LMPS 2007

Content-Guidance in Formal Problem Solving Processes

Diderik Batens
Centre for Logic and Philosophy of Science
Ghent University, Belgium
Diderik.Batens@UGent.be
http://logica.UGent.be/dirk/
http://logica.UGent.be/centrum/
http://logica.UGent.be/adlog/

Outline

Some Background (1)

- Vienna Circle: (degenerated to) a priori methodology
- Historicism (Kuhn, . . .): relativism
- today: scientific problem-solving is content-guided "what we have learned, including what we have learned about how to learn" (Shapere)

all possibilities / historicism

Aim of this paper:

- formal approach to problem solving
- in which there is large room for content-guidance

(⇒ example)

Some Background (2)

direct sources of inspiration:

- philosophy of science/epistemology:
  - problem solving: Kuhn, Laudan, *Nickles*, . . .
  - 'contextual': specific certainties, relevant data, methodological instructions, . . . for each problem
- logic:
  - procedures
  - prospective dynamics
  - adaptive logics
  - erotetic logic (varying on Wiśniewski)

problem determined by (changing) *constraints*

- conditions on solution
- methodological instructions / heuristics / examples
- certainties (conceptual system, . . .)

Some Background (3)

some elements of the plot:

- aim: *explication* for problem solving processes (psps)
- *backbone*:
  solve $\{?\{A, \sim A\}\}$ by **CL**-deriving A or $\sim A$ from Γ
- empirical means: observation and experiment guided by psp
- new available information (not originally seen as relevant) guided by psp
- adaptive logics:
  corrective: handling inconsistency (and similar), ambiguity, vagueness, . . .
  ampliative: inductive generalization, abduction, . . .

  control defeasible inferences by conditions and marking
- model-based reasoning (mainly future research)

Some Background (4)

from the general plot (consolations?)

- in the end all knowledge defeasible
- going through different 'contexts'
  $\Rightarrow \begin{cases} \text{conceptual change} \\ \text{generation of new concepts} \end{cases}$
- many ampliative mechanisms
  (abduction overestimated)
- independent of ontological debates

Elements of a psp backbone

lines of psp

- problem lines: example $\{?\{p \vee q, \sim p \vee q\}, ?\{r, \sim r\}\}$

  problem = non-empty set of questions
- declarative lines
  · conditional: $[B_1, \ldots, B_n]\ A$
  · unconditional: $[\emptyset]\ A$, viz. A

psp

- stage of a psp: sequence of lines
- psp: chain of stages
- next stage: add new line + apply marking definitions
- governed by *procedure*

procedure (set of instructions)

- *rule of inference*: to derive A from $B_1, \ldots, B_n$
- *instruction*:
  rule of inference + permissions/obligations

  permissions/obligations depend on the present stage (lines + marks)
- below: instructions (including the rules)

varying on Smullyan

$\mathfrak{a}$	$\mathfrak{a}_1$	$\mathfrak{a}_2$		$\mathfrak{b}$	$\mathfrak{b}_1$	$\mathfrak{b}_2$
$A \wedge B$	A	B		$\sim(A \wedge B)$	$*A$	$*B$
$A \equiv B$	$A \supset B$	$B \supset A$		$\sim(A \equiv B)$	$\sim(A \supset B)$	$\sim(B \supset A)$
$\sim(A \vee B)$	$*A$	$*B$		$A \vee B$	A	B
$\sim(A \supset B)$	A	$*B$		$A \supset B$	$*A$	B
$\sim\sim A$	A	A				

complement of A: $*A = B$ if $A = \sim B$; otherwise $*A = \sim A$

$\mathfrak{a}$	$\mathfrak{a}_1$	$\mathfrak{a}_2$		$\mathfrak{b}$	$\mathfrak{b}_1$	$\mathfrak{b}_2$
$A \wedge B$	A	B		$\sim(A \wedge B)$	$*A$	$*B$
$A \equiv B$	$A \supset B$	$B \supset A$		$\sim(A \equiv B)$	$\sim(A \supset B)$	$\sim(B \supset A)$
$\sim(A \vee B)$	$*A$	$*B$		$A \vee B$	A	B
$\sim(A \supset B)$	A	$*B$		$A \supset B$	$*A$	B
$\sim\sim A$	A	A				

formula analysing rules:

$$\frac{[\Delta]\,\mathfrak{a}}{[\Delta]\,\mathfrak{a}_1 \quad [\Delta]\,\mathfrak{a}_2} \qquad \frac{[\Delta]\,\mathfrak{b}}{[\Delta \cup \{*\mathfrak{b}_2\}]\,\mathfrak{b}_1 \quad [\Delta \cup \{*\mathfrak{b}_1\}]\,\mathfrak{b}_2}$$

condition analysing rules:

$$\frac{[\Delta \cup \{\mathfrak{a}\}]\,A}{[\Delta \cup \{\mathfrak{a}_1, \mathfrak{a}_2\}]\,A} \qquad \frac{[\Delta \cup \{\mathfrak{b}\}]\,A}{[\Delta \cup \{\mathfrak{b}_1\}]\,A \quad [\Delta \cup \{\mathfrak{b}_2\}]\,A}$$

$\mathfrak{a}$	$\mathfrak{a}_1$	$\mathfrak{a}_2$		$\mathfrak{b}$	$\mathfrak{b}_1$	$\mathfrak{b}_2$
$A \wedge B$	A	B		$\sim(A \wedge B)$	$*A$	$*B$
$A \equiv B$	$A \supset B$	$B \supset A$		$\sim(A \equiv B)$	$\sim(A \supset B)$	$\sim(B \supset A)$
$\sim(A \vee B)$	$*A$	$*B$		$A \vee B$	A	B
$\sim(A \supset B)$	A	$*B$		$A \supset B$	$*A$	B
$\sim\sim A$	A	A				

positive part relation

1. pp(A, A).
2. pp(A, $\mathfrak{a}$) if pp(A, $\mathfrak{a}_1$) or pp(A, $\mathfrak{a}_2$).
3. pp(A, $\mathfrak{b}$) if pp(A, $\mathfrak{b}_1$) or pp(A, $\mathfrak{b}_2$).
4. If pp(A, B) and pp(B, C), then pp(A, C).

The instructions

Main Start a psp with the line:

1 $\{?\{M, \sim M\}\}$ Main

Target If P is the problem of an unmarked problem line, and A is a direct answer of a member of P, then one may add:

k $[A]\,A$ Target

Prem If A is an unmarked target, $B \in \Gamma$, and pp(A, B), then one may add:

k B Prem

Formula analysing rules:

$$\frac{[\Delta]\,\alpha}{[\Delta]\,\alpha_1 \quad [\Delta]\,\alpha_2} \qquad \frac{[\Delta]\,\beta}{[\Delta \cup \{*\beta_2\}]\,\beta_1 \quad [\Delta \cup \{*\beta_1\}]\,\beta_2}$$

FAR If C is an unmarked target, $[\Delta]\,A$ is the formula of an unmarked line i, $[\Delta]\,A$ / $[\Delta \cup \Delta']\,B$ is a formula analysing rule, and pp(C, B), then one may add:

k $[\Delta \cup \Delta']\,B$ i; R

in which R is the name of the formula analysing rule.

Condition analysing rules:

$$\frac{[\Delta \cup \{\alpha\}]\,A}{[\Delta \cup \{\alpha_1, \alpha_2\}]\,A} \qquad \frac{[\Delta \cup \{\beta\}]\,A}{[\Delta \cup \{\beta_1\}]\,A \quad [\Delta \cup \{\beta_2\}]\,A}$$

CAR If A is an unmarked target, $[\Delta \cup \{B\}]\,A$ is the formula of an unmarked line i, and $[\Delta \cup \{B\}]\,A$ / $[\Delta \cup \Delta']\,A$ is a condition analysing rule, then one may add:

k $[\Delta \cup \Delta']\,A$ i; R

in which R is the name of the condition analysing rule.

Eliminate some problems without answering them:

EM0 If $[\Delta \cup \{*A\}]\,A$ is the formula of a line i that is neither R-marked nor I-marked, then one may add:

k $[\Delta]\,A$ i; EM0

EM If A is an unmarked target, $[\Delta \cup \{B\}]\,A$ and $[\Delta' \cup \{\sim B\}]\,A$ are the respective formulas of the unmarked or only D-marked lines i and j, and $\Delta \subseteq \Delta'$ or $\Delta' \subseteq \Delta$, then one may add:

k $[\Delta \cup \Delta']\,A$ i, j; EM

eliminate solved questions from a problem and summarize remaining problems (and paths):

Trans If A is an unmarked target, and $[\Delta \cup \{B\}]\,A$ and $[\Delta']\,B$ are the respective formulas of the at most S-marked lines i and j, then one may add:

k $[\Delta \cup \Delta']\,A$ i, j; Trans handle derived problems:

DP If A is an unmarked target from problem line i and $[B_1, \ldots, B_n]\, A$ is the formula of an unmarked line j, then one may add:

$k \quad \{?\{B_1, \sim B_1\}, \ldots, ?\{B_n, \sim B_n\}\} \quad i, j$; DP

Remark:

no instruction for applying EFQ

in view of the intended applications
(deriving predictions, explanations, etc.)

Marking definitions

redundant lines are R-marked:

Definition
An at most S-marked declarative line i that has $[\Delta]\, A$ as its formula is R-marked at a stage iff, at that stage, $[\Theta]\, A$ is the formula of a line for some $\Theta \subset \Delta$.

Definition
An unmarked problem line i is R-marked at a stage iff, at that stage, a direct answer A of a question of line i is the formula of a line.

· target from a problem line
· resolution line
· direct target from
· target sequence
· grounded target sequence

inoperative lines are I-marked (not useful for extant problem):

Definition
An at most S-marked target line that has $[A]\, A$ as its formula is I-marked at a stage iff every problem line from which A is a target is marked at that stage.

Definition
An at most S-marked resolution line of which $[\Delta^1]\, A^1$ is the formula and $\Delta^1 \neq \emptyset$ is *I-marked* at a stage iff, at that stage, for every grounded target sequence $\langle [\Delta^n]\, A^n, \ldots, [\Delta^1]\, A^1 \rangle$,

(i) some target $[A^i]\, A^i$ $(1 \leq i \leq n)$ is marked, or
(ii) $\{A^n, \ldots, A^1\} \cap \Delta^1 \neq \emptyset$, or
(iii) $\Delta^1 \cup \ldots \cup \Delta^n \cup \Gamma^\circ_s$ is flatly inconsistent.

Definition
An unmarked problem line is I-marked iff no unmarked resolution line generates it.

Dead end lines are D-marked (no further action from such line)

- A is a *dead end* (A is literal and not positive part of premise)
- CAR-descendant of $[\Delta]\, B$

Definition
An at most S-marked resolution line with formula $[\Delta]\, A$ is D-marked at a stage iff some $B \in \Delta$ is a dead end or, at that stage, all CAR-descendants of $[\Delta]\, A$ occur in the psp and are D-marked.

Definition
An at most S-marked target line with formula $[A]\, A$ is D-marked at a stage iff A is a dead end or no further action can be taken in view of target A.

for all consistent Γ:

if $\Gamma \vdash A$, then the procedure applied to Γ and $\{?\{A, \sim A\}\}$ results in the answer A,

and

if $\Gamma \nvdash A$, then the procedure applied to Γ and $\{?\{A, \sim A\}\}$ stops without the main problem being answered or results in the answer $\sim A$.

Speed up the procedure by S-marks

- Γ°_s union of the Γ and of the set of the conditionless formulas that occur at stage s of the psp

Definition
A R-unmarked resolution line in which $[\Delta^1]\, A^1$ is derived is S-marked iff

(i) $\Delta^1 \cap \Gamma^\circ_s \neq \emptyset$, or
(ii) for some target sequence $\langle [\Delta^n]\, A^n, \ldots, [\Delta^1]\, A^1 \rangle$, $\{A^n\} \cup \Delta^1$ is flatly inconsistent whereas Δ^1 is not flatly inconsistent, or
(iii) $\Delta_1 \subset \Delta^n \cup \ldots \cup \Delta^2$ for some target sequence $\langle [\Delta^n]\, A^n, \ldots, [\Delta^1]\, A^1 \rangle$.

instruction: operate on S-marked lines before doing anything else

An Example of the Backbone

main problem: $?\{p \vee q, \sim(p \vee q)\}$

premise set: $\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

logic: $\mathbf{CL}^-$ + an erotetic logic (fixed by the procedure)

procedure: prospective dynamics + problems

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
2	$[\sim(p \vee q)]\ \sim(p \vee q)$	Target	D^3
3	$[\sim p, \sim q]\ \sim(p \vee q)$	2; C∼∨E	D^3
4	$[p \vee q]\ p \vee q$	Target	
5	$[p]\ p \vee q$	4; C∨E	D^5
6	$[q]\ p \vee q$	4; C∨E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q]\ q$	Target	
9	$(q \vee u) \supset (\sim t \vee q)$	Prem	
10	$[q \vee u]\ \sim t \vee q$	9; ⊃E	
11	$[q]\ \sim t \vee q$	10; C∨E	
12	$[q, t]\ q$	11; ∨E	I^{12}
13	$[u]\ \sim t \vee q$	10; C∨E	
14	$[u, t]\ q$	13; ∨E	
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	
16	$[t]\ t$	Target	

1, 6, 14–16

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
...			
6	$[q]\ p \vee q$	4; C∨E	
...			
14	$[u, t]\ q$	13; ∨E	S^{23} R^{24}
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	R^{23}
16	$[t]\ t$	Target	R^{23}
17	$(r \wedge t) \vee s$	Prem	
18	$[\sim s]\ r \wedge t$	17; ∨E	
19	$[\sim s]\ t$	18; ∧E	S^{22} R^{23}
20	$\{?\{s, \sim s\}\}$	16, 19; DP	R^{22}
21	$[\sim s]\ \sim s$	Target	R^{22}
22	$\sim s$	Prem	
23	t	19, 22; Trans	
24	$[u]\ q$	14, 23; Trans	
25	$\{?\{u, \sim u\}\}$	8, 24; DP	

1, 6, 23–25

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	R^{31}
...			
6	$[q]\ p \vee q$	4; C∨E	S^{30} R^{31}
...			
23	t	19, 22; Trans	
24	$[u]\ q$	14, 23; Trans	S^{29} R^{30}
25	$\{?\{u, \sim u\}\}$	8, 24; DP	R^{29}
26	$[u]\ u$	Target	R^{29}
27	$t \supset u$	Prem	
28	$[t]\ u$	27; ⊃E	S^{28} R^{29}
29	u	23, 28; Trans	
30	q	24, 29; Trans	
31	$p \vee q$	6, 30; Trans	

problem solved

Adaptive logics (only Standard Format)

characterization

- *lower limit logic* **LLL**
  monotonic, compact, ... logic
- *set of abnormalities* Ω
  characterized by a (possibly restricted) logical form
- *strategy*
  Reliability, Minimal Abnormality, ...

upper limit logic:
syntax: **ULL** = **LLL** + axiom/rule
semantics: the **LLL**-models that verify no abnormality

general idea behind adaptive logics:
$Cn_{\mathbf{AL}}(\Gamma)$: $Cn_{\mathbf{LLL}}(\Gamma)$ + what follows if as many members of Ω are false as the premises permit

Example: the inconsistency-adaptive $\mathbf{CLuN}^r$

- *lower limit logic*: **CLuN**
- *set of abnormalities*: $\Omega = \{\exists(A \wedge \sim A) \mid A \in \mathcal{F}\}$
- *strategy*: Reliability

upper limit logic:
CL = **CLuN** + $(A \wedge \sim A) \supset B$
semantically: the **CLuN**-models that verify no inconsistency

corrective adaptive logic (if **CL** is the standard)

Example: logic of inductive generalization: $\mathbf{IL}^m$

- *lower limit logic*: **CL**
- *set of abnormalities*: $\Omega = \{\exists A \wedge \exists \sim A \mid A \in \mathcal{F}^{\circ}\}$
- *strategy*: Minimal Abnormality

upper limit logic:
UCL = **CL** + $\exists \alpha A(\alpha) \supset \forall \alpha A(\alpha)$
semantically: the uniform **CL**-models ($v(\pi^r) \in \{\emptyset, D^{(r)}\}$)

ampliative adaptive logic (if **CL** is the standard)

standard format provides

- proofs
- semantics
- most of metatheory (including soundness and completeness)
- prospective dynamics (published for Reliability)

further examples (relevant for philosophy of science)

- many other inconsistency-handling (+ other logical symbols)
- ambiguity-adaptive
- vagueness-adaptive
- corrective deontic logics
- paraconsistent compatibility
- ...

- plausibility-adaptive
- compatibility
- diagnosis
- abduction
- analogies, metaphors
- erotetic evocation/implication (problem solving)
- ...

Some Extra-Logical Extensions (1)

answerable questions

$\mathbb{A}$ is a set of couples $(\Delta : Q)$ in which Δ is a set of statements and Q is a question

idea: if the members of Δ are true, Q can be answered by observational/experimental means
(not Hintikka's oracle)

New If A is an unmarked target, pp(A, B) for some direct answer B of Q, $(\Delta : Q)$ and all members of Δ occur in the fpsp, then one may add, for some direct answer C of Q:

k C i; New

psp guides (which observations/experiments should be carried out)

Some Extra-Logical Extensions (2)

bringing in available information (formerly judged irrelevant)

one tries to solve problem from theory T and set of data
later a theory T' turns out to be relevant
(because a target is a positive part of an axiom of T')

psp guides (which further theories are relevant?)

Some Extra-Logical Extensions (3)

plausible conjectures

where A is an abnormality, introduce $\Diamond^i A$ or $\Diamond^i \neg A$

- basis: worldview, personal constraint, study of situation, blind guess
- thus reducing a disjunction of abnormalities
- = defeasibly obtaining more consequences (plausibility-adaptive logic)

psp guides (which disjunctions of abnormalities may be reduced?)

...

Comments (1)

framework that contains **open slots**

these make content guidance possible

but the framework is **formal**

prospective dynamics pushes the 'logical' part of the heuristics into the proof

part of remaining heuristics is fixed by procedure

still remaining heuristics

Comments (2)

content-guided

- 'language' of a scientific discipline (not typical)
- adaptive logics validate *applications* of rules that transcend the lower limit logic
- multiplicity of adaptive logics for every purpose (to be justified)
- multiplicity of erotetic logics (to be justified)
- multiplicity of procedures for prospective dynamics (to be justified)
- take background theories serious + several forms of defeasibility
- ./.

Comments (3)

content-guided

- ./.
- 'guesses': world-view, personal constraints, ..., blind
  (which guesses useful: determined by disjunctions of abnormalities)
  (extra logical origin; logic guides handling of the guesses)
- local selection of adaptive logics
  (abd./ind.; inconsistency; replace lower limit logic; plausibilities; ...)
- *heuristics* of psp
  - road followed to derive conclusion
  - observation / experiment / theoretical derivation
  - [use of models]

  to be decided in view of what was learned about world/learning in specific domain/context

Comments (4)

conclusion

framework that contains **open slots**

these make content guidance possible

but the framework is **formal**

status of the approach itself: provisional hypothesis