

Content-Guidance in Formal Problem Solving Processes

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Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

Outline

Some Background

Elements of a psp backbone

procedure

rules

instructions

marking

An Example of the Backbone

Adaptive logics

Some Extra-Logical Extensions

Comments

Some Background

Elements of a psp backbone

procedure

rules

instructions

marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Some Background (1)

- ▶ Vienna Circle: (degenerated to) a priori methodology
- ▶ Historicism (Kuhn, . . .): relativism
- ▶ today: scientific problem-solving is **content-guided**
“what we have learned, including what we have learned about how to learn” (Shapere)

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

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all possibilities / historicism

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

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“what we have learned, including what we have learned about how to learn” (Shapere)

all possibilities / historicism

Aim of this paper:

- ▶ formal approach to problem solving
- ▶ in which there is large room for content-guidance

(\Rightarrow example)

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

Some Background (2)

direct sources of inspiration:

- ▶ philosophy of science/epistemology:
 - ▶ problem solving: Kuhn, Laudan, *Nickles*, ...
 - ▶ 'contextual': specific certainties, relevant data, methodological instructions, ... for each problem
- ▶ logic:
 - ▶ procedures
 - ▶ prospective dynamics
 - ▶ adaptive logics
 - ▶ erotetic logic (varying on Wiśniewski)

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

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problem determined by (changing) *constraints*

- ▶ conditions on solution
- ▶ methodological instructions / heuristics / examples
- ▶ certainties (conceptual system, ...)

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

Some Background (3)

some elements of the plot:

- ▶ aim: *explication* for problem solving processes (psps)

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

Some Background (3)

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- ▶ aim: *explication* for problem solving processes (psps)
- ▶ *backbone*:
solve $\{?\{A, \sim A\}\}$ by **CL**-deriving A or $\sim A$ from Γ

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

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- ▶ *backbone*:
solve $\{?\{A, \sim A\}\}$ by **CL**-deriving A or $\sim A$ from Γ
- ▶ empirical means: observation and experiment guided by psp

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

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- ▶ new available information (not originally seen as relevant) guided by psp

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

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- ▶ empirical means: observation and experiment guided by psp
- ▶ new available information (not originally seen as relevant) guided by psp
- ▶ adaptive logics:
corrective: handling inconsistency (and similar), ambiguity, vagueness, ...
ampliative: inductive generalization, abduction, ...
control defeasible inferences by conditions and marking

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

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- ▶ empirical means: observation and experiment guided by psp
- ▶ new available information (not originally seen as relevant) guided by psp
- ▶ adaptive logics:
corrective: handling inconsistency (and similar), ambiguity, vagueness, ...
ampliative: inductive generalization, abduction, ...
control defeasible inferences by conditions and marking
- ▶ model-based reasoning (mainly future research)

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

Some Background (4)

from the general plot (consolutions?)

- ▶ in the end all knowledge defeasible

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

Some Background (4)

from the general plot (consolutions?)

- ▶ in the end all knowledge defeasible
- ▶ going through different 'contexts'
⇒ { conceptual change
generation of new concepts

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Some Background (4)

from the general plot (consolations?)

- ▶ in the end all knowledge defeasible
- ▶ going through different 'contexts'
 - ⇒ { conceptual change
 - { generation of new concepts
- ▶ many ampliative mechanisms (abduction overestimated)

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Some Background (4)

from the general plot (consolutions?)

- ▶ in the end all knowledge defeasible
- ▶ going through different 'contexts'
 - ⇒ { conceptual change
 - generation of new concepts
- ▶ many ampliative mechanisms (abduction overestimated)
- ▶ independent of ontological debates

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Elements of a psp backbone

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

lines of psp

- ▶ problem lines: example $\{?\{p \vee q, \sim p \vee q\}, ?\{r, \sim r\}\}$
problem = non-empty set of questions
- ▶ declarative lines
 - conditional: $[B_1, \dots, B_n] A$
 - unconditional: $[\emptyset] A$, viz. A

Outline

Some Background

Elements of a psp backbone

procedure

rules

instructions

marking

An Example of the Backbone

Adaptive logics

Some Extra-Logical Extensions

Comments

Some Background

Elements of a psp backbone

procedure

rules

instructions

marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

psp

- ▶ stage of a psp: sequence of lines
- ▶ psp: chain of stages
- ▶ next stage: add new line + apply marking definitions
- ▶ governed by *procedure*

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

psp

- ▶ stage of a psp: sequence of lines
- ▶ psp: chain of stages
- ▶ next stage: add new line + apply marking definitions
- ▶ governed by *procedure*

procedure (set of instructions)

- ▶ *rule of inference*: to derive A from B_1, \dots, B_n
- ▶ *instruction*:
rule of inference + permissions/obligations
permissions/obligations depend on the present stage
(lines + marks)
- ▶ below: instructions (including the rules)

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

varying on Smullyan

a	a_1	a_2		b	b_1	b_2
$A \wedge B$	A	B		$\sim(A \wedge B)$	$*A$	$*B$
$A \equiv B$	$A \supset B$	$B \supset A$		$\sim(A \equiv B)$	$\sim(A \supset B)$	$\sim(B \supset A)$
$\sim(A \vee B)$	$*A$	$*B$		$A \vee B$	A	B
$\sim(A \supset B)$	A	$*B$		$A \supset B$	$*A$	B
$\sim\sim A$	A	A				

complement of A : $*A = B$ if $A = \sim B$; otherwise $*A = \sim A$

Outline

Some Background

Elements of a psp backbone

procedure

rules

instructions

marking

An Example of the Backbone

Adaptive logics

Some Extra-Logical Extensions

Comments

Some Background

Elements of a psp backbone

procedure

rules

instructions

marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

a	a_1	a_2		b	b_1	b_2
$A \wedge B$	A	B		$\sim(A \wedge B)$	$*A$	$*B$
$A \equiv B$	$A \supset B$	$B \supset A$		$\sim(A \equiv B)$	$\sim(A \supset B)$	$\sim(B \supset A)$
$\sim(A \vee B)$	$*A$	$*B$		$A \vee B$	A	B
$\sim(A \supset B)$	A	$*B$		$A \supset B$	$*A$	B
$\sim\sim A$	A	A				

formula analysing rules:

$$\frac{[\Delta] a}{[\Delta] a_1 \quad [\Delta] a_2} \qquad \frac{[\Delta] b}{[\Delta \cup \{ *b_2 \}] b_1 \quad [\Delta \cup \{ *b_1 \}] b_2}$$

α	α_1	α_2		β	β_1	β_2
$A \wedge B$	A	B		$\sim(A \wedge B)$	$*A$	$*B$
$A \equiv B$	$A \supset B$	$B \supset A$		$\sim(A \equiv B)$	$\sim(A \supset B)$	$\sim(B \supset A)$
$\sim(A \vee B)$	$*A$	$*B$		$A \vee B$	A	B
$\sim(A \supset B)$	A	$*B$		$A \supset B$	$*A$	B
$\sim\sim A$	A	A				

formula analysing rules:

$$\frac{[\Delta] \alpha}{[\Delta] \alpha_1 \quad [\Delta] \alpha_2} \qquad \frac{[\Delta] \beta}{[\Delta \cup \{*\beta_2\}] \beta_1 \quad [\Delta \cup \{*\beta_1\}] \beta_2}$$

condition analysing rules:

$$\frac{[\Delta \cup \{\alpha\}] A}{[\Delta \cup \{\alpha_1, \alpha_2\}] A} \qquad \frac{[\Delta \cup \{\beta\}] A}{[\Delta \cup \{\beta_1\}] A \quad [\Delta \cup \{\beta_2\}] A}$$

a	a_1	a_2		b	b_1	b_2
$A \wedge B$	A	B		$\sim(A \wedge B)$	$*A$	$*B$
$A \equiv B$	$A \supset B$	$B \supset A$		$\sim(A \equiv B)$	$\sim(A \supset B)$	$\sim(B \supset A)$
$\sim(A \vee B)$	$*A$	$*B$		$A \vee B$	A	B
$\sim(A \supset B)$	A	$*B$		$A \supset B$	$*A$	B
$\sim\sim A$	A	A				

positive part relation

1. $pp(A, A)$.
2. $pp(A, a)$ if $pp(A, a_1)$ or $pp(A, a_2)$.
3. $pp(A, b)$ if $pp(A, b_1)$ or $pp(A, b_2)$.
4. If $pp(A, B)$ and $pp(B, C)$, then $pp(A, C)$.

Outline

Some Background

Elements of a psp backbone

procedure

rules

instructions

marking

An Example of the Backbone

Adaptive logics

Some Extra-Logical Extensions

Comments

Some Background

Elements of a psp backbone

procedure

rules

instructions

marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

The instructions

Main Start a psp with the line:

1 $\{?\{M, \sim M\}\}$

Main

Some Background

Elements of a psp
backbone

procedure
rules

instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

The instructions

Main Start a psp with the line:

1 $\{?\{M, \sim M\}\}$

Main

Target If P is the problem of an unmarked problem line,
and A is a direct answer of a member of P, then
one may add:

k $[A] A$

Target

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

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Main Start a psp with the line:

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Target If P is the problem of an unmarked problem line, and A is a direct answer of a member of P , then one may add:

k $[A] A$

Target

Prem If A is an unmarked target, $B \in \Gamma$, and $pp(A, B)$, then one may add:

k B

Prem

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Formula analysing rules:

$$\frac{[\Delta] \alpha}{[\Delta] \alpha_1 \quad [\Delta] \alpha_2} \qquad \frac{[\Delta] \beta}{[\Delta \cup \{*\beta_2\}] \beta_1 \quad [\Delta \cup \{*\beta_1\}] \beta_2}$$

FAR If C is an unmarked target, $[\Delta] A$ is the formula of an unmarked line i , $[\Delta] A / [\Delta \cup \Delta'] B$ is a formula analysing rule, and $\text{pp}(C, B)$, then one may add:

$$k \qquad [\Delta \cup \Delta'] B \qquad i; R$$

in which R is the name of the formula analysing rule.

Some Background

Elements of a psp backbone

procedure

rules

instructions

marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Condition analysing rules:

$$\frac{[\Delta \cup \{\alpha\}] A}{[\Delta \cup \{\alpha_1, \alpha_2\}] A} \quad \frac{[\Delta \cup \{\beta\}] A}{[\Delta \cup \{\beta_1\}] A \quad [\Delta \cup \{\beta_2\}] A}$$

CAR If A is an unmarked target, $[\Delta \cup \{B\}] A$ is the formula of an unmarked line i , and $[\Delta \cup \{B\}] A / [\Delta \cup \Delta'] A$ is a condition analysing rule, then one may add:

$$k \quad [\Delta \cup \Delta'] A \quad i; R$$

in which R is the name of the condition analysing rule.

Some Background

Elements of a psp backbone

procedure

rules

instructions

marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Eliminate some problems without answering them:

EM0 If $[\Delta \cup \{*A\}] A$ is the formula of a line i that is neither R-marked nor I-marked, then one may add:

k $[\Delta] A$ i ; EM0

EM If A is an unmarked target, $[\Delta \cup \{B\}] A$ and $[\Delta' \cup \{\sim B\}] A$ are the respective formulas of the unmarked or only D-marked lines i and j , and $\Delta \subseteq \Delta'$ or $\Delta' \subseteq \Delta$, then one may add:

k $[\Delta \cup \Delta'] A$ i, j ; EM

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

eliminate solved questions from a problem and summarize remaining problems (and paths):

Trans If A is an unmarked target, and $[\Delta \cup \{B\}] A$ and $[\Delta'] B$ are the respective formulas of the at most S -marked lines i and j , then one may add:

$k \quad [\Delta \cup \Delta'] A \quad i, j; \text{Trans}$

Some Background

Elements of a psp backbone

procedure

rules

instructions

marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

handle derived problems:

DP If A is an unmarked target from problem line i and $[B_1, \dots, B_n]$ A is the formula of an unmarked line j , then one may add:

$$k \quad \{?\{B_1, \sim B_1\}, \dots, ?\{B_n, \sim B_n\}\} \quad i, j; \text{ DP}$$

Remark:

no instruction for applying EFQ

in view of the intended applications
(deriving predictions, explanations, etc.)

Some Background

Elements of a psp
backbone

procedure

rules

instructions

marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

Outline

Some Background

Elements of a psp backbone

procedure

rules

instructions

marking

An Example of the Backbone

Adaptive logics

Some Extra-Logical Extensions

Comments

Some Background

Elements of a psp backbone

procedure

rules

instructions

marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Marking definitions

redundant lines are R-marked:

Definition

An at most S-marked declarative line i that has $[\Delta] A$ as its formula is R-marked at a stage iff, at that stage, $[\Theta] A$ is the formula of a line for some $\Theta \subset \Delta$.

Definition

An unmarked problem line i is R-marked at a stage iff, at that stage, a direct answer A of a question of line i is the formula of a line.

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

- target from a problem line
- resolution line
- direct target from
- target sequence
- grounded target sequence

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

inoperative lines are I-marked (not useful for extant problem):

Definition

An at most S-marked target line that has $[A] A$ as its formula is I-marked at a stage iff every problem line from which A is a target is marked at that stage.

Definition

An at most S-marked resolution line of which $[\Delta^1] A^1$ is the formula and $\Delta^1 \neq \emptyset$ is *I-marked* at a stage iff, at that stage, for every grounded target sequence

$\langle [\Delta^n] A^n, \dots, [\Delta^1] A^1 \rangle$,

- (i) some target $[A^i] A^i$ ($1 \leq i \leq n$) is marked, or
- (ii) $\{A^n, \dots, A^1\} \cap \Delta^1 \neq \emptyset$, or
- (iii) $\Delta^1 \cup \dots \cup \Delta^n \cup \Gamma_s^\circ$ is flatly inconsistent.

Definition

An unmarked problem line is I-marked iff no unmarked resolution line generates it.

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Dead end lines are D-marked (no further action from such line)

- A is a *dead end* (A is literal and not positive part of premise)
- CAR-descendant of $[\Delta] B$

Definition

An at most S-marked resolution line with formula $[\Delta] A$ is D-marked at a stage iff some $B \in \Delta$ is a dead end or, at that stage, all CAR-descendants of $[\Delta] A$ occur in the psp and are D-marked.

Definition

An at most S-marked target line with formula $[A] A$ is D-marked at a stage iff A is a dead end or no further action can be taken in view of target A .

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

for all consistent Γ :

if $\Gamma \vdash A$, then the procedure applied to Γ and $\{?\{A, \sim A\}\}$ results in the answer A ,

and

if $\Gamma \not\vdash A$, then the procedure applied to Γ and $\{?\{A, \sim A\}\}$ stops without the main problem being answered or results in the answer $\sim A$.

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Speed up the procedure by S-marks

- Γ_s° union of the Γ and of the set of the conditionless formulas that occur at stage s of the psp

Definition

A R-unmarked resolution line in which $[\Delta^1] A^1$ is derived is S-marked iff

- $\Delta^1 \cap \Gamma_s^\circ \neq \emptyset$, or
- for some target sequence $\langle [\Delta^n] A^n, \dots, [\Delta^1] A^1 \rangle$, $\{A^n\} \cup \Delta^1$ is flatly inconsistent whereas Δ^1 is not flatly inconsistent, or
- $\Delta_1 \subset \Delta^n \cup \dots \cup \Delta^2$ for some target sequence $\langle [\Delta^n] A^n, \dots, [\Delta^1] A^1 \rangle$.

instruction: operate on S-marked lines before doing anything else

Some Background

Elements of a psp
backbone

procedure

rules

instructions

marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

An Example of the Backbone

main problem: $\{p \vee q, \sim(p \vee q)\}$

premise set:

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

logic: **CL**⁻ + an erotetic logic (fixed by the procedure)

procedure: prospective dynamics + problems

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1 $\{?\{p \vee q, \sim(p \vee q)\}\}$ Main

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1 $\{?\{p \vee q, \sim(p \vee q)\}\}$

2 $[\sim(p \vee q)] \sim(p \vee q)$

Main

Target

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1 $\{?\{p \vee q, \sim(p \vee q)\}\}$

2 $[\sim(p \vee q)] \sim(p \vee q)$

3 $[\sim p, \sim q] \sim(p \vee q)$

Main

Target

2; C \sim \vee E

D³

D³

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1 $\{?\{p \vee q, \sim(p \vee q)\}\}$

2 $[\sim(p \vee q)] \sim(p \vee q)$

3 $[\sim p, \sim q] \sim(p \vee q)$

4 $[p \vee q] p \vee q$

Main

Target

2; C \sim \vee E

Target

D³

D³

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1 $\{?\{p \vee q, \sim(p \vee q)\}\}$

2 $[\sim(p \vee q)] \sim(p \vee q)$

3 $[\sim p, \sim q] \sim(p \vee q)$

4 $[p \vee q] p \vee q$

5 $[p] p \vee q$

Main

Target D^3

2; $C\sim\vee E$ D^3

Target

4; $C\vee E$ D^5

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1 $\{?\{p \vee q, \sim(p \vee q)\}\}$

2 $[\sim(p \vee q)] \sim(p \vee q)$

3 $[\sim p, \sim q] \sim(p \vee q)$

4 $[p \vee q] p \vee q$

5 $[p] p \vee q$

6 $[q] p \vee q$

Main

Target

2; C \sim \vee E

Target

4; C \vee E

4; C \vee E

D³

D³

D⁵

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1 $\{?\{p \vee q, \sim(p \vee q)\}\}$

2 $[\sim(p \vee q)] \sim(p \vee q)$

3 $[\sim p, \sim q] \sim(p \vee q)$

4 $[p \vee q] p \vee q$

5 $[p] p \vee q$

6 $[q] p \vee q$

7 $\{?\{q, \sim q\}\}$

Main

Target D^3

2; $C\sim\vee E$ D^3

Target

4; $C\vee E$ D^5

4; $C\vee E$

4, 6; DP

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
2	$[\sim(p \vee q)] \sim(p \vee q)$	Target	D ³
3	$[\sim p, \sim q] \sim(p \vee q)$	2; C \sim \vee E	D ³
4	$[p \vee q] p \vee q$	Target	
5	$[p] p \vee q$	4; C \vee E	D ⁵
6	$[q] p \vee q$	4; C \vee E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q] q$	Target	

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
2	$[\sim(p \vee q)] \sim(p \vee q)$	Target	D ³
3	$[\sim p, \sim q] \sim(p \vee q)$	2; C \sim \vee E	D ³
4	$[p \vee q] p \vee q$	Target	
5	$[p] p \vee q$	4; C \vee E	D ⁵
6	$[q] p \vee q$	4; C \vee E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q] q$	Target	
9	$(q \vee u) \supset (\sim t \vee q)$	Prem	

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
2	$[\sim(p \vee q)] \sim(p \vee q)$	Target	D ³
3	$[\sim p, \sim q] \sim(p \vee q)$	2; C \sim \vee E	D ³
4	$[p \vee q] p \vee q$	Target	
5	$[p] p \vee q$	4; C \vee E	D ⁵
6	$[q] p \vee q$	4; C \vee E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q] q$	Target	
9	$(q \vee u) \supset (\sim t \vee q)$	Prem	
10	$[q \vee u] \sim t \vee q$	9; \supset E	

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
2	$[\sim(p \vee q)] \sim(p \vee q)$	Target	D ³
3	$[\sim p, \sim q] \sim(p \vee q)$	2; C \sim \vee E	D ³
4	$[p \vee q] p \vee q$	Target	
5	$[p] p \vee q$	4; C \vee E	D ⁵
6	$[q] p \vee q$	4; C \vee E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q] q$	Target	
9	$(q \vee u) \supset (\sim t \vee q)$	Prem	
10	$[q \vee u] \sim t \vee q$	9; \supset E	
11	$[q] \sim t \vee q$	10; C \vee E	

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
2	$[\sim(p \vee q)] \sim(p \vee q)$	Target	D ³
3	$[\sim p, \sim q] \sim(p \vee q)$	2; C \sim VE	D ³
4	$[p \vee q] p \vee q$	Target	
5	$[p] p \vee q$	4; C \vee E	D ⁵
6	$[q] p \vee q$	4; C \vee E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q] q$	Target	
9	$(q \vee u) \supset (\sim t \vee q)$	Prem	
10	$[q \vee u] \sim t \vee q$	9; \supset E	
11	$[q] \sim t \vee q$	10; C \vee E	
12	$[q, t] q$	11; \vee E	

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
2	$[\sim(p \vee q)] \sim(p \vee q)$	Target	D ³
3	$[\sim p, \sim q] \sim(p \vee q)$	2; C \sim VE	D ³
4	$[p \vee q] p \vee q$	Target	
5	$[p] p \vee q$	4; C \vee E	D ⁵
6	$[q] p \vee q$	4; C \vee E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q] q$	Target	
9	$(q \vee u) \supset (\sim t \vee q)$	Prem	
10	$[q \vee u] \sim t \vee q$	9; \supset E	
11	$[q] \sim t \vee q$	10; C \vee E	
12	$[q, t] q$	11; \vee E	I ¹²
13	$[u] \sim t \vee q$	10; C \vee E	

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
2	$[\sim(p \vee q)] \sim(p \vee q)$	Target	D ³
3	$[\sim p, \sim q] \sim(p \vee q)$	2; C \sim VE	D ³
4	$[p \vee q] p \vee q$	Target	
5	$[p] p \vee q$	4; C \vee E	D ⁵
6	$[q] p \vee q$	4; C \vee E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q] q$	Target	
9	$(q \vee u) \supset (\sim t \vee q)$	Prem	
10	$[q \vee u] \sim t \vee q$	9; \supset E	
11	$[q] \sim t \vee q$	10; C \vee E	
12	$[q, t] q$	11; \vee E	I ¹²
13	$[u] \sim t \vee q$	10; C \vee E	
14	$[u, t] q$	13; \vee E	

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
2	$[\sim(p \vee q)] \sim(p \vee q)$	Target	D ³
3	$[\sim p, \sim q] \sim(p \vee q)$	2; C \sim VE	D ³
4	$[p \vee q] p \vee q$	Target	
5	$[p] p \vee q$	4; C \vee E	D ⁵
6	$[q] p \vee q$	4; C \vee E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q] q$	Target	
9	$(q \vee u) \supset (\sim t \vee q)$	Prem	
10	$[q \vee u] \sim t \vee q$	9; \supset E	
11	$[q] \sim t \vee q$	10; C \vee E	
12	$[q, t] q$	11; \vee E	I ¹²
13	$[u] \sim t \vee q$	10; C \vee E	
14	$[u, t] q$	13; \vee E	
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
2	$[\sim(p \vee q)] \sim(p \vee q)$	Target	D ³
3	$[\sim p, \sim q] \sim(p \vee q)$	2; C \sim VE	D ³
4	$[p \vee q] p \vee q$	Target	
5	$[p] p \vee q$	4; C \vee E	D ⁵
6	$[q] p \vee q$	4; C \vee E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q] q$	Target	
9	$(q \vee u) \supset (\sim t \vee q)$	Prem	
10	$[q \vee u] \sim t \vee q$	9; \supset E	
11	$[q] \sim t \vee q$	10; C \vee E	
12	$[q, t] q$	11; \vee E	I ¹²
13	$[u] \sim t \vee q$	10; C \vee E	
14	$[u, t] q$	13; \vee E	
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	
16	$[t] t$	Target	

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
2	$[\sim(p \vee q)] \sim(p \vee q)$	Target	D ³
3	$[\sim p, \sim q] \sim(p \vee q)$	2; C \sim VE	D ³
4	$[p \vee q] p \vee q$	Target	
5	$[p] p \vee q$	4; C \vee E	D ⁵
6	$[q] p \vee q$	4; C \vee E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q] q$	Target	
9	$(q \vee u) \supset (\sim t \vee q)$	Prem	
10	$[q \vee u] \sim t \vee q$	9; \supset E	
11	$[q] \sim t \vee q$	10; C \vee E	
12	$[q, t] q$	11; \vee E	I ¹²
13	$[u] \sim t \vee q$	10; C \vee E	
14	$[u, t] q$	13; \vee E	
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	
16	$[t] t$	Target	

1, 6, 14–16

[61 80 117]

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1 $\{?\{p \vee q, \sim(p \vee q)\}\}$

Main

...

6 $[q] p \vee q$

4; $\text{C}\vee\text{E}$

...

14 $[u, t] q$

13; $\vee\text{E}$

15 $\{?\{u, \sim u\}, ?\{t, \sim t\}\}$

8, 14; DP

16 $[t] t$

Target

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
...		
6	$[q] p \vee q$	4; $\text{C}\vee\text{E}$
...		
14	$[u, t] q$	13; $\vee\text{E}$
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP
16	$[t] t$	Target
17	$(r \wedge t) \vee s$	Prem

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
...		
6	$[q] p \vee q$	4; $\text{C}\vee\text{E}$
...		
14	$[u, t] q$	13; $\vee\text{E}$
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP
16	$[t] t$	Target
17	$(r \wedge t) \vee s$	Prem
18	$[\sim s] r \wedge t$	17; $\vee\text{E}$

[Some Background](#)

[Elements of a psp backbone](#)

[procedure](#)

[rules](#)

[instructions](#)

[marking](#)

[An Example of the Backbone](#)

[Adaptive logics](#)

[Extra-Logical Extensions](#)

[Comments](#)

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
...		
6	$[q] p \vee q$	4; $\text{C}\vee\text{E}$
...		
14	$[u, t] q$	13; $\vee\text{E}$
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP
16	$[t] t$	Target
17	$(r \wedge t) \vee s$	Prem
18	$[\sim s] r \wedge t$	17; $\vee\text{E}$
19	$[\sim s] t$	18; $\wedge\text{E}$

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
...		
6	$[q] p \vee q$	4; $\text{C}\vee\text{E}$
...		
14	$[u, t] q$	13; $\vee\text{E}$
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP
16	$[t] t$	Target
17	$(r \wedge t) \vee s$	Prem
18	$[\sim s] r \wedge t$	17; $\vee\text{E}$
19	$[\sim s] t$	18; $\wedge\text{E}$
20	$\{?\{s, \sim s\}\}$	16, 19; DP

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
...		
6	$[q] p \vee q$	4; $\text{C}\vee\text{E}$
...		
14	$[u, t] q$	13; $\vee\text{E}$
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP
16	$[t] t$	Target
17	$(r \wedge t) \vee s$	Prem
18	$[\sim s] r \wedge t$	17; $\vee\text{E}$
19	$[\sim s] t$	18; $\wedge\text{E}$
20	$\{?\{s, \sim s\}\}$	16, 19; DP
21	$[\sim s] \sim s$	Target

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
...			
6	$[q] p \vee q$	4; C \vee E	
...			
14	$[u, t] q$	13; \vee E	
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	
16	$[t] t$	Target	
17	$(r \wedge t) \vee s$	Prem	
18	$[\sim s] r \wedge t$	17; \vee E	
19	$[\sim s] t$	18; \wedge E	S ²²
20	$\{?\{s, \sim s\}\}$	16, 19; DP	R ²²
21	$[\sim s] \sim s$	Target	R ²²
22	$\sim s$	Prem	

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
...			
6	$[q] p \vee q$	4; C \vee E	
...			
14	$[u, t] q$	13; \vee E	S ²³
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	R ²³
16	$[t] t$	Target	R ²³
17	$(r \wedge t) \vee s$	Prem	
18	$[\sim s] r \wedge t$	17; \vee E	
19	$[\sim s] t$	18; \wedge E	S ²² R ²³
20	$\{?\{s, \sim s\}\}$	16, 19; DP	R ²²
21	$[\sim s] \sim s$	Target	R ²²
22	$\sim s$	Prem	
23	t	19, 22; Trans	

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
...			
6	$[q] p \vee q$	4; C \vee E	
...			
14	$[u, t] q$	13; \vee E	S ²³ R ²⁴
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	R ²³
16	$[t] t$	Target	R ²³
17	$(r \wedge t) \vee s$	Prem	
18	$[\sim s] r \wedge t$	17; \vee E	
19	$[\sim s] t$	18; \wedge E	S ²² R ²³
20	$\{?\{s, \sim s\}\}$	16, 19; DP	R ²²
21	$[\sim s] \sim s$	Target	R ²²
22	$\sim s$	Prem	
23	t	19, 22; Trans	
24	$[u] q$	14, 23; Trans	

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
...			
6	$[q] p \vee q$	4; C \vee E	
...			
14	$[u, t] q$	13; \vee E	S ²³ R ²⁴
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	R ²³
16	$[t] t$	Target	R ²³
17	$(r \wedge t) \vee s$	Prem	
18	$[\sim s] r \wedge t$	17; \vee E	
19	$[\sim s] t$	18; \wedge E	S ²² R ²³
20	$\{?\{s, \sim s\}\}$	16, 19; DP	R ²²
21	$[\sim s] \sim s$	Target	R ²²
22	$\sim s$	Prem	
23	t	19, 22; Trans	
24	$[u] q$	14, 23; Trans	
25	$\{?\{u, \sim u\}\}$	8, 24; DP	

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
...			
6	$[q] p \vee q$	4; C \vee E	
...			
14	$[u, t] q$	13; \vee E	S ²³ R ²⁴
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	R ²³
16	$[t] t$	Target	R ²³
17	$(r \wedge t) \vee s$	Prem	
18	$[\sim s] r \wedge t$	17; \vee E	
19	$[\sim s] t$	18; \wedge E	S ²² R ²³
20	$\{?\{s, \sim s\}\}$	16, 19; DP	R ²²
21	$[\sim s] \sim s$	Target	R ²²
22	$\sim s$	Prem	
23	t	19, 22; Trans	
24	$[u] q$	14, 23; Trans	
25	$\{?\{u, \sim u\}\}$	8, 24; DP	

1, 6, 23–25

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
...		
6	$[q] p \vee q$	4; C \vee E
...		
23	t	19, 22; Trans
24	$[u] q$	14, 23; Trans
25	$\{?\{u, \sim u\}\}$	8, 24; DP

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
...		
6	$[q] p \vee q$	4; C \vee E
...		
23	t	19, 22; Trans
24	$[u] q$	14, 23; Trans
25	$\{?\{u, \sim u\}\}$	8, 24; DP
26	$[u] u$	Target

Some Background

Elements of a psp
backbone

procedure

rules

instructions

marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
...		
6	$[q] p \vee q$	4; C \vee E
...		
23	t	19, 22; Trans
24	$[u] q$	14, 23; Trans
25	$\{?\{u, \sim u\}\}$	8, 24; DP
26	$[u] u$	Target
27	$t \supset u$	Prem

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

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1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
...			
6	$[q] p \vee q$	4; $\text{C}\vee\text{E}$	
...			
23	t	19, 22; Trans	
24	$[u] q$	14, 23; Trans	
25	$\{?\{u, \sim u\}\}$	8, 24; DP	
26	$[u] u$	Target	
27	$t \supset u$	Prem	
28	$[t] u$	27; $\supset\text{E}$	S^{28}

[Some Background](#)

[Elements of a psp backbone](#)

[procedure](#)

[rules](#)

[instructions](#)

[marking](#)

[An Example of the Backbone](#)

[Adaptive logics](#)

[Extra-Logical Extensions](#)

[Comments](#)

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1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
...			
6	$[q] p \vee q$	4; C \vee E	
...			
23	t	19, 22; Trans	
24	$[u] q$	14, 23; Trans	S ²⁹
25	$\{?\{u, \sim u\}\}$	8, 24; DP	R ²⁹
26	$[u] u$	Target	R ²⁹
27	$t \supset u$	Prem	
28	$[t] u$	27; \supset E	S ²⁸ R ²⁹
29	u	23, 28; Trans	

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

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1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
...			
6	$[q] p \vee q$	4; C \vee E	S ³⁰
...			
23	t	19, 22; Trans	
24	$[u] q$	14, 23; Trans	S ²⁹ R ³⁰
25	$\{?\{u, \sim u\}\}$	8, 24; DP	R ²⁹
26	$[u] u$	Target	R ²⁹
27	$t \supset u$	Prem	
28	$[t] u$	27; \supset E	S ²⁸ R ²⁹
29	u	23, 28; Trans	
30	q	24, 29; Trans	

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

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1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	R^{31}
...			
6	$[q] p \vee q$	4; C \vee E	$S^{30} R^{31}$
...			
23	t	19, 22; Trans	
24	$[u] q$	14, 23; Trans	$S^{29} R^{30}$
25	$\{?\{u, \sim u\}\}$	8, 24; DP	R^{29}
26	$[u] u$	Target	R^{29}
27	$t \supset u$	Prem	
28	$[t] u$	27; \supset E	$S^{28} R^{29}$
29	u	23, 28; Trans	
30	q	24, 29; Trans	
31	$p \vee q$	6, 30; Trans	

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

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1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	R^{31}
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28	$[t] u$	27; \supset E	$S^{28} R^{29}$
29	u	23, 28; Trans	
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problem solved

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

Adaptive logics (only Standard Format)

characterization

- ▶ *lower limit logic* **LLL**
- ▶ *set of abnormalities* Ω
- ▶ *strategy*

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

Adaptive logics (only Standard Format)

characterization

- ▶ *lower limit logic* **LLL**
monotonic, compact, ... logic
- ▶ *set of abnormalities* Ω
characterized by a (possibly restricted) logical form
- ▶ *strategy*
Reliability, Minimal Abnormality, ...

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

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monotonic, compact, ... logic
- ▶ *set of abnormalities* Ω
characterized by a (possibly restricted) logical form
- ▶ *strategy*
Reliability, Minimal Abnormality, ...

upper limit logic:

syntax: **ULL** = **LLL** + axiom/rule

semantics: the **LLL**-models that verify no abnormality

general idea behind adaptive logics:

$Cn_{AL}(\Gamma) : Cn_{LLL}(\Gamma) +$ what follows if as many members of Ω are false as the premises permit

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Example: the inconsistency-adaptive **CLuN**^r

- ▶ *lower limit logic*: **CLuN**
- ▶ *set of abnormalities*: $\Omega = \{\exists(A \wedge \sim A) \mid A \in \mathcal{F}\}$
- ▶ *strategy*: Reliability

upper limit logic:

$$\mathbf{CL} = \mathbf{CLuN} + (A \wedge \sim A) \supset B$$

semantically: the **CLuN**-models that verify no inconsistency

corrective adaptive logic (if **CL** is the standard)

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

Example: logic of inductive generalization: \mathbf{IL}^m

- ▶ *lower limit logic*: \mathbf{CL}
- ▶ *set of abnormalities*: $\Omega = \{\exists A \wedge \exists \sim A \mid A \in \mathcal{F}^\circ\}$
- ▶ *strategy*: Minimal Abnormality

upper limit logic:

$$\mathbf{UCL} = \mathbf{CL} + \exists \alpha A(\alpha) \supset \forall \alpha A(\alpha)$$

semantically: the uniform \mathbf{CL} -models ($v(\pi^r) \in \{\emptyset, D^{(r)}\}$)

ampliative adaptive logic (if \mathbf{CL} is the standard)

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

standard format provides

- ▶ proofs
- ▶ semantics
- ▶ most of metatheory (including soundness and completeness)
- ▶ **prospective dynamics** (published for Reliability)

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

further examples (relevant for philosophy of science)

- ▶ many other inconsistency-handling (+ other logical symbols)
- ▶ ambiguity-adaptive
- ▶ vagueness-adaptive
- ▶ corrective deontic logics
- ▶ paraconsistent compatibility
- ▶ ...

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

further examples (relevant for philosophy of science)

- ▶ many other inconsistency-handling (+ other logical symbols)
- ▶ ambiguity-adaptive
- ▶ vagueness-adaptive
- ▶ corrective deontic logics
- ▶ paraconsistent compatibility
- ▶ ...

- ▶ plausibility-adaptive
- ▶ compatibility
- ▶ diagnosis
- ▶ abduction
- ▶ analogies, metaphors
- ▶ erotetic evocation/implication (problem solving)
- ▶ ...

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Some Extra-Logical Extensions (1)

answerable questions

\mathbb{A} is a set of couples $(\Delta : Q)$ in which Δ is a set of statements and Q is a question

idea: if the members of Δ are true, Q can be answered
by observational/experimental means
(not Hintikka's oracle)

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

Some Extra-Logical Extensions (1)

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idea: if the members of Δ are true, Q can be answered by observational/experimental means
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New If A is an unmarked target, $pp(A, B)$ for some direct answer B of Q , $(\Delta : Q)$ and all members of Δ occur in the f_{psp}, then one may add, for some direct answer C of Q :

k C

i ; New

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Some Extra-Logical Extensions (1)

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k C i ; New

psp guides (which observations/experiments should be carried out)

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Some Extra-Logical Extensions (2)

bringing in available information (formerly judged irrelevant)

one tries to solve problem from theory T and set of data
later a theory T' turns out to be relevant
(because a target is a positive part of an axiom of T')

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Some Extra-Logical Extensions (2)

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one tries to solve problem from theory T and set of data
later a theory T' turns out to be relevant
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psp guides (which further theories are relevant?)

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Some Extra-Logical Extensions (3)

plausible conjectures

where A is an abnormality, introduce $\diamond^i A$ or $\diamond^i \neg A$

- ▶ basis: worldview, personal constraint, study of situation, blind guess
- ▶ thus reducing a disjunction of abnormalities
- ▶ = defeasibly obtaining more consequences (plausibility-adaptive logic)

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

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Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

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...

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Comments (1)

framework that contains **open slots**

these make content guidance possible

but the framework is **formal**

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

Comments (1)

framework that contains **open slots**

these make content guidance possible

but the framework is **formal**

prospective dynamics pushes the 'logical' part of the heuristics into the proof

part of remaining heuristics is fixed by procedure

still remaining heuristics

Some Background

Elements of a psp backbone

- procedure
- rules
- instructions
- marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Comments (2)

content-guided

Some Background

Elements of a psp
backbone

procedure

rules

instructions

marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

Comments (2)

content-guided

- ▶ 'language' of a scientific discipline (not typical)

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

Comments (2)

content-guided

- ▶ 'language' of a scientific discipline (not typical)
- ▶ adaptive logics validate *applications* of rules that transcend the lower limit logic

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Comments (2)

content-guided

- ▶ 'language' of a scientific discipline (not typical)
- ▶ adaptive logics validate *applications* of rules that transcend the lower limit logic
- ▶ multiplicity of adaptive logics for every purpose (to be justified)

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

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- ▶ adaptive logics validate *applications* of rules that transcend the lower limit logic
- ▶ multiplicity of adaptive logics for every purpose (to be justified)
- ▶ multiplicity of erotetic logics (to be justified)

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

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content-guided

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- ▶ multiplicity of erotetic logics (to be justified)
- ▶ multiplicity of procedures for prospective dynamics (to be justified)

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

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content-guided

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- ▶ take background theories serious + several forms of defeasibility

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Comments (2)

content-guided

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- ▶ ./.

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Comments (3)

content-guided

▶ ./.

Some Background

Elements of a psp
backbone

procedure

rules

instructions

marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

Comments (3)

content-guided

- ▶ ./.
- ▶ 'guesses': world-view, personal constraints, . . . , blind (which guesses useful: determined by disjunctions of abnormalities)
(extra logical origin; logic guides handling of the guesses)

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Comments (3)

content-guided

- ▶ ./.
- ▶ ‘guesses’: world-view, personal constraints, . . . , blind (which guesses useful: determined by disjunctions of abnormalities)
(extra logical origin; logic guides handling of the guesses)
- ▶ **local** selection of adaptive logics
(abd./ind.; inconsistency; replace lower limit logic; plausibilities; . . .)

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

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- ▶ *heuristics* of psp

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Comments (3)

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(extra logical origin; logic guides handling of the guesses)
- ▶ **local** selection of adaptive logics
(abd./ind.; inconsistency; replace lower limit logic; plausibilities; . . .)
- ▶ *heuristics* of psp
 - ▶ road followed to derive conclusion

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

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- ▶ *heuristics* of psp
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 - ▶ observation / experiment / theoretical derivation

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Comments (3)

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- ▶ ‘guesses’: world-view, personal constraints, . . . , blind (which guesses useful: determined by disjunctions of abnormalities)
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 - ▶ [use of models]

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Comments (3)

content-guided

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- ▶ ‘guesses’: world-view, personal constraints, . . . , blind (which guesses useful: determined by disjunctions of abnormalities)
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- ▶ **local** selection of adaptive logics
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- ▶ *heuristics* of psp
 - ▶ road followed to derive conclusion
 - ▶ observation / experiment / theoretical derivation
 - ▶ [use of models]

to be decided in view of what was learned about world/learning in specific domain/context

Some Background

Elements of a psp backbone

procedure
rules
instructions
marking

An Example of the Backbone

Adaptive logics

Extra-Logical Extensions

Comments

Comments (4)

conclusion

framework that contains **open slots**

these make content guidance possible

but the framework is **formal**

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments

Comments (4)

conclusion

framework that contains **open slots**

these make content guidance possible

but the framework is **formal**

status of the approach itself: provisional hypothesis

Some Background

Elements of a psp
backbone

procedure
rules
instructions
marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments



Questions?

Some Background

Elements of a psp
backbone

- procedure
- rules
- instructions
- marking

An Example of the
Backbone

Adaptive logics

Extra-Logical
Extensions

Comments