Adaptive Logics

The Logics You Always Wanted

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Outline

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Introductory Remarks (1)

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Adaptive logics broaden the domain of logic:

grasp a large set of reasoning forms often considered
- as mistaken
 or
- as too indistinct to allow for formal treatment

Introductory Remarks (2)

adaptive logics explicate reasoning processes that display an internal (and possibly an external) dynamics

- external dynamics: non-monotonicity
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technical reason for dynamics:

absence of positive test for derivability (at predicative level)

- decision procedure *vs.* positive test

Introductory Remarks (3)

- many reasoning patterns explicated by an adaptive logic
- many known inference relations characterized by an adaptive logic

Outline

Incomplete Survey

(propaganda)

- Corrective
- Ampliative (+ ampliative and corrective)
- Incorporation
- Applications

take **CL** as the standard of deduction

Incomplete Survey: ▶ Corrective

- inconsistency-adaptive logics (adapt to *negation gluts*): **CLuN**$^{\mathbf{r}}$ and **CLuN**$^{\mathbf{m}}$,
  those based on other paraconsistent logics, including **CLuNs** (**LP**, . . .), **ANA**, Jaśkowski's **D2**, . . .
- negation gaps
- gluts/gaps for other logical symbols
- ambiguity adaptive logics
- adaptive zero logic
- corrective deontic logics, . . .
- prioritized ial
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- compatibility with inconsistent premises
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- inductive generalization
- abduction and inference to the best explanation
- analogies, metaphors
- erotetic evocation and erotetic inference
- tentatively eliminating abnormalities
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- flat Rescher–Manor consequence relations (+ extensions)
- prioritized Rescher–Manor consequence relations
- partial structures and pragmatic truth
- circumscription, defaults, negation as failure, ...
- dynamic characterization of $\mathbf{R}_{\rightarrow}$
- signed systems (Besnard et. al.)
- logics that are adaptive with respect to *rules* instead of abnormalities
- ...

Incomplete Survey: ▶ Applications

- scientific discovery and creativity
- scientific explanation
- diagnosis
- changing positions in discussions
- positions defended / agreed upon in discussions
- belief revision (predicative / inconsistent contexts)
- inconsistent arithmetic
- evocation of questions from inconsistent premises
- inductive statistical explanation
- inductive conjectures of sorts
- Gricean maxims
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Outline

General Characterization

- Introductory Remarks
- Incomplete Survey
- Ordering the Domain
- Why Integration?
- Combining adaptive logics

Some Specific Topics (for the Standard Format)

- Proof Theory (1)
- Semantics
- Metatheory
- Proof Theory (2)

More General Framework

References

Ordering the Domain (most of it)

large diversity and every new adaptive logic requires:

- syntax (proof theory)
- semantics (models)
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  - (especially hard bit)

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- *set of abnormalities* Ω
- *strategy*

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  reflexive, . . . , monotonic and compact logic
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- *strategy*
  Reliability, Minimal Abnormality, . . .

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“abnormality” is technical term

only abnormalities of corrective adaptive logics are **CL**-impossible general idea behind adaptive logics:

whereas **ULL** extends **LLL**
by validating some further rules,

AL extends **LLL**
by validating certain *applications* of those **ULL**-rules

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in other words:

$Cn_{\mathbf{AL}}(\Gamma)$: $Cn_{\mathbf{LLL}}(\Gamma)$ + what follows if as many abnormalities are false as the premises permit

Conventions

to simplify the metatheoretic proofs, add all logical symbols of **CL** to the **LLL**

- notation: $\check{\sim}$, $\check{\supset}$, $\check{\wedge}$, $\check{\vee}$, $\check{\forall}$, . . .
- these symbols need not occur in the premises or conclusion
- harmless

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Dab-formula: classical disjunction of the members of a finite $\Delta \subset \Omega$ notation: $Dab(\Delta)$

Example: the inconsistency-adaptive $\mathbf{CLuN}^m$

· *lower limit logic*: **CLuN**

· *set of abnormalities*: $\Omega = \{\exists(A \wedge \sim A) \mid A \in \mathcal{F}\}$

· *strategy*: Minimal Abnormality

upper limit logic:
CL = **CLuN** + $(A \wedge \sim A) \supset B$
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corrective adaptive logic

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Example: logic of inductive generalization: $\mathbf{IL}^m$

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· *set of abnormalities*: $\Omega = \{\exists A \wedge \exists{\sim}A \mid A \in \mathcal{F}^{\circ}\}$

· *strategy*: Minimal Abnormality

upper limit logic:
UCL = **CL** + $\exists\alpha A(\alpha) \supset \forall\alpha A(\alpha)$
semantically: the uniform **CL**-models
$(v(\pi^r) \in \{\emptyset, D^{(r)}\})$

ampliative adaptive logic

Example: Strong Consequence Relation (Rescher–Manor)

let Γ' comprise the members of Γ with $\sim$ replaced by $\check{\sim}$

let $\Gamma^{\sim\check{\sim}} = \{\sim\check{\sim}A \mid A \in \Gamma'\}$

Theorem:
$\Gamma \vdash_{Strong} A$ iff $\Gamma^{\sim\check{\sim}} \models_{\mathbf{CLuN}^m} A$

corrective consequence relation characterized by an adaptive logic (under a translation)

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the SF provides a guide in devising new adaptive logics

if a new adaptive logic is in SF, most of the hard work can be skipped

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or whether (at least) the number of schemes can be reduced

which schemes are most unifying cannot be settled today

the unifying power of adaptive logics should be studied
because there is a clear underlying concept

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Combining adaptive logics

- ‘union’ of abnormalities: $\Omega_1 \cup \Omega_2$
- sequential combination:
  $\ldots Cn_{\mathbf{AL3}}(Cn_{\mathbf{AL2}}(Cn_{\mathbf{AL1}}(\Gamma)))\ldots$

example: combing a set of adaptive logics: **AT^i**

- *lower limit logic*: **T**
- *set of abnormalities*: $\Omega^i = \{\Diamond^i A \wedge {\sim}A \mid A \in \mathcal{W}^a\}$
  $\Diamond^i$ abbreviates sequence of *i* diamonds
  (abnormality is falsehood of an expectancy)
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many possible variants examples:

$$\Omega^i = \{\Diamond^i A \wedge \Diamond^i {\sim}A \mid A \in \mathcal{W}^a\}$$

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diagnosis applies $\mathbf{AT}^P$:
  data + accepted generalizations (**CL**)
  + generalizations accepted with a degree of plausibility

A further example

inductive generalization in the presence of background theories:

data (governed by **CL**)
+
background theories that are defeasible in different senses

(sundry preferential systems combined)

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Standard format and proof theory: part 1

· rules of inference (determined by **LLL** and Ω)

· a marking definition (determined by Ω and the stategy)

dynamics of the proofs controlled by attaching *conditions* (finite subsets of Ω) to derived formulas

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marking definition:
determines for every line *i* at every stage *s* of a proof whether *i* is unmarked/marked (IN/OUT) in view of
{ the condition of *i*
{ the *Dab*-formulas derived

Rules of inference

(depend on **LLL** and Ω, *not* on the strategy)

PREM If $A \in \Gamma$:

$$\begin{array}{ll} \ldots & \ldots \\ \hline A & \emptyset \end{array}$$

RU If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B$:

$$\begin{array}{ll} A_1 & \Delta_1 \\ \ldots & \ldots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \end{array}$$

RC If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B \check{\vee} Dab(\Theta)$

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for example:

$p,\ p \supset q \vdash_{\mathbf{CLuN}} q$

$p,\ {\sim}p \vee q \vdash_{\mathbf{CLuN}} q \vee (p \wedge {\sim}p)$

Marking definitions

proceed in terms of the *minimal Dab-formulas* derived at the stage of the proof

$Dab(\Delta)$ is a *minimal Dab-formula* at stage s iff, at s,
$Dab(\Delta)$ is derived with condition $\emptyset$
no $Dab(\Delta')$ with $\Delta' \subset \Delta$ is derived with condition $\emptyset$

Marking Definition for Reliability

where $Dab(\Delta_1), \ldots, Dab(\Delta_n)$ are the minimal *Dab*-formulas derived on condition $\emptyset$ at stage s,

$$U_s(\Gamma) = \Delta_1 \cup \ldots \cup \Delta_n$$

Definition

where Δ is the condition of line i,
line i is marked at stage s iff $\Delta \cap U_s(\Gamma) \neq \emptyset$

Marking Definition for Minimal Abnormality

where $Dab(\Delta_1), \ldots, Dab(\Delta_n)$ are the minimal *Dab*-formulas derived on condition $\emptyset$ at stage s,

$\Phi_s(\Gamma)$:
the minimal choice sets of $\{Dab(\Delta_1), \ldots, Dab(\Delta_n)\}$

the minimal sets of abnormalities that should be true
in order for all *Dab*-formulas derived at stage s to be true

Definition
where A is the formula and Δ is the condition of line i, line i is marked at stage s iff,

(i) there is no $\varphi \in \Phi_s(\Gamma)$ such that $\varphi \cap \Delta = \emptyset$, *or*
(ii) for some $\varphi \in \Phi_s(\Gamma)$, there is no line at which A is derived on a condition Θ for which $\varphi \cap \Theta = \emptyset$

Marking Definition for the Simple strategy

Definition
where Δ is the condition of line i,
line i is marked at stage s iff some $A \in \Delta$ is derived on condition $\emptyset$

only suitable iff, for all Γ,

$$\Gamma \vdash_{\mathbf{LLL}} Dab(\Delta) \quad \text{iff} \quad \text{for some } A \in \Delta, \Gamma \vdash_{\mathbf{LLL}} A.$$

in other words: if $Dab(\Delta)$ is derived on condition $\emptyset$,
then, for some $A \in \Delta$, A is derivable on condition $\emptyset$

in this case, Reliability and Minimal Abnormality both coincide with the Simple Strategy

Derivability at a stage vs. **final derivability**

idea: *A* derived on an unmarked line *i*
and the proof is *stable* with respect to *i*
(line *i* not marked in any extension)

stability concerns a specific line

Extremely simple propositional example for $\mathbf{CLuN}^r$ (and $\mathbf{CLuN}^m$)

1	$(p \wedge q) \wedge t$	PREM	$\emptyset$
2	$\sim p \vee r$	PREM	$\emptyset$
3	$\sim q \vee s$	PREM	$\emptyset$
4	$\sim p \vee \sim q$	PREM	$\emptyset$
5	$t \supset \sim p$	PREM	$\emptyset$

Extremely simple propositional example for $\mathbf{CLuN}^r$ (and $\mathbf{CLuN}^m$)

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4	$\sim p \vee \sim q$	PREM	$\emptyset$
5	$t \supset \sim p$	PREM	$\emptyset$
6	r	1, 2; RC	$\{p \wedge \sim p\}$

Extremely simple propositional example for $\mathbf{CLuN}^r$ (and $\mathbf{CLuN}^m$)

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3	$\sim q \vee s$	PREM	$\emptyset$
4	$\sim p \vee \sim q$	PREM	$\emptyset$
5	$t \supset \sim p$	PREM	$\emptyset$
6	r	1, 2; RC	$\{p \wedge \sim p\}$
7	s	1, 3; RC	$\{q \wedge \sim q\}$

Extremely simple propositional example for **CLuN**^{r} (and **CLuN**^{m})

1	$(p \wedge q) \wedge t$	PREM	$\emptyset$	
2	$\sim p \vee r$	PREM	$\emptyset$	
3	$\sim q \vee s$	PREM	$\emptyset$	
4	$\sim p \vee \sim q$	PREM	$\emptyset$	
5	$t \supset \sim p$	PREM	$\emptyset$	
6	r	1, 2; RC	$\{p \wedge \sim p\}$	$\surd$
7	s	1, 3; RC	$\{q \wedge \sim q\}$	$\surd$
8	$(p \wedge \sim p) \vee (q \wedge \sim q)$	1, 4; RU	$\emptyset$	

Extremely simple propositional example for $\mathbf{CLuN}^r$ (and $\mathbf{CLuN}^m$)

1	$(p \wedge q) \wedge t$	PREM	$\emptyset$	
2	$\sim p \vee r$	PREM	$\emptyset$	
3	$\sim q \vee s$	PREM	$\emptyset$	
4	$\sim p \vee \sim q$	PREM	$\emptyset$	
5	$t \supset \sim p$	PREM	$\emptyset$	
6	r	1, 2; RC	$\{p \wedge \sim p\}$	$\surd$
7	s	1, 3; RC	$\{q \wedge \sim q\}$	
8	$(p \wedge \sim p) \vee (q \wedge \sim q)$	1, 4; RU	$\emptyset$	
9	$p \wedge \sim p$	1, 5; RU	$\emptyset$	

Extremely simple propositional example for $\mathbf{CLuN}^r$ (and $\mathbf{CLuN}^m$)

1	$(p \wedge q) \wedge t$	PREM	$\emptyset$	
2	$\sim p \vee r$	PREM	$\emptyset$	
3	$\sim q \vee s$	PREM	$\emptyset$	
4	$\sim p \vee \sim q$	PREM	$\emptyset$	
5	$t \supset \sim p$	PREM	$\emptyset$	
6	r	1, 2; RC	$\{p \wedge \sim p\}$	$\surd$
7	s	1, 3; RC	$\{q \wedge \sim q\}$	
8	$(p \wedge \sim p) \vee (q \wedge \sim q)$	1, 4; RU	$\emptyset$	
9	$p \wedge \sim p$	1, 5; RU	$\emptyset$	

nothing interesting happens when the proof is continued
no mark will be removed or added

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$

number of data of each form immaterial: same generalizations derivable from $\{Pa\}$ and from $\{Pa, Pb\}$

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
5	$\forall x(Qx \supset Rx)$	2; RC	$\{!(Qx \supset Rx)\}$

Let $!(A \supset B)$ abbreviate $\exists(A \supset B) \wedge \exists\sim(A \supset B)$

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
5	$\forall x(Qx \supset Rx)$	2; RC	$\{!(Qx \supset Rx)\}$
6	Rd	4, 5; RU	$\{!(Qx \supset Rx)\}$

Let $!(A \supset B)$ abbreviate $\exists(A \supset B) \wedge \exists\sim(A \supset B)$

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge {\sim}Qa) \wedge {\sim}Ra$	PREM	$\emptyset$
2	${\sim}Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge {\sim}Pe$	PREM	$\emptyset$
5	$\forall x(Qx \supset Rx)$	2; RC	$\{!(Qx \supset Rx)\}$
6	Rd	4, 5; RU	$\{!(Qx \supset Rx)\}$
7	$\forall x({\sim}Px \supset Qx)$	2; RC	$\{!({\sim}Px \supset Qx)\}$

Let $!(A \supset B)$ abbreviate $\exists(A \supset B) \wedge \exists{\sim}(A \supset B)$

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
5	$\forall x(Qx \supset Rx)$	2; RC	$\{!(Qx \supset Rx)\}$
6	Rd	4, 5; RU	$\{!(Qx \supset Rx)\}$
7	$\forall x(\sim Px \supset Qx)$	2; RC	$\{!(\sim Px \supset Qx)\}$
8	Qe	4, 7; RU	$\{!(\sim Px \supset Qx)\}$

Let $!(A \supset B)$ abbreviate $\exists(A \supset B) \wedge \exists\sim(A \supset B)$

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$

$\ldots$

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge {\sim}Qa) \wedge {\sim}Ra$	PREM	$\emptyset$
2	${\sim}Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge {\sim}Pe$	PREM	$\emptyset$
...			
9	$\forall x(Px \supset {\sim}Rx)$	1; RC	$\{!(Px \supset {\sim}Rx)\}$

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
9	$\forall x(Px \supset \sim Rx)$	1; RC	$\{!(Px \supset \sim Rx)\}$ $\surd^{10}$
10	$!(Px \supset \sim Rx)$	1, 3; RU	$\emptyset$

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$

...

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
11	$\forall x(Px \supset \sim Qx)$	1; RC	$\{!(Px \supset \sim Qx)\}$

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
11	$\forall x(Px \supset \sim Qx)$	1; RC	$\{!(Px \supset \sim Qx)\}$
12	$\sim Qc$	3, 11; RU	$\{!(Px \supset \sim Qx)\}$

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
11	$\forall x(Px \supset \sim Qx)$	1; RC	$\{!(Px \supset \sim Qx)\}$
12	$\sim Qc$	3, 11; RU	$\{!(Px \supset \sim Qx)\}$
13	$\forall x(Rx \supset Qx)$	2; RC	$\{!(Rx \supset Qx)\}$

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
11	$\forall x(Px \supset \sim Qx)$	1; RC	$\{!(Px \supset \sim Qx)\}$
12	$\sim Qc$	3, 11; RU	$\{!(Px \supset \sim Qx)\}$
13	$\forall x(Rx \supset Qx)$	2; RC	$\{!(Rx \supset Qx)\}$
14	Qc	3, 13; RU	$\{!(Rx \supset Qx)\}$

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
. . .			
11	$\forall x(Px \supset \sim Qx)$	1; RC	$\{!(Px \supset \sim Qx)\}$
12	$\sim Qc$	3, 11; RU	$\{!(Px \supset \sim Qx)\}$
13	$\forall x(Rx \supset Qx)$	2; RC	$\{!(Rx \supset Qx)\}$
14	Qc	3, 13; RU	$\{!(Rx \supset Qx)\}$
15	$\exists x \sim (Px \supset \sim Qx) \vee \exists x \sim (Rx \supset Qx)$	3; RU	$\emptyset$

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
11	$\forall x(Px \supset \sim Qx)$	1; RC	$\{!(Px \supset \sim Qx)\}$
12	$\sim Qc$	3, 11; RU	$\{!(Px \supset \sim Qx)\}$
13	$\forall x(Rx \supset Qx)$	2; RC	$\{!(Rx \supset Qx)\}$
14	Qc	3, 13; RU	$\{!(Rx \supset Qx)\}$
15	$\exists x \sim (Px \supset \sim Qx) \vee \exists x \sim (Rx \supset Qx)$	3; RU	$\emptyset$
16	$\exists x(Px \supset \sim Qx) \wedge \exists x(Rx \supset Qx)$	1, 2; RU	$\emptyset$

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$	
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$	
3	$Pc \wedge Rc$	PREM	$\emptyset$	
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$	
...				
11	$\forall x(Px \supset \sim Qx)$	1; RC	$\{!(Px \supset \sim Qx)\}$	$\surd^{17}$
12	$\sim Qc$	3, 11; RU	$\{!(Px \supset \sim Qx)\}$	$\surd^{17}$
13	$\forall x(Rx \supset Qx)$	2; RC	$\{!(Rx \supset Qx)\}$	$\surd^{17}$
14	Qc	3, 13; RU	$\{!(Rx \supset Qx)\}$	$\surd^{17}$
15	$\exists x{\sim}(Px \supset \sim Qx) \vee \exists x{\sim}(Rx \supset Qx)$	3; RU	$\emptyset$	
16	$\exists x(Px \supset \sim Qx) \wedge \exists x(Rx \supset Qx)$	1, 2; RU	$\emptyset$	
17	$!(Px \supset \sim Qx) \vee !(Rx \supset Qx)$	15, 16; RU	$\emptyset$	

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$

...

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
18	$\forall x(Px \supset Sx)$	4; RC	$\{!(Px \supset Sx)\}$
19	Sa	1, 18; RU	$\{!(Px \supset Sx)\}$

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
18	$\forall x(Px \supset Sx)$	4; RC	$\{!(Px \supset Sx)\}$
19	Sa	1, 18; RU	$\{!(Px \supset Sx)\}$
20	$\exists x \sim (Px \supset Sx) \vee \exists x \sim (Px \supset \sim Sx)$	3; RU	$\emptyset$
21	$\exists x(Px \supset Sx) \wedge \exists x(Px \supset \sim Sx)$	4; RU	$\emptyset$

Extremely simple example for $\mathbf{IL}^r$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$	
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$	
3	$Pc \wedge Rc$	PREM	$\emptyset$	
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$	
...				
18	$\forall x(Px \supset Sx)$	4; RC	$\{!(Px \supset Sx)\}$	$\surd^{22}$
19	Sa	1, 18; RU	$\{!(Px \supset Sx)\}$	$\surd^{22}$
20	$\exists x \sim (Px \supset Sx) \vee \exists x \sim (Px \supset \sim Sx)$	3; RU	$\emptyset$	
21	$\exists x(Px \supset Sx) \wedge \exists x(Px \supset \sim Sx)$	4; RU	$\emptyset$	
22	$!(Px \supset Sx) \vee !(Px \supset \sim Sx)$	20, 21; RU	$\emptyset$	

Outline

Standard format and semantics

$Dab(\Delta)$ is a *minimal Dab-consequence of* Γ:

$\Gamma \vDash_{\mathbf{LLL}} Dab(\Delta)$ and, for all $\Delta' \subset \Delta$, $\Gamma \nvDash_{\mathbf{LLL}} Dab(\Delta')$

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where $Dab(\Delta_1),\ Dab(\Delta_2),\ \ldots$ are the minimal *Dab*-consequences of Γ,

$U(\Gamma) = \Delta_1 \cup \Delta_2 \cup \ldots$

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where $Dab(\Delta_1), \ Dab(\Delta_2), \ \ldots$ are the minimal *Dab*-consequences of Γ,

$U(\Gamma) = \Delta_1 \cup \Delta_2 \cup \ldots$

where M is a **LLL**-model: $Ab(M) = \{A \in \Omega \mid M \models A\}$

Standard format and semantics

Dab(Δ) is a *minimal Dab-consequence of* Γ:

$\Gamma \vDash_{\mathbf{LLL}} Dab(\Delta)$ and, for all $\Delta' \subset \Delta$, $\Gamma \nvDash_{\mathbf{LLL}} Dab(\Delta')$

where $Dab(\Delta_1),\ Dab(\Delta_2),\ \ldots$ are the minimal *Dab*-consequences of Γ,

$U(\Gamma) = \Delta_1 \cup \Delta_2 \cup \ldots$

where M is a **LLL**-model: $Ab(M) = \{A \in \Omega \mid M \models A\}$

the **AL**-semantics selects some **LLL**-models of Γ as **AL**-models *of* Γ

the selection depends on Ω and on the strategy

Reliability

a **LLL**-model M of Γ is *reliable* iff $Ab(M) \subseteq U(\Gamma)$

$\Gamma \vDash_{\mathbf{AL}^r} A$ iff all reliable models of Γ verify A

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Minimal Abnormality

a **LLL**-model M of Γ is *minimally abnormal*
iff
there is no **LLL**-model M' of Γ for which $Ab(M') \subset Ab(M)$

$\Gamma \vDash_{\mathbf{AL}^m} A$ iff all minimally abnormal models of Γ verify A

Reliability

a **LLL**-model M of Γ is *reliable* iff $Ab(M) \subseteq U(\Gamma)$

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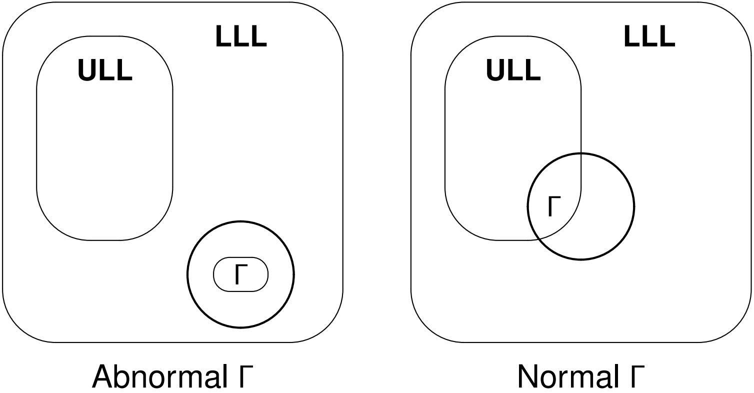
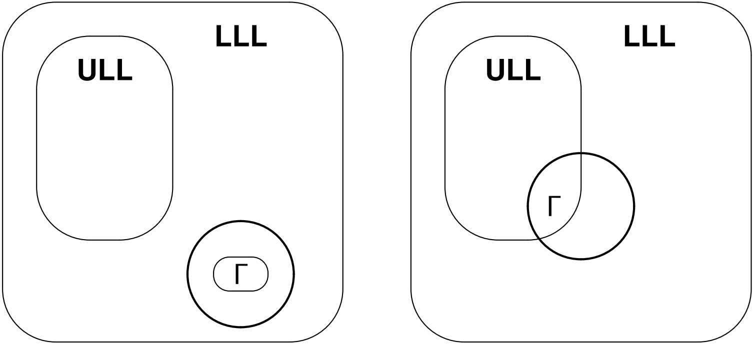
$\Gamma \vDash_{\mathbf{AL}^m} A$ iff all minimally abnormal models of Γ verify A

Simple strategy

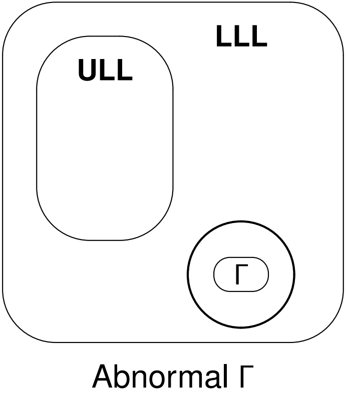
either of the above if the Simple strategy is suitable

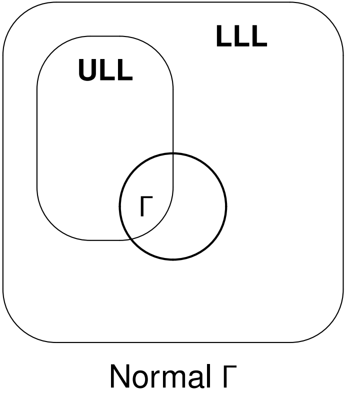

LLL
ULL
Γ
Abnormal Γ
LLL
ULL
Γ
Normal Γ



Abnormal Γ Normal Γ

flip-flop (if Ω not suitably restricted or because of strategy)





flip-flop (if Ω not suitably restricted or because of strategy)

there are no **AL**-models, but only **AL**-models *of some* Γ

Outline

Standard format and metatheory

Theorem
$\Gamma \vDash_{\mathbf{AL}^r} A$ iff $\Gamma \vDash_{\mathbf{LLL}} A \check{\vee} Dab(\Delta)$ and $\Delta \cap U(\Gamma) = \emptyset$ for a finite $\Delta \subset \Omega$.

...

Corollary
$\Gamma \vdash_{\mathbf{AL}^r} A$ iff $\Gamma \vDash_{\mathbf{AL}^r} A$. (Soundness and Completeness)

Standard format and metatheory

Theorem
$\Gamma \vDash_{\mathbf{AL}^r} A$ iff $\Gamma \vDash_{\mathbf{LLL}} A \check{\vee} Dab(\Delta)$ and $\Delta \cap U(\Gamma) = \emptyset$ for a finite $\Delta \subset \Omega$.

...

Corollary
$\Gamma \vdash_{\mathbf{AL}^r} A$ iff $\Gamma \vDash_{\mathbf{AL}^r} A$. (Soundness and Completeness)

Lemma
$M \in \mathcal{M}^m_\Gamma$ iff $M \in \mathcal{M}^{\mathbf{LLL}}_\Gamma$ and $Ab(M) \in \Phi_\Gamma$.

...

Theorem
$\Gamma \vdash_{\mathbf{AL}^m} A$ iff $\Gamma \vDash_{\mathbf{AL}^m} A$. (Soundness and Completeness)

Strong Reassurance (Stopperedness, Smoothness)

if a model of the premisses is not selected, this is justified by the fact that a selected model of the premisses is less abnormal

Strong Reassurance (Stopperedness, Smoothness)

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Theorem
If $M \in \mathcal{M}_\Gamma^{\mathbf{LLL}} - \mathcal{M}_\Gamma^m$, then there is a $M' \in \mathcal{M}_\Gamma^m$ such that $Ab(M') \subset Ab(M)$. (Strong Reassurance for Minimal Abnormality.)

Theorem
If $M \in \mathcal{M}_\Gamma^{\mathbf{LLL}} - \mathcal{M}_\Gamma^r$, then there is a $M' \in \mathcal{M}_\Gamma^r$ such that $Ab(M') \subset Ab(M)$. (Strong Reassurance for Reliability.)

Theorem each of the following obtains:

1. $\mathcal{M}^m_\Gamma \subseteq \mathcal{M}^r_\Gamma$. Hence $Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma)$. •
2. If $A \in \Omega - U(\Gamma)$, then $\check{\sim} A \in Cn_{\mathbf{AL}^r}(\Gamma)$.
3. If $Dab(\Delta)$ is a minimal Dab-consequence of Γ and $A \in \Delta$, then some $M \in \mathcal{M}^m_\Gamma$ verifies A and falsifies all members (if any) of $\Delta - \{A\}$.
4. $\mathcal{M}^m_\Gamma = \mathcal{M}^m_{Cn_{\mathbf{AL}^m}(\Gamma)}$ whence $Cn_{\mathbf{AL}^m}(\Gamma) = Cn_{\mathbf{AL}^m}(Cn_{\mathbf{AL}^m}(\Gamma))$. •(Fixed Point.)
5. $\mathcal{M}^r_\Gamma = \mathcal{M}^r_{Cn_{\mathbf{AL}^r}(\Gamma)}$ whence $Cn_{\mathbf{AL}^r}(\Gamma) = Cn_{\mathbf{AL}^r}(Cn_{\mathbf{AL}^r}(\Gamma))$. •(Fixed Point.)
6. For all $\Delta \subseteq \Omega$, $Dab(\Delta) \in Cn_{\mathbf{AL}}(\Gamma)$ iff $Dab(\Delta) \in Cn_{\mathbf{LLL}}(\Gamma)$. (Immunity.)
7. If $\Gamma \vDash_{\mathbf{AL}} A$ for every $A \in \Gamma'$, and $\Gamma \cup \Gamma' \vDash_{\mathbf{AL}} B$, then $\Gamma \vDash_{\mathbf{AL}} B$. •(Cautious Cut.)
8. If $\Gamma \vDash_{\mathbf{AL}} A$ for every $A \in \Gamma'$, and $\Gamma \vDash_{\mathbf{AL}} B$, then $\Gamma \cup \Gamma' \vDash_{\mathbf{AL}} B$. •(Cautious Monotonicity.)

Theorem each of the following obtains:

1. If Γ is normal, then $\mathcal{M}^{\mathbf{ULL}}_\Gamma = \mathcal{M}^m_\Gamma = \mathcal{M}^r_\Gamma$ whence $Cn_{\mathbf{AL}^r}(\Gamma) = Cn_{\mathbf{AL}^m}(\Gamma) = Cn_{\mathbf{ULL}}(\Gamma)$. •
2. If Γ is abnormal and $\mathcal{M}^{\mathbf{LLL}}_\Gamma \neq \emptyset$, then $\mathcal{M}^{\mathbf{ULL}}_\Gamma \subset \mathcal{M}^m_\Gamma$ and hence $Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma) \subset Cn_{\mathbf{ULL}}(\Gamma)$.
3. $\mathcal{M}^{\mathbf{ULL}}_\Gamma \subseteq \mathcal{M}^m_\Gamma \subseteq \mathcal{M}^r_\Gamma \subseteq \mathcal{M}^{\mathbf{LLL}}_\Gamma$ whence $Cn_{\mathbf{LLL}}(\Gamma) \subseteq Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma) \subseteq Cn_{\mathbf{ULL}}(\Gamma)$. •
4. $\mathcal{M}^r_\Gamma \subset \mathcal{M}^{\mathbf{LLL}}_\Gamma$ iff $\Gamma \cup \{A\}$ is **LLL**-satisfiable for some $A \in \Omega - U(\Gamma)$.
5. $Cn_{\mathbf{LLL}}(\Gamma) \subset Cn_{\mathbf{AL}^r}(\Gamma)$ iff $\mathcal{M}^r_\Gamma \subset \mathcal{M}^{\mathbf{LLL}}_\Gamma$.
6. $\mathcal{M}^m_\Gamma \subset \mathcal{M}^{\mathbf{LLL}}_\Gamma$ iff there is a (possibly infinite) $\Delta \subseteq \Omega$ such that $\Gamma \cup \Delta$ is **LLL**-satisfiable and there is no $\varphi \in \Phi_\Gamma$ for which $\Delta \subseteq \varphi$.
7. If there are $A_1, \ldots, A_n \in \Omega$ ($n \geq 1$) such that $\Gamma \cup \{A_1, \ldots, A_n\}$ is **LLL**-satisfiable and, for every $\varphi \in \Phi_\Gamma$, $\{A_1, \ldots, A_n\} \not\subseteq \varphi$, then $Cn_{\mathbf{LLL}}(\Gamma) \subset Cn_{\mathbf{AL}^m}(\Gamma)$.
8. $Cn_{\mathbf{AL}^m}(\Gamma)$ and $Cn_{\mathbf{AL}^r}(\Gamma)$ are non-trivial iff $Cn_{\mathbf{AL}^m}(\Gamma)$ is non-trivial. • (Reassurance)

Theorem
If $\Gamma \vdash_{\mathbf{AL}} A$, then every **AL**-proof from Γ can be extended in such a way that A is finally derived in it. (Proof Invariance)

…

Outline

Standard format and proof theory: part 2

final derivability:
A derived on unmarked line *i* *and* proof *stable* with respect to *i*

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Note:
- proof in weak sense: correct applications of the rules
- proof in strong sense: establishes *by itself* that $\Gamma \vdash A$

mind applications !

more handy definition (provably equivalent):

Definition

A is *finally derived* from Γ at line *i* of a proof at stage *s* iff *A* is derived at line *i* of a proof P at a stage and every extension of P may be further extended in such a way that line *i* is unmarked.

Definition

$\Gamma \vdash_{\mathbf{AL}} A$ (*A* is *finally* **AL**-*derivable* from Γ) iff *A* is finally derived at a line of a proof from Γ.

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$\Gamma \vdash_{\mathbf{AL}} A$ (*A* is *finally* **AL**-*derivable* from Γ) iff *A* is finally derived at a line of a proof from Γ.

game-theoretic interpretation (variants possible)

burden of proof on proponent / burden of defeating it on opponent

propositional fragment:
final derivability from finite premise set: decidable

full predicative logics: not even positive test

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even at the predicative level, there are *criteria* for final derivability

- blocks
- tableau methods
- prospective dynamics
  (proof procedure that provides criterion)
  solves the problem *whenever it is solvable*

What if no criterion applies?

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(1) *Does the dynamics of the proofs go anywhere?*

in view of the block analysis of proofs (and the block semantics):

· a stage of a proof provides a certain insight in the premises

· every step of the proof is informative or non-informative

  · if informative: more insight in the premises gained

  · if non-informative: no insight lost (sq)

· sensible proofs converge toward maximal insight

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(2) *application context may not require final derivability*

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certain abnormalities located
clear idea for replacement
may be sufficient to launch hypothesis for replacement

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cf. Clausius

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(2) *application context may not require final derivability*

example 1: inconsistency-adaptive

certain abnormalities located
clear idea for replacement
may be sufficient to launch hypothesis for replacement

cf. Frege's set theory (Russell paradox / Curry paradox)
cf. Clausius

example 2: inductive generalization + background knowledge

certain abnormalities located
abnormalities narrowed down in view of personal constraints etc.
clear idea for theory
may be sufficient to launch theory (obviously defeasible)

aim of applications: to arrive at sensible hypothetical proposals aim of applications: to arrive at sensible hypothetical proposals

in that respect $Cn_{\mathbf{AL}}(\Gamma)$ is ideal

study it to show that the applied mechanism
is coherent and conceptually sound
even if $Cn_{\mathbf{AL}}(\Gamma)$ is beyond reach

More General Framework

content-guided formal approach to problem solving

prospective procedure handling:
- set of declarative (adaptive) logics (set itself defeasible)
- erotetic logic to handle problems
- external means (oracle, other theories, . . .)

procedure guides observation and experiment

allows to consider *all* knowledge as defeasible,
methods and logic included

content-guidance can be demonstrated / further studied

References

see the urls listed on the font page:

`http://logica.UGent.be/dirk/`
papers by Diderik

`http://logica.UGent.be/centrum/`
papers by Ghent Centre (1995–)

`http://logica.UGent.be/adlog/`
specific papers on adaptive logics (needs updating)

Questions?