Spoiled for Choice?

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Outline

Unexpected Inconsistency

The Problem

Gluts and Gaps

All Together

Ambiguity-adaptive Logics

Adaptive Zero Logic
Handling Unexpected Inconsistency

inconsistency-adaptive logics (the oldest adaptive logics) devised for specific application type:

$T$, intended as consistent, turns out to be inconsistent
Handling Unexpected Inconsistency

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⇒ reason from ‘the theory’ in search of consistent replacement
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‘the theory’ = ‘$T$ in its full richness, except for the pernicious consequences of its inconsistency’
Handling Unexpected Inconsistency

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\[ T, \text{ intended as consistent, turns out to be inconsistent} \]

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‘the theory’ = ‘\( T \) in its full richness, except for the pernicious consequences of its inconsistency’

interpret \( T \) ‘as consistently as possible’
consider inconsistencies as false, except where \( T \) prevents this

adapt to the specific inconsistencies of \( T \)
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CL too strong, paraconsistent logics too weak
Solution: inconsistency-adaptive logics

special case of adaptive logic

standard format

· lower limit logic

· set of abnormalities $\Omega$

· strategy
Solution: inconsistency-adaptive logics

special case of adaptive logic

standard format

- lower limit logic
  reflexive, transitive, monotonic, uniform, and compact logic, for which there is a positive test
- set of abnormalities $\Omega$
  characterized by a (possibly restricted) logical form
- strategy
  Reliability, Minimal Abnormality, . . .
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upper limit logic:

$\text{ULL} = \text{LLL} +$ axiom/rule that trivializes abnormalities
semantically: the $\text{LLL}$-models that verify no abnormality
Solution: inconsistency-adaptive logics

special case of adaptive logic

**standard format**

- **lower limit logic**  \( \text{CLuN} \)
  reflexive, transitive, monotonic, uniform, and compact logic, for which there is a positive test
- **set of abnormalities** \( \Omega \)
  \( \exists (A \land \neg A) \)
  characterized by a (possibly restricted) logical form
- **strategy**
  Reliability, Minimal Abnormality, . . .

upper limit logic:  \( \text{CL} \)
\( \text{ULL} = \text{LLL} + \) axiom/rule that trivializes abnormalities
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Some Notes

“abnormality” is technical term
only abnormalities of corrective adaptive logics CL-impossible
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standard format provides adaptive logic with
\begin{itemize}
  \item dynamic proofs
  \item selection semantics (selects \textbf{LLL}-models of $\Gamma$)
  \item most of the metatheory
    (soundness, completeness, all central properties)
\end{itemize}
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classical logical symbols added (\(~\), \(\exists\), ... , \(\exists\), \(\exists\))
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adaptive logics are formal characterizations of methods (not
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classical logical symbols added ($\neg$, $\land$, $\ldots$, $\exists$, $\models$)
(not in premises or conclusion)
drastically simplify metatheoretic proofs
simplify proof theory
The Problem

approach in terms of inconsistency-adaptive logics the only correct one?
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claim in 1997:

classical logicians obsessed by contradictions
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deep, many more **LLL**-variants possible

- negation gaps
- other gluts and gaps
- ambiguity
- combinations (up to zero logic)
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all offer minimally abnormal interpretation of some $\Gamma$
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all offer minimally abnormal interpretation of some $\Gamma$

in CL all of those abnormalities surface as inconsistencies
example: conjunction gap $\{p, q, \neg(p \land q)\}$
Gluts and Gaps

semantically:

glut: $\nu(A) = 1$ and $\nu$-values of subformulas sufficient for $\nu(A) = 0$ in CL-semantics
Gluts and Gaps

semantically:

glut: $v(A) = 1$ and $v$-values of subformulas sufficient for $v(A) = 0$ in $\text{CL}$-semantics

gap: $v(A) = 0$ and $v$-values of subformulas sufficient for $v(A) = 1$ in $\text{CL}$-semantics
Gluts and Gaps

semantically:

**glut:** $v(A) = 1$ and $v$-values of subformulas sufficient for $v(A) = 0$ in CL-semantics

**gap:** $v(A) = 0$ and $v$-values of subformulas sufficient for $v(A) = 1$ in CL-semantics

**negation gap:** $v_M(A) = 0$ and $v_M(\neg A) = 1$ and $v_M(\neg A) = 0$

**negation glut:** $v_M(A) = 1$ and $v_M(\neg A) = 0$ and $v_M(\neg A) = 1$
Gluts and Gaps

semantically:

glut: \( \nu(A) = 1 \) and 
\( \nu \)-values of subformulas sufficient for \( \nu(A) = 0 \) in CL-semantics

gap: \( \nu(A) = 0 \) and 
\( \nu \)-values of subformulas sufficient for \( \nu(A) = 1 \) in CL-semantics

negation gap: \([\nu_M(A) = 0] \nu_M(\Diamond A) = 1 \) and \( \nu_M(\lnot A) = 0 \)
negation glut: \([\nu_M(A) = 1] \nu_M(\Diamond A) = 0 \) and \( \nu_M(\lnot A) = 1 \)

predicative and in terms of the checked symbols:

negation gap: \( \Diamond (\Diamond A \land \lnot \lnot A) \)
negation glut: \( \Diamond (\Diamond A \land \lnot A) \) (new formulation!)
More Gluts and Gaps
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conjunction:

\[ \exists ((A \land B) \land \neg (A \land B)) \]

\[ \exists (\neg (A \land B) \land (A \land B)) \]
More Gluts and Gaps

conjunction:
\[ \exists ((A \land B) \land \neg (A \land B)) \]

identity:
\[ \exists (\alpha \equiv \beta \land \neg \alpha = \beta) \]
More Gluts and Gaps

conjunction:
\[ \exists((A \land B) \land \neg (A \land B)) \]
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identity:
\[ \exists(\alpha \equiv \beta \land \neg \alpha = \beta) \]
\[ \exists(\neg \alpha \equiv \beta \land \alpha = \beta) \]

existential quantifier:
\[ \exists(\exists \alpha A(\alpha) \land \neg \exists \alpha A(\alpha)) \]
\[ \exists(\neg \exists \alpha A(\alpha) \land \exists \alpha A(\alpha)) \]
More Gluts and Gaps

conjunction:
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\[ \exists(\exists \alpha A(\alpha) \land \neg \exists \alpha A(\alpha)) \]
\[ \exists(\neg \exists \alpha A(\alpha) \land \exists \alpha A(\alpha)) \]

example:
\[ \{ p, r, (p \lor q) \supset s, (p \lor t) \supset \neg r, (p \land r) \supset \neg s, (p \land s) \supset t \} \]
has models
if negation gluts allowed
More Gluts and Gaps

conjunction:
\[ \exists ((A \land B) \land \neg (A \land B)) \quad \exists (\neg (A \land B) \land (A \land B)) \]

identity:
\[ \exists (\alpha \rightleftharpoons \beta \land \neg \alpha = \beta) \quad \exists (\neg \alpha \rightleftharpoons \beta \land \alpha = \beta) \]

existential quantifier:
\[ \exists (\exists \alpha A(\alpha) \land \neg \exists \alpha A(\alpha)) \quad \exists (\neg \exists \alpha A(\alpha) \land \exists \alpha A(\alpha)) \]

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More Gluts and Gaps

conjunction:
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identity:
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\[ \exists(\exists\alpha A(\alpha) \land \neg\exists\alpha A(\alpha)) \]  
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has models
if negation gluts allowed
if negation gaps allowed
if conjunction gaps and disjunction gaps allowed
More Gluts and Gaps

conjunction:
\[ \exists((A \land B) \land \neg(A \land B)) \]
\[ \exists((\neg(A \land B) \land (A \land B)) \]

identity:
\[ \exists(\alpha \equiv \beta \land \neg \alpha = \beta) \]
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existential quantifier:
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\[ \{ p, r, (p \lor q) \supset s, (p \lor t) \supset \neg r, (p \land r) \supset \neg s, (p \land s) \supset t \} \]
has models
if negation gluts allowed
if negation gaps allowed
if conjunction gaps and disjunction gaps allowed
if implication gluts allowed

. . .
Axiomatization of the Logics

**CL** axioms for the classical symbols
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axiom for each standard symbol: example *disjunction*

no gluts or gaps: \((A \lor B) \equiv (A \uparrow B)\)
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no gluts or gaps: \((A \lor B) \equiv (A \山路 B)\)

gluts no gaps: \((A \山路 B) \山路 (A \lor B)\)


**Axiomatization of the Logics**

**CL** axioms for the classical symbols

axiom for each standard symbol: example *disjunction*

no gluts or gaps: \((A \lor B) \equiv (A \triangleright B)\)

gluts no gaps: \((A \triangleright B) \supset (A \lor B)\)

gaps no gluts: \((A \lor B) \supset (A \triangleright B)\)
Axiomatization of the Logics

**CL** axioms for the classical symbols

axiom for each standard symbol: example *disjunction*

- no gluts or gaps: \((A \lor B) \equiv (A \uparrow B)\)
- gluts no gaps: \((A \uparrow B) \supset (A \lor B)\)
- gaps no gluts: \((A \lor B) \supset (A \uparrow B)\)
- both: nothing
Axiomatization of the Logics

**CL** axioms for the classical symbols

axiom for each standard symbol: example *disjunction*

no gluts or gaps: \((A \lor B) \equiv (A \triangledown B)\)

gluts no gaps: \((A \triangledown B) \supset (A \lor B)\)

gaps no gluts: \((A \lor B) \supset (A \triangledown B)\)

both: nothing

naming: **CLuD**, **CLaD**, **CLoD**, ...
Combinations and Adaptive Logics

combinations of gluts and gaps: \textbf{CLaNoC, CLaCuX}, \ldots,
Combinations and Adaptive Logics

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Adaptive

any of those Tarski logics may will function as \text{LLL}
suitable $\Omega$ determines which abnormalities minimized by AL
(normal case: minimize all permitted gluts and gaps
warrants \text{ULL} = \text{CL})

and combine with strategy: \text{CLaCuX}^m, \ldots
All Together

CLo
All Together

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meaning of every logical symbol arbitrary  (seems nonsense)
All Together

CLo

meaning of every logical symbol arbitrary \textit{(seems nonsense)}

adaptive: CLo^r, CLo^m, \ldots

meaning of every logical symbol contingent on the premises \textit{(seems very interesting)}
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meaning of every logical symbol arbitrary \textit{ (seems nonsense) }

adaptive: CLo\(^r\), CLo\(^m\), \ldots

meaning of every logical symbol contingent on the premises \textit{ (seems very interesting) }

moreover:

a CLo\(^m\)-proof from \(\Gamma\) reveals which (combinations of) gluts/gaps lead to a sensible minimally abnormal interpretation of \(\Gamma\)

each of these may be the basis for a transformation to a consistent theory
All Together

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better is coming up
Ambiguity-adaptive Logics

rough idea CLI: occurrences of non-logical term may have a different index, which may reveal a difference in meaning
Ambiguity-adaptive Logics

rough idea **CLI**: occurrences of non-logical term may have a different index, which may reveal a difference in meaning

example: \( p^1 \land q^2 \not\vdash_{\text{CLI}} p^3 \)
(some sophistication required at predicative level)
Ambiguity-adaptive Logics

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example: $p^1 \land q^2 \not\vdash_{\text{CLI}} p^3$
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abnormalities: $\nvdash p^i \equiv p^j$ (for $i, j \in \mathbb{N}$), . . .
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$\text{CLI}^\times$: ambiguities are minimized
Ambiguity-adaptive Logics

rough idea **CLI**: occurrences of non-logical term may have a different index, which may reveal a difference in meaning

example: $p^1 \land q^2 \not\models^*_\text{CLI} p^3$

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**CLI**$^\times$: ambiguities are minimized

\[
\Gamma \vdash^{\text{CLA}_m} A \text{ iff } \Gamma^\dagger \vdash^{\text{CLI}_m} A^\dagger
\]
Adaptive Zero Logic

\( \text{CL}\emptyset I \): all gluts/gaps + non-logical terms indexed
Adaptive Zero Logic

**CL₀I**: all gluts/gaps + non-logical terms indexed

\[ \Gamma \vdash_{\text{CL₀I}} A \iff \Gamma^\dagger \vdash_{\text{CL₀I}} A^\dagger \]
Adaptive Zero Logic

\( \text{CL}^{\emptyset} \text{I}: \text{all gluts/gaps + non-logical terms indexed} \)

\[ \Gamma \vdash_{\text{CL}^{\emptyset}} A \iff \Gamma^{\dagger} \vdash_{\text{CL}^{\emptyset}} A^{\dagger} \]

meaning of every logical symbol is contingent
any 2 occurrences of non-logical symbol may have different meaning
Adaptive Zero Logic

**CL₀I**: all gluts/gaps + non-logical terms indexed

\[ \Gamma \vdash_{CL_0} A \text{ iff } \Gamma^\dagger \vdash_{CL_0 I} A^\dagger \]

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(post-modernist logic)
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meaning of every logical symbol is contingent

any 2 occurrences of non-logical symbol may have different meaning

(post-modernist logic)

for all \( \Gamma \) and \( A \), \( \Gamma \not\vdash_{CL₀} A \)
Adaptive Zero Logic

**CL∅I**: all gluts/gaps + non-logical terms indexed

\[ \Gamma \vdash_{\text{CL∅}} A \text{ iff } \Gamma^\dagger \vdash_{\text{CL∅I}} A^\dagger \]

meaning of every logical symbol is contingent
any 2 occurrences of non-logical symbol may have different meaning
(post-modernist logic)

for all \( \Gamma \) and \( A \)

\[ \Gamma \vdash_{\text{CL∅}^m} A \text{ iff } \Gamma^\dagger \vdash_{\text{CL∅I}^m} A^\dagger \]
Adaptive Zero Logic

\( \text{CL}_{I}: \) all gluts/gaps + non-logical terms indexed

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for all \( \Gamma \) and \( A \), \( \Gamma \not\vdash_{\text{CL}_0} A \)

\[ \Gamma \vdash_{\text{CL}_0} A \iff \Gamma^{\dagger} \vdash_{\text{CL}_0} A^{\dagger} \]

meaning of logical symbols depends on premise set
ambiguities minimized (in function of premises)
Some Comments

heuristic value:
\[ \text{CL}^0_m \]-proofs reveal which (combination of)
gluts/gaps/ambiguities lead to a sensible minimally abnormal
interpretation of \( \Gamma \)
Some Comments

heuristic value:
\( \text{CL} \emptyset^m \)-proofs reveal which (combination of) gluts/gaps/ambiguities lead to a sensible minimally abnormal interpretation of \( \Gamma \)

\( \text{CL} \emptyset^m \) itself (permitting all reviewed abnormalities) is always an option, but a clumsy one if premise set has no \( \text{CL} \)-models (often only long disjunctions derivable: spoiled for choice)
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if \( \Gamma \) has \( \text{CL} \)-models, its \( \text{CL}^{\text{m}} \)-consequences are identical to its \( \text{CL} \)-consequences
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negation gluts and ambiguities suit every premise set but not always optimal (communication / minimal abnormal interpretation / . . .)
Some Comments

heuristic value:
\(\text{CL}\emptyset^m\)-proofs reveal which (combination of) gluts/gaps/ambiguities lead to a sensible minimally abnormal interpretation of \(\Gamma\)

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negation gluts and ambiguities suit every premise set but not always optimal (communication / minimal abnormal interpretation / . . .)

heuristic value makes extra-logical preferences applicable and suggests new ones
Questions?