamma \models_{AL^r} A \text{ iff } \Gamma \models_{LLL} A \cup Dab(\Delta) \text{ and } \Delta \cap U(\Gamma) = \emptyset \) for a finite \( \Delta \subset \Omega \).

Lemma
\( \Gamma \vdash_{LLL} A \cup Dab(\Delta) \text{ iff } \Gamma \models_{LLL} A \cup Dab(\Delta) \).

Corollary
\( \Gamma \vdash_{AL^r} A \text{ iff } \Gamma \models_{AL^r} A \).
5.5 Some Further properties

terminology

minimal \textit{Dab}-consequence of $\Gamma$

$\Phi^\circ(\Gamma)$: set of all sets that contain one member of each minimal \textit{Dab}-consequence of $\Gamma$

$\Phi^*(\Gamma)$: contains, for every $\varphi \in \Phi^\circ(\Gamma)$, $Cn_{LLL}(\varphi) \cap \Omega$

$\Phi(\Gamma)$: $\varphi \in \Phi^*(\Gamma)$ that are not proper supersets of a $\varphi' \in \Phi^*(\Gamma)$
5.5 Some Further properties

terminology

minimal $Dab$-consequence of $\Gamma$

$\Phi^o(\Gamma)$: set of all sets that contain one member of each minimal $Dab$-consequence of $\Gamma$

$\Phi^*(\Gamma)$: contains, for every $\varphi \in \Phi^o(\Gamma)$, $Cn_{LLL}(\varphi) \cap \Omega$

$\Phi(\Gamma)$: $\varphi \in \Phi^*(\Gamma)$ that are not proper supersets of a $\varphi' \in \Phi^*(\Gamma)$

Lemma

$M \in \mathcal{M}_\Gamma^m$ iff $M \in \mathcal{M}_\Gamma^{LLL}$ and $Ab(M) \in \Phi_\Gamma$.

Proof: long but perspicuous.
immediate or almost immediate consequences of the Lemma:

**Theorem 9**

Each of the following obtains:

1. \( \mathcal{M}_m^\Gamma \subseteq \mathcal{M}_r^\Gamma \). Hence \( \text{Cn}_{\text{AL}r}(\Gamma) \subseteq \text{Cn}_{\text{AL}m}(\Gamma) \).

2. If \( A \in \Omega - U(\Gamma) \), then \( \neg A \in \text{Cn}_{\text{AL}r}(\Gamma) \).

3. If \( Dab(\Delta) \) is a minimal \( Dab \)-consequence of \( \Gamma \) and \( A \in \Delta \), then some \( M \in \mathcal{M}_m^\Gamma \) verifies \( A \) and falsifies all members (if any) of \( \Delta - \{A\} \).

4. \( \mathcal{M}_m^\Gamma = \mathcal{M}_m^{\text{Cn}_{\text{AL}m}(\Gamma)} \) whence \( \text{Cn}_{\text{AL}m}(\Gamma) = \text{Cn}_{\text{AL}m}(\text{Cn}_{\text{AL}m}(\Gamma)) \). (Fixed Point.)

5. \( \mathcal{M}_r^\Gamma = \mathcal{M}_r^{\text{Cn}_{\text{AL}r}(\Gamma)} \) whence \( \text{Cn}_{\text{AL}r}(\Gamma) = \text{Cn}_{\text{AL}r}(\text{Cn}_{\text{AL}r}(\Gamma)) \). (Fixed Point.)

6. For all \( \Delta \subseteq \Omega \), \( Dab(\Delta) \in \text{Cn}_{\text{AL}}(\Gamma) \) iff \( Dab(\Delta) \in \text{Cn}_{\text{LLL}}(\Gamma) \). (Immunity.)

7. If \( \Gamma \models_{\text{AL}} A \) for every \( A \in \Gamma' \), and \( \Gamma \cup \Gamma' \models_{\text{AL}} B \), then \( \Gamma \models_{\text{AL}} B \). (Cautious Cut.)

8. If \( \Gamma \models_{\text{AL}} A \) for every \( A \in \Gamma' \), and \( \Gamma \models_{\text{AL}} B \), then \( \Gamma \cup \Gamma' \models_{\text{AL}} B \). (Cautious Monotonicity.)
Theorem 10  each of the following obtains:

1. If \( \Gamma \) is normal, then \( \mathcal{M}^{\text{ULL}}_\Gamma = \mathcal{M}^m_\Gamma = \mathcal{M}^r_\Gamma \) 
   whence \( Cn_{AL^r}(\Gamma) = Cn_{AL^m}(\Gamma) = Cn_{\text{ULL}}(\Gamma) \).

If \( \Gamma \) is normal, then \( U(\Gamma) = \emptyset \) and only ULL-models of \( \Gamma \) are minimally abnormal.
Theorem 10  each of the following obtains:

1. If $\Gamma$ is normal, then $M^\text{ULL}_\Gamma = M^m_\Gamma = M^r_\Gamma$
   whence $Cn_{\text{AL}^r}(\Gamma) = Cn_{\text{AL}^m}(\Gamma) = Cn_{\text{ULL}}(\Gamma)$.

2. If $\Gamma$ is abnormal and $M^\text{LLL}_\Gamma \neq \emptyset$, then $M^\text{ULL}_\Gamma \subset M^m_\Gamma$
   and hence $Cn_{\text{AL}^r}(\Gamma) \subseteq Cn_{\text{AL}^m}(\Gamma) \subset Cn_{\text{ULL}}(\Gamma)$.

If $\Gamma$ is abnormal, then $M^\text{ULL}_\Gamma = \emptyset$. 
Theorem 10  each of the following obtains:

1. If $\Gamma$ is normal, then $\mathcal{M}_\Gamma^{\text{ULL}} = \mathcal{M}_\Gamma^m = \mathcal{M}_\Gamma^r$
   whence $Cn_{\text{AL}^r}(\Gamma) = Cn_{\text{AL}^m}(\Gamma) = Cn_{\text{ULL}}(\Gamma)$.

2. If $\Gamma$ is abnormal and $\mathcal{M}_\Gamma^{\text{LLL}} \neq \emptyset$, then $\mathcal{M}_\Gamma^{\text{ULL}} \subset \mathcal{M}_\Gamma^m$
   and hence $Cn_{\text{AL}^r}(\Gamma) \subseteq Cn_{\text{AL}^m}(\Gamma) \subset Cn_{\text{ULL}}(\Gamma)$.

3. $\mathcal{M}_\Gamma^{\text{ULL}} \subset \mathcal{M}_\Gamma^m \subset \mathcal{M}_\Gamma^r \subset \mathcal{M}_\Gamma^{\text{LLL}}$
   whence $Cn_{\text{LLL}}(\Gamma) \subseteq Cn_{\text{AL}^r}(\Gamma) \subseteq Cn_{\text{AL}^m}(\Gamma) \subseteq Cn_{\text{ULL}}(\Gamma)$.

$\mathcal{M}_\Gamma^{\text{ULL}} \subset \mathcal{M}_\Gamma^m$: from 1 and 2. $\mathcal{M}_\Gamma^r \subset \mathcal{M}_\Gamma^{\text{LLL}}$ is immediate in view of the definition of a reliable model of $\Gamma$. $\mathcal{M}_\Gamma^m \subset \mathcal{M}_\Gamma^r$ is item 1 of the previous Theorem.
Theorem 10  each of the following obtains:

1. If $\Gamma$ is normal, then $\mathcal{M}_\Gamma^\text{ULL} = \mathcal{M}_\Gamma^m = \mathcal{M}_\Gamma^r$
   whence $Cn_{\text{AL}r}(\Gamma) = Cn_{\text{AL}m}(\Gamma) = Cn_{\text{ULL}}(\Gamma)$.

2. If $\Gamma$ is abnormal and $\mathcal{M}_\Gamma^\text{LLL} \neq \emptyset$, then $\mathcal{M}_\Gamma^\text{ULL} \subset \mathcal{M}_\Gamma^m$
   and hence $Cn_{\text{AL}r}(\Gamma) \subseteq Cn_{\text{AL}m}(\Gamma) \subset Cn_{\text{ULL}}(\Gamma)$.

3. $\mathcal{M}_\Gamma^\text{ULL} \subseteq \mathcal{M}_\Gamma^m \subseteq \mathcal{M}_\Gamma^r \subseteq \mathcal{M}_\Gamma^\text{LLL}$
   whence $Cn_{\text{LLL}}(\Gamma) \subseteq Cn_{\text{AL}r}(\Gamma) \subseteq Cn_{\text{AL}m}(\Gamma) \subseteq Cn_{\text{ULL}}(\Gamma)$.

4. $\mathcal{M}_\Gamma^r \subset \mathcal{M}_\Gamma^\text{LLL}$ iff $\Gamma \cup \{A\}$ is LLL-satisfiable for some $A \in \Omega - U(\Gamma)$.

Immediate in view of the definitions of a reliable model and $\Gamma \models_{\text{AL}r} A$. 
Theorem 10  each of the following obtains:

1. If \( \Gamma \) is normal, then \( \mathcal{M}_\Gamma^{\text{ULL}} = \mathcal{M}_\Gamma^m = \mathcal{M}_\Gamma^r \) whence \( \text{Cn}_{\text{AL}^r}(\Gamma) = \text{Cn}_{\text{AL}^m}(\Gamma) = \text{Cn}_{\text{ULL}}(\Gamma) \).

2. If \( \Gamma \) is abnormal and \( \mathcal{M}_\Gamma^{\text{LLL}} \neq \emptyset \), then \( \mathcal{M}_\Gamma^{\text{ULL}} \subset \mathcal{M}_\Gamma^m \) and hence \( \text{Cn}_{\text{AL}^r}(\Gamma) \subseteq \text{Cn}_{\text{AL}^m}(\Gamma) \subset \text{Cn}_{\text{ULL}}(\Gamma) \).

3. \( \mathcal{M}_\Gamma^{\text{ULL}} \subseteq \mathcal{M}_\Gamma^m \subseteq \mathcal{M}_\Gamma^r \subseteq \mathcal{M}_\Gamma^{\text{LLL}} \) whence \( \text{Cn}_{\text{LLL}}(\Gamma) \subseteq \text{Cn}_{\text{AL}^r}(\Gamma) \subseteq \text{Cn}_{\text{AL}^m}(\Gamma) \subseteq \text{Cn}_{\text{ULL}}(\Gamma) \).

4. \( \mathcal{M}_\Gamma^r \subset \mathcal{M}_\Gamma^{\text{LLL}} \) iff \( \Gamma \cup \{A\} \) is LLL-satisfiable for some \( A \in \Omega - U(\Gamma) \).

5. \( \text{Cn}_{\text{LLL}}(\Gamma) \subset \text{Cn}_{\text{AL}^r}(\Gamma) \) iff \( \mathcal{M}_\Gamma^r \subset \mathcal{M}_\Gamma^{\text{LLL}} \).

\[ \Rightarrow \] Suppose \( A \in \text{Cn}_{\text{LLL}}(\Gamma) - \text{Cn}_{\text{AL}^r}(\Gamma) \). So, for some \( A \in \Omega - U(\Gamma) \), all \( M \in \mathcal{M}_\Gamma^r \) falsify \( A \) whereas some \( M \in \mathcal{M}_\Gamma^{\text{LLL}} - \mathcal{M}_\Gamma^r \) verifies \( A \).

\[ \Leftarrow \] obvious.
Theorem 10 each of the following obtains:

1. If $\Gamma$ is normal, then $M_\Gamma^{\text{ULL}} = M_\Gamma^m = M_\Gamma^r$ whence $C_{n_{\text{AL}}r}(\Gamma) = C_{n_{\text{AL}}m}(\Gamma) = C_{n_{\text{ULL}}}(\Gamma)$.

2. If $\Gamma$ is abnormal and $M_\Gamma^{\text{LLL}} \neq \emptyset$, then $M_\Gamma^{\text{ULL}} \subset M_\Gamma^m$ and hence $C_{n_{\text{AL}}r}(\Gamma) \subseteq C_{n_{\text{AL}}m}(\Gamma) \subset C_{n_{\text{ULL}}}(\Gamma)$.

3. $M_\Gamma^{\text{ULL}} \subseteq M_\Gamma^m \subseteq M_\Gamma^r \subseteq M_\Gamma^{\text{LLL}}$ whence $C_{n_{\text{LLL}}}(\Gamma) \subseteq C_{n_{\text{AL}}r}(\Gamma) \subseteq C_{n_{\text{AL}}m}(\Gamma) \subseteq C_{n_{\text{ULL}}}(\Gamma)$.

4. $M_\Gamma^r \subset M_\Gamma^{\text{LLL}}$ iff $\Gamma \cup \{A\}$ is LLL-satisfiable for some $A \in \Omega - U(\Gamma)$.

5. $C_{n_{\text{LLL}}}(\Gamma) \subset C_{n_{\text{AL}}r}(\Gamma)$ iff $M_\Gamma^r \subset M_\Gamma^{\text{LLL}}$.

6. $M_\Gamma^m \subset M_\Gamma^{\text{LLL}}$ iff there is a (possibly infinite) $\Delta \subseteq \Omega$ such that $\Gamma \cup \Delta$ is LLL-satisfiable and there is no $\varphi \in \Phi_\Gamma$ for which $\Delta \subseteq \varphi$.

Immediate in view of the definitions of a Minimal Abnormal model and $\Gamma \models_{\text{AL}}^m A$. 
Theorem 10  each of the following obtains:

1. If $\Gamma$ is normal, then $\mathcal{M}_\Gamma^{\text{ULL}} = \mathcal{M}_\Gamma^m = \mathcal{M}_\Gamma^r$ whence $Cn_{\text{AL}^r}(\Gamma) = Cn_{\text{AL}^m}(\Gamma) = Cn_{\text{ULL}}(\Gamma)$.

2. If $\Gamma$ is abnormal and $\mathcal{M}_\Gamma^{\text{LLL}} \neq \emptyset$, then $\mathcal{M}_\Gamma^{\text{ULL}} \subset \mathcal{M}_\Gamma^m$ and hence $Cn_{\text{AL}^r}(\Gamma) \subseteq Cn_{\text{AL}^m}(\Gamma) \subset Cn_{\text{ULL}}(\Gamma)$.

3. $\mathcal{M}_\Gamma^{\text{ULL}} \subseteq \mathcal{M}_\Gamma^m \subseteq \mathcal{M}_\Gamma^r \subseteq \mathcal{M}_\Gamma^{\text{LLL}}$ whence $Cn_{\text{LLL}}(\Gamma) \subseteq Cn_{\text{AL}^r}(\Gamma) \subseteq Cn_{\text{AL}^m}(\Gamma) \subseteq Cn_{\text{ULL}}(\Gamma)$.

4. $\mathcal{M}_\Gamma^r \subset \mathcal{M}_\Gamma^{\text{LLL}}$ iff $\Gamma \cup \{A\}$ is LLL-satisfiable for some $A \in \Omega - U(\Gamma)$.

5. $Cn_{\text{LLL}}(\Gamma) \subset Cn_{\text{AL}^r}(\Gamma)$ iff $\mathcal{M}_\Gamma^r \subset \mathcal{M}_\Gamma^{\text{LLL}}$.

6. $\mathcal{M}_\Gamma^m \subset \mathcal{M}_\Gamma^{\text{LLL}}$ iff there is a (possibly infinite) $\Delta \subseteq \Omega$ such that $\Gamma \cup \Delta$ is LLL-satisfiable and there is no $\varphi \in \Phi_\Gamma$ for which $\Delta \subseteq \varphi$.

7. If there are $A_1, \ldots, A_n \in \Omega \ (n \geq 1)$ such that $\Gamma \cup \{A_1, \ldots, A_n\}$ is LLL-satisfiable and, for every $\varphi \in \Phi_\Gamma$, $\{A_1, \ldots, A_n\} \not\subseteq \varphi$, then $Cn_{\text{LLL}}(\Gamma) \subset Cn_{\text{AL}^m}(\Gamma)$.

Suppose the antecedent is true. Every $M \in \mathcal{M}_\Gamma^m$ falsifies some $A_i$ whereas some $M \in \mathcal{M}_\Gamma^{\text{LLL}}$ (viz. an $M \in \mathcal{M}_\Gamma^{\text{LLL}} \cup \{A_1, \ldots, A_n\}$) verifies $A_1 \cap \ldots \cap A_n$. 
Theorem 10  each of the following obtains:

1. If $\Gamma$ is normal, then $\mathcal{M}_\Gamma^\text{ULL} = \mathcal{M}_\Gamma^m = \mathcal{M}_\Gamma^r$
   whence $Cn_{\text{AL}r}(\Gamma) = Cn_{\text{AL}m}(\Gamma) = Cn_{\text{ULL}}(\Gamma)$.

2. If $\Gamma$ is abnormal and $\mathcal{M}_\Gamma^\text{LLL} \neq \emptyset$, then $\mathcal{M}_\Gamma^\text{ULL} \subset \mathcal{M}_\Gamma^m$
   and hence $Cn_{\text{AL}r}(\Gamma) \subseteq Cn_{\text{AL}m}(\Gamma) \subset Cn_{\text{ULL}}(\Gamma)$.

3. $\mathcal{M}_\Gamma^\text{ULL} \subseteq \mathcal{M}_\Gamma^m \subseteq \mathcal{M}_\Gamma^r \subseteq \mathcal{M}_\Gamma^\text{LLL}$
   whence $Cn_{\text{LLL}}(\Gamma) \subseteq Cn_{\text{AL}r}(\Gamma) \subseteq Cn_{\text{AL}m}(\Gamma) \subseteq Cn_{\text{ULL}}(\Gamma)$.

4. $\mathcal{M}_\Gamma^r \subset \mathcal{M}_\Gamma^\text{LLL}$ iff $\Gamma \cup \{A\}$ is LLL-satisfiable for some $A \in \Omega - U(\Gamma)$.

5. $Cn_{\text{LLL}}(\Gamma) \subset Cn_{\text{AL}r}(\Gamma)$ iff $\mathcal{M}_\Gamma^r \subset \mathcal{M}_\Gamma^\text{LLL}$.

6. $\mathcal{M}_\Gamma^m \subset \mathcal{M}_\Gamma^\text{LLL}$ iff there is a (possibly infinite) $\Delta \subseteq \Omega$ such that
   $\Gamma \cup \Delta$ is LLL-satisfiable and there is no $\varphi \in \Phi_\Gamma$ for which $\Delta \subseteq \varphi$.

7. If there are $A_1, \ldots, A_n \in \Omega$ ($n \geq 1$) such that $\Gamma \cup \{A_1, \ldots, A_n\}$ is
   LLL-satisfiable and, for every $\varphi \in \Phi_\Gamma$, $\{A_1, \ldots, A_n\} \not\subseteq \varphi$, then
   $Cn_{\text{LLL}}(\Gamma) \subset Cn_{\text{AL}m}(\Gamma)$.

8. $Cn_{\text{AL}m}(\Gamma)$ and $Cn_{\text{AL}r}(\Gamma)$ are non-trivial iff $\mathcal{M}_\Gamma^\text{LLL} \neq \emptyset$.

Immediate from Reassurance + no LLL-model trivial.
Theorem 11
If $\Gamma \vdash_{AL} A$, then every AL-proof from $\Gamma$ can be extended in such a way that $A$ is finally derived in it. (Proof Invariance)

etc.
6 Computability Matters

In view of the reasoning processes explicated by $\vdash_{AL}$,

- $\vdash_{AL}$ is not decidable
- there is no positive test for $\vdash_{AL}$
6 Computability Matters

In view of the reasoning processes explicated by $\vdash_{\text{AL}}$,

- $\vdash_{\text{AL}}$ is not decidable
- there is no positive test for $\vdash_{\text{AL}}$

Does the dynamics of the proofs go anywhere?

Are there criteria for final derivability?
Does the dynamics of the proofs go anywhere?

in view of the block analysis of proofs (and the block semantics):
• a stage of a proof provides a certain insight in the premises
• every step of the proof is informative or non-informative
  • if informative: more insight in the premises gained
  • if non-informative: no insight lost (sq)
• sensible proofs converge toward maximal insight
  (sensible proofs are obtained by the procedure on the next slides)
Are there criteria for final derivability?

- the block semantics
- tableau methods
- procedural criterion
Procedural criterion for Reliability

based on prospective dynamic proofs
  (goal-directed + most heuristics pushed into the proof)
Procedural criterion for Reliability

based on prospective dynamic proofs
  (goal-directed + most heuristics pushed into the proof)

3 phase procedure for testing whether $\Gamma \vdash_{\text{AL}^r} A$
Procedural criterion for Reliability

based on prospective dynamic proofs
  (goal-directed + most heuristics pushed into the proof)

3 phase procedure for testing whether $\Gamma \vdash_{AL^r} A$

if the procedure stops: answer is obtained (YES / NO)

(procedure at least as good as tableau methods)

pdp2.exe at http://logica.ugent.be/centrum/programs/ implements procedure for propositional ACLuN$^r$
Γ ⊬_{ALr} G?

*Phase 1*

try to derive $G$ on a condition

- no success: $\Gamma \not\vdash_{ALr} G$

- success: $G$ derived on a condition $\Delta$ at line $i$
  
  - $\Delta = \emptyset$: $\Gamma \vdash_{ALr} G$
  
  - $\Delta \neq \emptyset$:
    
    $\Rightarrow$ phase 2 $\Rightarrow$ phase 1

  - line $i$ not marked: $\Gamma \vdash_{ALr} G$

  - line $i$ marked: try to derive $G$ on a (different) condition
\[ \Gamma \vdash_{ALr} G? \]

\( G \) derived on condition \( \Delta \ (\neq \emptyset) \) at line \( i \)

**Phase 2**

try to derive \( Dab(\Delta) \) on a condition

- no success: \(^1\) return to phase 1 (line \( i \) is unmarked)
- success: \( Dab(\Delta) \) derived on condition \( \Theta \) at line \( j \)
  - \( \Theta = \emptyset \): mark line \( i \); return to phase 1
  - \( \Theta \neq \emptyset \):
    \[ \Rightarrow \text{ phase 3 } \Rightarrow \text{ phase 2 } \]
    - line \( j \) not marked: \(^2\) mark line \( i \); return to phase 1
    - line \( j \) marked: \(^3\) try to derive \( Dab(\Delta) \) on a (different) condition

1. \( \Delta \cap U(\Gamma) = \emptyset \)
2. \( \Theta \cap U(\Gamma) = \emptyset \) whence \( \Delta \cap U(\Gamma) \neq \emptyset \)
3. so \( \Gamma \vdash_{LLL} Dab(\Theta) \), so possibly \( \Delta \cap U(\Gamma) = \emptyset \)
\[ \Gamma \vdash_{\text{AL}} G? \]

\( G \) derived on condition \( \Delta \ (\neq \emptyset) \) at line \( i \)

\( Dab(\Delta) \) derived on condition \( \Theta \) at line \( j \)

**Phase 3**

try to derive \( Dab(\Theta) \) on a the condition \( \emptyset \)

- no success: return to phase 2 (line \( j \) is unmarked)
- success: mark line \( j \); return to phase 2

\[ \begin{align*}
1 & \text{ so } \Gamma \vdash_{\text{LLL}} Dab(\Delta \cup \Theta) \\
2 & \text{ so } \Gamma \not\vdash_{\text{LLL}} Dab(\Theta), \text{ whence } \Delta \cap U(\Gamma) \neq \emptyset \\
3 & \text{ so } \Gamma \vdash_{\text{LLL}} Dab(\Theta), \text{ so possibly } \Delta \cap U(\Gamma) = \emptyset
\end{align*} \]
Universal logic

the aim: characterize every reasoning form that displays the internal dynamics (including all defeasible reasoning) by an adaptive logic in SF
A  Further examples and applications

- Corrective
- Ampliative (+ ampliative and corrective)
- Incorporation
- Applications
Corrective

- inconsistency-adaptive logics (adapting to negation gluts): \( \text{ACLuN}^r \) and \( \text{ACLuN}^m \), those based on other paraconsistent logics, including \( \text{CLuN}s \) (LP, ...), \( \text{ANA} \), Jaśkowski’s \( \text{D2} \), ...

- negation gaps

- gluts/gaps for all logical symbols

- ambiguity adaptive logics

- adaptive zero logic

- corrective deontic logics

- prioritized ial

- ...
Ampliative (+ ampliative and corrective)

- compatibility (characterization)
- compatibility with inconsistent premises
- diagnosis
- prioritized adaptive logics
- inductive generalization
- abduction
- inference to the best explanation
- analogies, metaphors
- eroticetic evocation and eroticetic inference
- changing positions in discussions
- ...
Incorporation  (possibly + extension)

- flat Rescher–Manor consequence relations (+ extensions)
- prioritized Rescher–Manor consequence relations
- partial structures and pragmatic truth
- circumscription, defaults, negation as failure, . . .
- dynamic characterization of $\mathbf{R} \rightarrow$
- signed systems (Besnard & C°)
- . . .
Applications

- scientific discovery and creativity
- scientific explanation
- diagnosis
- positions defended / agreed upon in discussions
- changing positions in discussions
- belief revision in inconsistent contexts
- inconsistent arithmetic
- inductive statistical explanation
- tentatively eliminating abnormalities
- Gricean maxims
- ...