4 ENTER Adaptive Logics

4.1 The problem (repetition)

many reasoning processes in the sciences (and elsewhere) display

an external dynamics

non-monotonic

an internal dynamics

revise conclusions as insights in premises grow

$\Rightarrow$ gain technically sound control on the internal dynamics

examples

interpret as consistently as possible a theory that turned out inconsistent

inductive generalization

inductive prediction

compatibility

interpreting a person's position during an ongoing discussion

finding a (potential or actual) explanation no positive test for $\Gamma \vdash A$

$\vdash$ $\swarrow$ $\searrow$ reasoning

adaptive logic internal dynamics

$\downarrow$ explicate

dynamic proof theory of a.l.

What is an adaptive logic?

What is a dynamic proof theory?

4.2 Characterization of an adaptive Logic
(only the best studied kind)

- *lower limit logic*
  monotonic and compact logic

- *set of abnormalities* $\mathbf{\Omega}$:
  characterized by a (possibly restricted) logical form

- *strategy*:
  Reliability, Minimal Abnormality, . . .

idea: for each abnormality (separately):
consider it as false, unless this is impossible in view of the premises

example: $\sim p$, $p \vee r$, $\sim q$, $q \vee s$, p

strategy required because idea is ambiguous (see below)

upper limit logic **ULL**:
LLL + axiom warranting that members of Ω are logically false

the characterization provides **AL** with:

- a semantics
- a dynamic proof theory
- soundness and completeness proofs
- proofs of many other properties (Strong Reassurance, Proof Invariance, **ULL**-consequences for normal premise sets, . . .)

AL interprets the premises as ‘normally as possible’
(no positive test!)

Whence the need for a strategy?

definitions:

Dab-formula $Dab(\Delta)$: disjunction of finite $\Delta \subset \Omega$

$Dab(\Delta)$ is a minimal Dab-consequence of Γ:
$\Gamma \vdash_{\mathbf{LLL}} Dab(\Delta)$ and, for all $\Delta' \subset \Delta$, $\Gamma \nvdash_{\mathbf{LLL}} Dab(\Delta')$

Interpreting Γ as normally as possible

Simple strategy: take $A \in \Omega$ to be false, unless $\Gamma \vdash_{\mathbf{LLL}} A$

The Simple strategy is inadequate if
$Dab(\Delta)$ is a minimal Dab-consequence of Γ and Δ is not a singleton.

- Reliability strategy: consider all members of Δ as unreliable.
- Minimal Abnormality strategy (see below)
- . . .

Adaptive logics: example 1: $\mathbf{ACLuN^r}$

- *lower limit logic*: $\mathbf{CLuN}$ $(\mathbf{CL}^+ + A \vee \sim A + \neg)$
- *set of abnormalities*: $\Omega = \{\exists(A \wedge \sim A) \mid A \in \mathcal{F}^\circ\}$
  abnormality = occurrence of (existentially closed) contradiction
- *strategy*: Reliability

upper limit logic: $\mathbf{CL} = \mathbf{CLuN} + (A \wedge \sim A) \supset B$

semantically: the $\mathbf{CLuN}$-models that verify no inconsistency

Adaptive logics: example 2: $\mathbf{IL}$

- *lower limit logic*: $\mathbf{CL}$

- *set of abnormalities*: $\boldsymbol{\Omega} = \{\exists \boldsymbol{A} \wedge \exists{\sim}\boldsymbol{A} \mid \boldsymbol{A} \in \boldsymbol{\mathcal{F}}^{\circ}\}$
  abnormality = the absence of uniformity

- *strategy*: Reliability

upper limit logic: $\mathbf{UCL} = \mathbf{CL} + \exists \boldsymbol{A} \supset \forall \boldsymbol{A}$

semantically: the models in which, for all predicates $\boldsymbol{\pi}$ of rank $\boldsymbol{r}$, $\boldsymbol{v}(\boldsymbol{\pi}) \in \{\emptyset, \boldsymbol{D}^{\boldsymbol{r}}\}$

4.3 Annotated dynamic proofs: Reliability

(rules of inference + marking definition)

a *line* consists of

- a line number
- a formula
- a justification (line numbers + rule)
- a condition (finite subset of Ω)

for all adaptive logics of the described kind:

A is derivable on the condition Δ (in the dynamic proof)

iff

$A \vee Dab(\Delta)$ is derivable (on the condition $\emptyset$) (in the dynamic proof)

iff

$\Gamma \vdash_{\mathbf{LLL}} A \vee Dab(\Delta)$

Rules of inference (depend on $\mathbf{LLL}$ and $\mathbf{\Omega}$ *not* on the strategy)

PREM If $A \in \Gamma$:

$$\begin{array}{ll} \ldots & \ldots \\ \hline A & \emptyset \end{array}$$

RU If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B$:

$$\begin{array}{ll} A_1 & \Delta_1 \\ \ldots & \ldots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \end{array}$$

RC If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B \vee Dab(\Theta)$

$$\begin{array}{ll} A_1 & \Delta_1 \\ \ldots & \ldots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \cup \Theta \end{array}$$

Marking Definition for Reliability

where $Dab(\Delta_1), \ldots, Dab(\Delta_n)$ are the minimal Dab-formulas derived on the condition $\emptyset$ at stage s, $U_s(\Gamma) = \Delta_1 \cup \ldots \cup \Delta_n$

Definition
where Δ is the condition of line i, line i is marked (at stage s) iff $\Delta \cap U_s(\Gamma) \neq \emptyset$

$\Rightarrow$ idea for consequence set applied to stage of proof

Marking Definition for Minimal Abnormality: later

Derivability at a stage vs. final derivability

idea: A derived in line i and the proof is stable with respect to i

stability concerns a specific consequence and a specific line !

Definition

A is *finally derived* from Γ on line i of a proof at stage s iff

(i) A is the second element of line i,

(ii) line i is not marked at stage s, and

(iii) any extension of the proof may be further extended in such a way that line i is unmarked.

Definition

$\Gamma \vdash_{\mathbf{AL}} A$ (A is *finally* $\mathbf{AL}$-*derivable* from Γ) iff A is finally derived in a line of a proof from Γ.

Even at the predicative level, there are criteria for final derivability.

LLL invalidates certain rules of **ULL**

AL invalidates certain applications of rules of **ULL**

ULL extends **LLL** by validating some further rules

AL extends **LLL** by validating some applications of some further rules

example

adaptive logic: $\mathbf{IL}$

- *lower limit logic*: $\mathbf{CL}$
- *set of abnormalities*: $\Omega = \{\exists A \wedge \exists{\sim}A \mid A \in \mathcal{F}^{\circ}\}$
- *strategy*: Reliability

$$\Gamma = \{(Pa \wedge {\sim}Qa) \wedge {\sim}Ra, {\sim}Pb \wedge (Qb \wedge Rb), Pc \wedge Rc, Qd \wedge {\sim}Pe\}$$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
5	$(\forall x)(Qx \supset Rx)$	2; RC	$\{Qx \supset Rx\}$
6	Rd	4, 5; RU	$\{Qx \supset Rx\}$
7	$(\forall x)(\sim Px \supset Qx)$	2; RC	$\{\sim Px \supset Qx\}$
8	Qe	4, 7; RU	$\{\sim Px \supset Qx\}$

number of data of each form immaterial

$\Rightarrow$ same generalizations derivable from $\{Pa\}$ and from $\{Pa, Pb\}$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
9^{L10}	$(\forall x)(Px \supset \sim Rx)$	1; RC	$\{Px \supset \sim Rx\}$
10	$Dab(Px \supset \sim Rx)$	1, 3; RU	$\emptyset$

1	$(Pa \wedge {\sim}Qa) \wedge {\sim}Ra$	PREM	$\emptyset$
2	${\sim}Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge {\sim}Pe$	PREM	$\emptyset$
...			
11^{L17}	$(\forall x)(Px \supset {\sim}Qx)$	1; RC	$\{Px \supset {\sim}Qx\}$
12^{L17}	${\sim}Qc$	3, 11; RU	$\{Px \supset {\sim}Qx\}$
13^{L17}	$(\forall x)(Rx \supset Qx)$	2; RC	$\{Rx \supset Qx\}$
14^{L17}	Qc	3, 13; RU	$\{Rx \supset Qx\}$
15	$(\exists x){\sim}(Px \supset {\sim}Qx) \vee (\exists x){\sim}(Rx \supset Qx)$	3; RU	$\emptyset$
16	$(\exists x)(Px \supset {\sim}Qx) \wedge (\exists x)(Rx \supset Qx)$	1, 2; RU	$\emptyset$
17	$Dab\{Px \supset {\sim}Qx, Rx \supset Qx\}$	15, 16; RU	$\emptyset$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
18^{L22}	$(\forall x)(Px \supset Sx)$	4; RC	$\{Px \supset Sx\}$
19^{L22}	Sa	1, 18; RU	$\{Px \supset Sx\}$
20	$(\exists x)\sim(Px \supset Sx) \vee (\exists x)\sim(Px \supset \sim Sx)$	3; RU	$\emptyset$
21	$(\exists x)(Px \supset Sx) \wedge (\exists x)(Px \supset \sim Sx)$	4; RU	$\emptyset$
22	$Dab\{Px \supset Sx, Px \supset \sim Sx\}$	20, 21; RU	$\emptyset$

Some theoretical stuff

a stage (of a proof) is a sequence of lines

a proof is a chain of (1 or more) stages

a subsequent stage is obtained by adding a line to the stage

the marking definition determines which lines of the stage are marked (marks may come and go with the stage)

an extension of a proof P is a proof P′ that has P as its initial fragment

Definition (repetition)
A is *finally derived* from Γ on line i of a proof at stage s iff
(i) A is the second element of line i,
(ii) line i is not marked at stage s, and
(iii) any extension of the proof may be further extended in such a way that line i is unmarked.

for some logics (esp. Minimal Abnormality strategy), premise sets and conclusions, stability (final derivability) is reached only after infinitely many stages

if a stage has infinitely many lines, the next stage is reached by inserting a line (variant)

pace Leon Horsten (transfinite proofs)

Game theoretic approaches to final derivability

example:

proponent provides proof P in which A is derived in an unmarked line i

A is finally derived (in that line)

iff

any extension (by the opponent) of P into a P′ in which i is marked

can be extended (by the proponent) into a P″ in which i is unmarked

the proponent has an ‘answer’ to any ‘attack’

4.4 Semantics

$Dab(\Delta)$ is a minimal Dab-consequence of Γ:
$\Gamma \vDash_{\mathbf{LLL}} Dab(\Delta)$ and, for all $\Delta' \subset \Delta$, $\Gamma \nvDash_{\mathbf{LLL}} Dab(\Delta')$

where M is a **LLL**-model: $Ab(M) = \{A \in \Omega \mid M \models A\}$

Reliability

where $Dab(\Delta_1)$, $Dab(\Delta_2)$, ... are the minimal Dab-consequences of Γ,
$U(\Gamma) = \Delta_1 \cup \Delta_2 \cup \ldots$

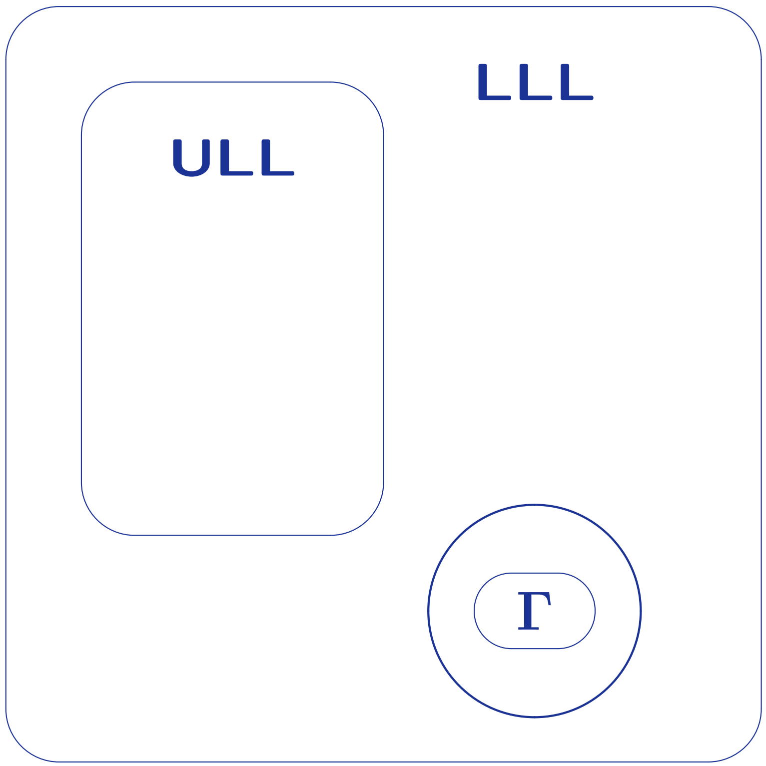
a **LLL**-model M of Γ is reliable iff $Ab(M) \subseteq U(\Gamma)$

$\Gamma \vDash_{\mathbf{AL}} A$ iff all reliable models of Γ verify A

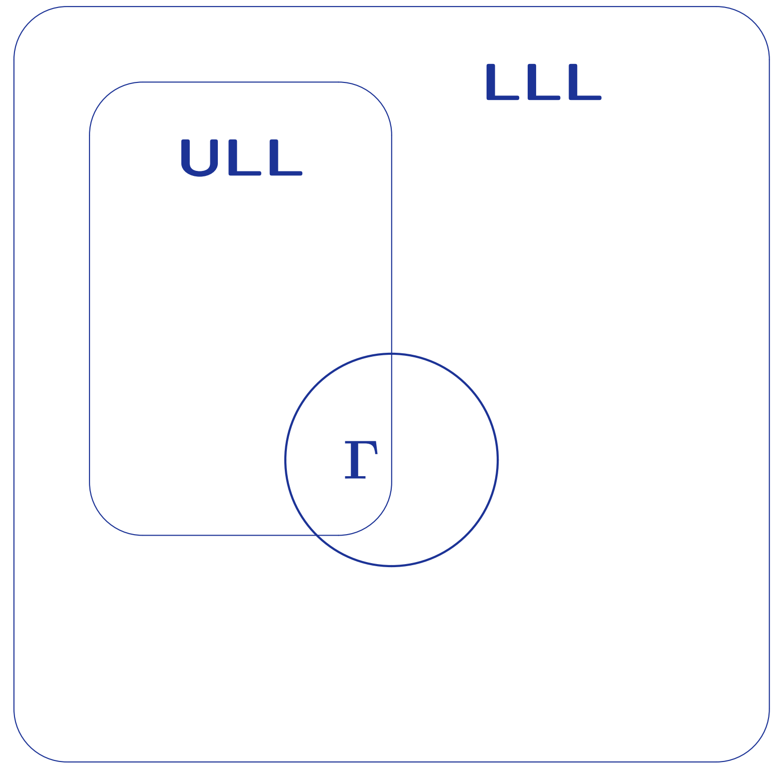
Minimal Abnormality

a $\mathbf{LLL}$-model M of Γ is minimally abnormal iff there is no $\mathbf{LLL}$-model M' for which $Ab(M') \subset Ab(M)$

$\Gamma \vDash_{\mathbf{AL}} A$ iff all minimally abnormal models of Γ verify A



Abnormal Γ



Normal Γ

there are no **AL**-models, but only **AL**-models of some $\mathbf{\Gamma}$

all **LLL**-models are **AL**-models of some $\mathbf{\Gamma}$

the **AL**-semantics selects some **LLL**-models of $\mathbf{\Gamma}$ as **AL**-models of $\mathbf{\Gamma}$

rules (as for Reliability) and marking definition

where $Dab(\Delta_1)$, ..., $Dab(\Delta_n)$ are the minimal Dab-formulas derived on the condition $\emptyset$ at stage s

$\Phi^{\circ}_s(\Gamma)$: the set of all sets that contain one member of each Δ_i

$\Phi^{\star}_s(\Gamma)$: contains, for any $\varphi \in \Phi^{\circ}_s(\Gamma)$, $Cn_{\mathbf{LLL}}(\varphi) \cap \Omega$

$\Phi_s(\Gamma)$: $\phi \in \Phi^{\star}_s(\Gamma)$ that are not proper supersets of a $\phi' \in \Phi^{\star}_s(\Gamma)$

Definition
where Δ is the condition of line i, line i is marked at stage s iff, where A derived on the condition Δ at line i, (i) there is no $\varphi \in \Phi_s(\Gamma)$ such that $\varphi \cap \Delta = \emptyset$, or (ii) for some $\varphi \in \Phi_s(\Gamma)$, there is no line at which A is derived on a condition Θ for which $\varphi \cap \Theta = \emptyset$

example: $\Gamma = \{\sim p,\ \sim q,\ p \vee q,\ p \vee r,\ q \vee s\}$

$\Gamma \vdash_{\mathbf{ACLuN}^{\mathrm{m}}} r \vee s$

$\Gamma \nvdash_{\mathbf{ACLuN}^{\mathrm{r}}} r \vee s$

4.6 Some further examples

corrective

- $\mathbf{ACLuN^r}$ and $\mathbf{ACLuN^m}$ (negation gluts)
- other paraconsistent logics as **LLL**, including **ANA** (J)
- negation gaps
- gluts/gaps for all logical symbols
- ambiguity adaptive logics (G)
- adaptive zero logic
- corrective deontic logics (J, **I**)
- (prioritized ial)
- ...

ampliative (+ ampliative and corrective)

- compatibility (characterization)
- compatibility with inconsistent premises (J, Da, L)
- diagnosis
- prioritized adaptive logics (L, E, Da, J)
- inductive generalization (D, Ln)
- abduction
- inference to the best explanation (J)
- analogies, metaphors
- erotetic inference
- discussions
- . . .

incorporation

- flat Rescher–Manor consequence relations (+ extensions)
- partial structures and pragmatic truth (J)
- prioritized Rescher–Manor consequence relations (L, Ti)
- circumscription, defaults, negation as failure, . . .
- dynamic characterization of $\mathbf{R}_{\rightarrow}$
- . . .

applications

- scientific discovery and creativity
- scientific explanation
- diagnosis (E, Da, L, J)
- positions defended / agreed upon in discussions
- changing positions in discussions
- belief revision (K)
- inconsistent arithmetic (Ti)
- evocation of questions from inconsistent premises (J, K)
- inductive statistical explanation (E)
- tentatively eliminating abnormalities
- ...

4.7 Some properties

Derivability Adjustment Theorem:

$A \in Cn_{\mathbf{ULL}}(\Gamma)$ iff $A \vee Dab(\Delta) \in Cn_{\mathbf{LLL}}(\Gamma)$ for some Δ.

Strong Reassurance

$$Cn_{\mathbf{LLL}}(\Gamma) \subseteq Cn_{\mathbf{AL}}(\Gamma) \subseteq Cn_{\mathbf{ULL}}(\Gamma)$$

...

4.8 Combined adaptive logics

- ‘union’: $\mathbf{\Omega_1} \cup \mathbf{\Omega_2}$

- sequential combination

...

example of a set of adaptive logics to combine: $\mathbf{AT^i}$

- *lower limit logic*: $\mathbf{T}$
- *set of abnormalities*: $\Omega^i = \{\lozenge^i A \wedge \sim A \mid A \in \mathcal{W}\}$
  (abnormality is falsehood of an expectancy)
- *strategy*: Reliability

upper limit logic: $\mathbf{Triv} = \mathbf{T} + \lozenge A \supset A$

many possible variants: e.g. $\Omega^i = \{\lozenge^i \forall A \wedge \sim\forall A \mid A \in \mathcal{F}^\circ\}$

we want $$\ldots Cn_{\mathbf{AT}^3}(Cn_{\mathbf{AT}^2}(Cn_{\mathbf{AT}^1}(\Gamma))) \quad (1)$$

Proofs: (skipping a couple of details)

- apply rules of $\mathbf{AT^1}$, $\mathbf{AT^2}$, ... in any order

- Marking definition: at any stage, mark for $\mathbf{AT^1}$, next for $\mathbf{AT^2}$, ...
up to the highest $\Diamond^i$ that occurs in the proof

Notwithstanding (1), a criterion may warrant final derivability after finitely many steps.