2 Prospective Dynamics: Pushing the Heuristics into the Proofs

2.1 Proofs and their Explications

CL is claimed to explicate actual proofs, for example in mathematics

This presupposes:

(1) specific meaning of the logical symbols in those contexts

2.1 Proofs and their Explications

CL is claimed to explicate actual proofs, for example in mathematics

This presupposes:

(1) specific meaning of the logical symbols in those contexts
not discussed here

2.1 Proofs and their Explications

CL is claimed to explicate actual proofs, for example in mathematics

This presupposes:

(1) specific meaning of the logical symbols in those contexts
not discussed here

(2) correct proofs classified as correct
proofs classified as correct are correct

2.1 Proofs and their Explications

CL is claimed to explicate actual proofs, for example in mathematics

This presupposes:

(1) specific meaning of the logical symbols in those contexts
not discussed here

(2) correct proofs classified as correct OK
proofs classified as correct are correct

2.1 Proofs and their Explications

CL is claimed to explicate actual proofs, for example in mathematics

This presupposes:

(1) specific meaning of the logical symbols in those contexts
not discussed here

(2) correct proofs classified as correct OK
proofs classified as correct are correct yes, but . . .

Actual proofs:

actually produced	actually published
—	—
—	—
—	—
—	—
—	—
—	—
—	—
—	—
—	
result of search process	presentation

Actual proofs:

actually produced		actually published
—		—
—		—
—	⟶	—
—	skip dead ends	—
—	skip detours	—
—	skip obvious steps	—
—	...	—
—		—
—		
result of search process		presentation

Actual proofs: result from goal-directed process

actually produced		actually published
—		—
—		—
—	⟶	—
—	skip dead ends	—
—	skip detours	—
—	skip obvious steps	—
—	...	—
—		—
—		
result of search process		presentation

Neither produced nor published proofs are explicated adequately by **CL**:

CL is too permissive, viz. not goal directed

Neither produced nor published proofs are explicated adequately by **CL**:

CL is too permissive, viz. not goal directed

for example

1	p	Prem
2	$p \vee q$	1; Add
3	$p \vee r$	1; Add
4	$p \vee s$	1; Add
...	...	

2.2 Instructions vs. rules

rule: preserves truth

instruction: permission/obligation to apply a rule
(depending on stage of the proof)

2.2 Instructions vs. rules

rule: preserves truth

instruction: permission/obligation to apply a rule
(depending on stage of the proof)

official proof: procedure = rules + universal permission

— not goal-directed

— does not explicate actually produced / published proofs

— is border case of procedure

2.2 Instructions vs. rules

rule: preserves truth

instruction: permission/obligation to apply a rule
(depending on stage of the proof)

official proof: procedure = rules + universal permission

– not goal-directed

– does not explicate actually produced / published proofs

– is border case of procedure

some **procedures** explicate actual proofs

2.3 Prospective dynamics: idea and examples

— idea:
if one looks for $\boldsymbol{A}$
and, e.g., $\boldsymbol{B} \supset \boldsymbol{A}$ was derived
then look for $\boldsymbol{B}$

2.3 Prospective dynamics: idea and examples

— idea:
if one looks for A
and, e.g., $B \supset A$ was derived
then look for B

— pushing (part of) the heuristics in the proof:
if one looks for A
and, e.g., $B \supset A$ was derived
then derive $[B]\, A$
indicating that one should look for B
(given the premises, obtaining B is sufficient to obtain A)

$$t \vee q,\ \ p \supset (q \vee {\sim}r),\ \ r \wedge s,\ \ s \supset p \ \vdash \ q$$

$$t \vee q,\ \ p \supset (q \vee {\sim}r),\ \ r \wedge s,\ \ s \supset p\ \ \vdash\ \ q$$

1	$[q]\ q$	Goal

$t \vee q,\ p \supset (q \vee \sim r),\ r \wedge s,\ s \supset p \ \vdash\ q$

1	$[q]\ q$	Goal
2	$t \vee q$	Prem

$t \vee q,\ p \supset (q \vee \sim r),\ r \wedge s,\ s \supset p \ \vdash\ q$

1	$[q]\, q$	Goal
2	$t \vee q$	Prem
3	$[\sim t]\, q$	2; ∨E

$$t \vee q,\ p \supset (q \vee \sim r),\ r \wedge s,\ s \supset p\ \vdash\ q$$

1	$[q]\, q$	Goal	
2	$t \vee q$	Prem	
3	$[\sim t]\, q$	2; $\vee$E	$\sim t \vert$

$t \vee q,\ p \supset (q \vee {\sim}r),\ r \wedge s,\ s \supset p \ \vdash\ q$

1	$[q]\ q$	Goal	
2	$t \vee q$	Prem	
3	$[{\sim}t]\ q$	2; $\vee$E	${\sim}t\vert$
4	$p \supset (q \vee {\sim}r)$	Prem	

$t \lor q,\ p \supset (q \lor {\sim}r),\ r \land s,\ s \supset p \ \vdash\ q$

1	$[q]\ q$	Goal	
2	$t \lor q$	Prem	
3	$[{\sim}t]\ q$	2; $\lor$E	${\sim}t\vert$
4	$p \supset (q \lor {\sim}r)$	Prem	
5	$[p]\ q \lor {\sim}r$	4; $\supset$E	

$t \vee q,\ p \supset (q \vee {\sim}r),\ r \wedge s,\ s \supset p \ \vdash\ q$

1	$[q]\ q$	Goal	
2	$t \vee q$	Prem	
3	$[{\sim}t]\ q$	2; $\vee$E	${\sim}t \vert$
4	$p \supset (q \vee {\sim}r)$	Prem	
5	$[p]\ q \vee {\sim}r$	4; $\supset$E	
6	$s \supset p$	Prem	

$t \vee q,\ p \supset (q \vee \sim r),\ r \wedge s,\ s \supset p\ \vdash\ q$

1	$[q]\, q$	Goal	
2	$t \vee q$	Prem	
3	$[\sim t]\, q$	2; $\vee$E	$\sim t \vert$
4	$p \supset (q \vee \sim r)$	Prem	
5	$[p]\, q \vee \sim r$	4; $\supset$E	
6	$s \supset p$	Prem	
7	$[s]\, p$	6; $\supset$E	

$t \vee q,\ p \supset (q \vee \sim r),\ r \wedge s,\ s \supset p \ \vdash\ q$

1	$[q]\ q$	Goal	
2	$t \vee q$	Prem	
3	$[\sim t]\ q$	2; $\vee$E	$\sim t\vert$
4	$p \supset (q \vee \sim r)$	Prem	
5	$[p]\ q \vee \sim r$	4; $\supset$E	
6	$s \supset p$	Prem	
7	$[s]\ p$	6; $\supset$E	
8	$r \wedge s$	Prem	

$t \vee q,\ p \supset (q \vee \sim r),\ r \wedge s,\ s \supset p \ \vdash\ q$

1	$[q]\ q$	Goal	
2	$t \vee q$	Prem	
3	$[\sim t]\ q$	2; $\vee$E	$\sim t\vert$
4	$p \supset (q \vee \sim r)$	Prem	
5	$[p]\ q \vee \sim r$	4; $\supset$E	
6	$s \supset p$	Prem	
7	$[s]\ p$	6; $\supset$E	
8	$r \wedge s$	Prem	
9	s	8; $\wedge$E	

$t \lor q,\ p \supset (q \lor {\sim}r),\ r \land s,\ s \supset p \ \vdash \ q$

1	$[q]\, q$	Goal	
2	$t \lor q$	Prem	
3	$[{\sim}t]\, q$	2; $\lor$E	${\sim}t\vert$
4	$p \supset (q \lor {\sim}r)$	Prem	
5	$[p]\, q \lor {\sim}r$	4; $\supset$E	
6	$s \supset p$	Prem	
7	$[s]\, p$	6; $\supset$E	
8	$r \land s$	Prem	
9	s	8; $\land$E	
10	p	7, 9; Trans	

$t \vee q,\; p \supset (q \vee {\sim}r),\; r \wedge s,\; s \supset p \;\vdash\; q$

1	$[q]\, q$	Goal	
2	$t \vee q$	Prem	
3	$[{\sim}t]\, q$	2; $\vee$E	${\sim}t\vert$
4	$p \supset (q \vee {\sim}r)$	Prem	
5	$[p]\, q \vee {\sim}r$	4; $\supset$E	
6	$s \supset p$	Prem	
7	$[s]\, p$	6; $\supset$E	R^{10}
8	$r \wedge s$	Prem	
9	s	8; $\wedge$E	
10	p	7, 9; Trans	

$\boldsymbol{t \vee q,\ p \supset (q \vee \sim r),\ r \wedge s,\ s \supset p\ \vdash\ q}$

1	$[\boldsymbol{q}]\ \boldsymbol{q}$	Goal	
2	$\boldsymbol{t} \vee \boldsymbol{q}$	Prem	
3	$[\sim\boldsymbol{t}]\ \boldsymbol{q}$	2; ∨E	$\sim\boldsymbol{t}\vert$
4	$\boldsymbol{p} \supset (\boldsymbol{q} \vee \sim\boldsymbol{r})$	Prem	
5	$[\boldsymbol{p}]\ \boldsymbol{q} \vee \sim\boldsymbol{r}$	4; ⊃E	
6	$\boldsymbol{s} \supset \boldsymbol{p}$	Prem	
7	$[\boldsymbol{s}]\ \boldsymbol{p}$	6; ⊃E	$\mathrm{R}^{\mathbf{10}}$
8	$\boldsymbol{r} \wedge \boldsymbol{s}$	Prem	
9	$\boldsymbol{s}$	8; ∧E	
10	$\boldsymbol{p}$	7, 9; Trans	
11	$\boldsymbol{q} \vee \sim\boldsymbol{r}$	5, 10; Trans	

$t \vee q,\ p \supset (q \vee {\sim}r),\ r \wedge s,\ s \supset p \ \vdash\ q$

1	$[q]\, q$	Goal	
2	$t \vee q$	Prem	
3	$[{\sim}t]\, q$	2; $\vee$E	${\sim}t\vert$
4	$p \supset (q \vee {\sim}r)$	Prem	
5	$[p]\, q \vee {\sim}r$	4; $\supset$E	R^{11}
6	$s \supset p$	Prem	
7	$[s]\, p$	6; $\supset$E	R^{10}
8	$r \wedge s$	Prem	
9	s	8; $\wedge$E	
10	p	7, 9; Trans	
11	$q \vee {\sim}r$	5, 10; Trans	

$t \vee q,\ p \supset (q \vee \sim r),\ r \wedge s,\ s \supset p\ \vdash\ q$

1	$[q]\ q$	Goal	
2	$t \vee q$	Prem	
3	$[\sim t]\ q$	2; ∨E	$\sim t \vert$
4	$p \supset (q \vee \sim r)$	Prem	
5	$[p]\ q \vee \sim r$	4; ⊃E	R^{11}
6	$s \supset p$	Prem	
7	$[s]\ p$	6; ⊃E	R^{10}
8	$r \wedge s$	Prem	
9	s	8; ∧E	
10	p	7, 9; Trans	
11	$q \vee \sim r$	5, 10; Trans	
12	$[r]\ q$	11; ∨E	

$t \vee q,\ p \supset (q \vee \sim r),\ r \wedge s,\ s \supset p \ \vdash \ q$

1	$[q]\, q$	Goal	
2	$t \vee q$	Prem	
3	$[\sim t]\, q$	2; $\vee$E	$\sim t\vert$
4	$p \supset (q \vee \sim r)$	Prem	
5	$[p]\, q \vee \sim r$	4; $\supset$E	R^{11}
6	$s \supset p$	Prem	
7	$[s]\, p$	6; $\supset$E	R^{10}
8	$r \wedge s$	Prem	
9	s	8; $\wedge$E	
10	p	7, 9; Trans	
11	$q \vee \sim r$	5, 10; Trans	
12	$[r]\, q$	11; $\vee$E	
13	r	8; $\wedge$E	

$t \lor q,\ p \supset (q \lor {\sim}r),\ r \land s,\ s \supset p\ \vdash\ q$

1	$[q]\, q$	Goal	
2	$t \lor q$	Prem	
3	$[{\sim}t]\, q$	2; $\lor$E	${\sim}t\vert$
4	$p \supset (q \lor {\sim}r)$	Prem	
5	$[p]\, q \lor {\sim}r$	4; $\supset$E	R^{11}
6	$s \supset p$	Prem	
7	$[s]\, p$	6; $\supset$E	R^{10}
8	$r \land s$	Prem	
9	s	8; $\land$E	
10	p	7, 9; Trans	
11	$q \lor {\sim}r$	5, 10; Trans	
12	$[r]\, q$	11; $\lor$E	
13	r	8; $\land$E	
14	q	12, 13; Trans	

$t \vee q,\ p \supset (q \vee {\sim}r),\ r \wedge s,\ s \supset p \ \vdash\ q$

1	$[q]\, q$	Goal	R^{14}
2	$t \vee q$	Prem	
3	$[{\sim}t]\, q$	2; $\vee$E	${\sim}t\vert$
4	$p \supset (q \vee {\sim}r)$	Prem	
5	$[p]\, q \vee {\sim}r$	4; $\supset$E	R^{11}
6	$s \supset p$	Prem	
7	$[s]\, p$	6; $\supset$E	R^{10}
8	$r \wedge s$	Prem	
9	s	8; $\wedge$E	
10	p	7, 9; Trans	
11	$q \vee {\sim}r$	5, 10; Trans	
12	$[r]\, q$	11; $\vee$E	R^{14}
13	r	8; $\wedge$E	
14	q	12, 13; Trans	

Incidentally:

algorithm: prospective proofs $\Rightarrow$ Fitch-style proofs

1	$p \supset (q \vee \sim r)$	Prem
2	$s \supset p$	Prem
3	$r \wedge s$	Prem
4	s	3; Sim
5	p	2, 4; MP
6	$q \vee \sim r$	1, 5; MP
7	r	3; Sim
8	q	6, 7; DS

$$\sim p \lor q \ \vdash \ p \supset q$$

$\sim p \vee q \ \vdash \ p \supset q$

1 $[p \supset q]\, p \supset q$ Goal

$\sim p \vee q \ \vdash \ p \supset q$

1	$[p \supset q]\, p \supset q$	Goal
2	$[q]\, p \supset q$	1; C⊃E

$\sim p \vee q \;\vdash\; p \supset q$

1	$[p \supset q]\, p \supset q$	Goal
2	$[q]\, p \supset q$	1; C⊃E
3	$\sim p \vee q$	Prem

$\sim p \vee q \ \vdash \ p \supset q$

1	$[p \supset q]\, p \supset q$	Goal
2	$[q]\, p \supset q$	1; C⊃E
3	$\sim p \vee q$	Prem
4	$[p]\, q$	3; ∨E

$\sim p \vee q \ \vdash \ p \supset q$

1	$[p \supset q]\, p \supset q$	Goal	
2	$[q]\, p \supset q$	1; C⊃E	$q\vert$
3	$\sim p \vee q$	Prem	
4	$[p]\, q$	3; ∨E	$p\vert$

$\sim p \vee q \ \vdash \ p \supset q$

1	$[p \supset q]\ p \supset q$	Goal	
2	$[q]\ p \supset q$	1; C⊃E	$q\vert$
3	$\sim p \vee q$	Prem	
4	$[p]\ q$	3; ∨E	$p\vert$
5	$[\sim p]\ p \supset q$	1; C⊃E	

$\sim p \vee q \ \vdash \ p \supset q$

1	$[p \supset q]\ p \supset q$	Goal	
2	$[q]\ p \supset q$	1; C⊃E	$q\vert$
3	$\sim p \vee q$	Prem	
4	$[p]\ q$	3; ∨E	$p\vert$
5	$[\sim p]\ p \supset q$	1; C⊃E	
6	$[\sim q]\ \sim p$	3; ∨E	

$\sim p \vee q \ \vdash \ p \supset q$

1	$[p \supset q]\ p \supset q$	Goal	
2	$[q]\ p \supset q$	1; C⊃E	$q\vert$
3	$\sim p \vee q$	Prem	
4	$[p]\ q$	3; ∨E	$p\vert$
5	$[\sim p]\ p \supset q$	1; C⊃E	$\sim p\vert$
6	$[\sim q]\ \sim p$	3; ∨E	$\sim q\vert$

$\sim p \vee q \;\vdash\; p \supset q$

1	$[p \supset q]\; p \supset q$	Goal	
2	$[q]\; p \supset q$	1; C⊃E	$q\vert$
3	$\sim p \vee q$	Prem	
4	$[p]\; q$	3; ∨E	$p\vert$
5	$[\sim p]\; p \supset q$	1; C⊃E	$\sim p\vert$
6	$[\sim q]\; \sim p$	3; ∨E	$\sim q\vert$
7	$[p]\; p \supset q$	2, 4; Trans	

obtain the Goal on all non-redundant conditions

$\sim p \vee q \ \vdash \ p \supset q$

1	$[p \supset q]\ p \supset q$	Goal	
2	$[q]\ p \supset q$	1; C⊃E	$q\vert$
3	$\sim p \vee q$	Prem	
4	$[p]\ q$	3; ∨E	$p\vert$
5	$[\sim p]\ p \supset q$	1; C⊃E	$\sim p\vert$
6	$[\sim q]\ \sim p$	3; ∨E	$\sim q\vert$
7	$[p]\ p \supset q$	2, 4; Trans	$p\vert$

obtain the Goal on all non-redundant conditions

$\sim p \vee q \ \vdash \ p \supset q$

1	$[p \supset q]\, p \supset q$	Goal	
2	$[q]\, p \supset q$	1; C⊃E	$q\vert$
3	$\sim p \vee q$	Prem	
4	$[p]\, q$	3; ∨E	$p\vert$
5	$[\sim p]\, p \supset q$	1; C⊃E	$\sim p\vert$
6	$[\sim q]\, \sim p$	3; ∨E	$\sim q\vert$
7	$[p]\, p \supset q$	2, 4; Trans	$p\vert$
8	$p \supset q$	5, 7; EM	

obtain the Goal on all non-redundant conditions

$\sim p \vee q \ \vdash \ p \supset q$

1	$[p \supset q]\, p \supset q$	Goal	R^8
2	$[q]\, p \supset q$	1; C⊃E	$q\vert$ R^8
3	$\sim p \vee q$	Prem	
4	$[p]\, q$	3; ∨E	$p\vert$
5	$[\sim p]\, p \supset q$	1; C⊃E	$\sim p\vert$ R^8
6	$[\sim q] \sim p$	3; ∨E	$\sim q\vert$
7	$[p]\, p \supset q$	2, 4; Trans	$p\vert$ R^8
8	$p \supset q$	5, 7; EM	

obtain the Goal on all non-redundant conditions

2.4 Prospective dynamics: characterization

Rules (prospective proof for $\Gamma \vdash G$)

Goal To introduce $[G]\,G$.

Prem To introduce A for an $A \in \Gamma$.

Trans

$$\frac{[\Delta \cup \{B\}]\,A \qquad [\Delta']\,B}{[\Delta \cup \Delta']\,A}$$

EM

$$\frac{[\Delta \cup \{B\}]\,A \qquad [\Delta' \cup \{\sim B\}]\,A}{[\Delta \cup \Delta']\,A}$$

Note: the complement of a formula:

if A has the form $\sim B$, then $*A = B$

otherwise $*A = \sim A$

Note: the complement of a formula:

if A has the form $\sim B$, then $*A = B$

otherwise $*A = \sim A$

$$\begin{aligned} *p &= \sim p \\ *\sim p &= p \\ *\sim\sim p &= \sim p \end{aligned} \qquad \begin{aligned} **p &= p \\ **\sim p &= \sim p \\ **\sim\sim p &= p \end{aligned}$$

α	α_1	α_2		β	β_1	β_2
$A \wedge B$	A	B		$\sim(A \wedge B)$	$*A$	$*B$
$A \equiv B$	$A \supset B$	$B \supset A$		$\sim(A \equiv B)$	$\sim(A \supset B)$	$\sim(B \supset A)$
$\sim(A \vee B)$	$*A$	$*B$		$A \vee B$	A	B
$\sim(A \supset B)$	A	$*B$		$A \supset B$	$*A$	B
$\sim\sim A$	A	A				

Formula analysing rules:

$$\frac{[\Delta]\,\alpha}{[\Delta]\,\alpha_1 \quad [\Delta]\,\alpha_2} \qquad \frac{[\Delta]\,\beta}{[\Delta \cup \{*\beta_2\}]\,\beta_1 \quad [\Delta \cup \{*\beta_1\}]\,\beta_2}$$

α	α_1	α_2		β	β_1	β_2
$A \wedge B$	A	B		$\sim(A \wedge B)$	$*A$	$*B$
$A \equiv B$	$A \supset B$	$B \supset A$		$\sim(A \equiv B)$	$\sim(A \supset B)$	$\sim(B \supset A)$
$\sim(A \vee B)$	$*A$	$*B$		$A \vee B$	A	B
$\sim(A \supset B)$	A	$*B$		$A \supset B$	$*A$	B
$\sim\sim A$	A	A				

Formula analysing rules:

$$\frac{[\Delta]\,\alpha}{[\Delta]\,\alpha_1 \quad [\Delta]\,\alpha_2} \qquad \frac{[\Delta]\,\beta}{[\Delta \cup \{*\beta_2\}]\,\beta_1 \quad [\Delta \cup \{*\beta_1\}]\,\beta_2}$$

Example:

$$\frac{[\Delta]\,p \wedge q}{[\Delta]\,p \quad [\Delta]\,q} \qquad \frac{[\Delta]\,p \vee q}{[\Delta \cup \{\sim q\}]\,p \quad [\Delta \cup \{\sim p\}]\,q}$$

α	α_1	α_2		β	β_1	β_2
$A \wedge B$	A	B		$\sim(A \wedge B)$	$*A$	$*B$
$A \equiv B$	$A \supset B$	$B \supset A$		$\sim(A \equiv B)$	$\sim(A \supset B)$	$\sim(B \supset A)$
$\sim(A \vee B)$	$*A$	$*B$		$A \vee B$	A	B
$\sim(A \supset B)$	A	$*B$		$A \supset B$	$*A$	B
$\sim\sim A$	A	A				

Condition analysing rules:

$$\frac{[\Delta \cup \{\alpha\}]\, A}{[\Delta \cup \{\alpha_1, \alpha_2\}]\, A} \qquad \frac{[\Delta \cup \{\beta\}]\, A}{[\Delta \cup \{\beta_1\}]\, A \quad [\Delta \cup \{\beta_2\}]\, A}$$

α	α_1	α_2		β	β_1	β_2
$A \wedge B$	A	B		$\sim(A \wedge B)$	$*A$	$*B$
$A \equiv B$	$A \supset B$	$B \supset A$		$\sim(A \equiv B)$	$\sim(A \supset B)$	$\sim(B \supset A)$
$\sim(A \vee B)$	$*A$	$*B$		$A \vee B$	A	B
$\sim(A \supset B)$	A	$*B$		$A \supset B$	$*A$	B
$\sim\sim A$	A	A				

Condition analysing rules:

$$\frac{[\Delta \cup \{\alpha\}]\, A}{[\Delta \cup \{\alpha_1, \alpha_2\}]\, A} \qquad \frac{[\Delta \cup \{\beta\}]\, A}{[\Delta \cup \{\beta_1\}]\, A \quad [\Delta \cup \{\beta_2\}]\, A}$$

Example:

$$\frac{[\Delta \cup \{q \wedge r\}]\, p}{[\Delta \cup \{q, r\}]\, p} \qquad \frac{[\Delta \cup \{q \vee r\}]\, p}{[\Delta \cup \{q\}]\, p \quad [\Delta \cup \{r\}]\, p}$$

The permissions and obligations

positive part:

1. $\mathrm{pp}(\boldsymbol{A}, \boldsymbol{A})$.
2. $\mathrm{pp}(\boldsymbol{A}, \boldsymbol{\alpha})$ if $\mathrm{pp}(\boldsymbol{A}, \boldsymbol{\alpha_1})$ or $\mathrm{pp}(\boldsymbol{A}, \boldsymbol{\alpha_2})$.
3. $\mathrm{pp}(\boldsymbol{A}, \boldsymbol{\beta})$ if $\mathrm{pp}(\boldsymbol{A}, \boldsymbol{\beta_1})$ or $\mathrm{pp}(\boldsymbol{A}, \boldsymbol{\beta_2})$.

The permissions and obligations

positive part:

1. pp(A, A).
2. pp(A, α) if pp(A, α_1) or pp(A, α_2).
3. pp(A, β) if pp(A, β_1) or pp(A, β_2).

A line with second element $[\Delta]\, A$ is marked as a dead end iff an element of Δ is not a pp of any premise.

The permissions and obligations

positive part:

1. pp(A, A).
2. pp(A, α) if pp(A, α_1) or pp(A, α_2).
3. pp(A, β) if pp(A, β_1) or pp(A, β_2).

A line with second element $[\Delta]\, A$ is marked as a dead end iff an element of Δ is not a pp of any premise.

A line with second element $[\Delta]\, A$ is marked as a redundant iff
(i) $A \in \Delta$ (not the Goal line) or
(ii) a line with second element $[\Delta']\, A$ occurs and $\Delta' \subset \Delta$.

The permissions and obligations

positive part:

1. pp(A, A).
2. pp(A, α) if pp(A, α_1) or pp(A, α_2).
3. pp(A, β) if pp(A, β_1) or pp(A, β_2).

A line with second element $[\Delta]\, A$ is marked as a dead end iff an element of Δ is not a pp of any premise.

A line with second element $[\Delta]\, A$ is marked as a redundant iff
(i) $A \in \Delta$ (not the Goal line) or
(ii) a line with second element $[\Delta']\, A$ occurs and $\Delta' \subset \Delta$.

more marks possible (e.g., inconsistent paths)

The target is the first formula in the condition of the last unmarked line.   (alternatives possible)

Phase 1:

– start with Goal rule
– apply FAR only to formula of line that has Prem-line in its path
– derive $[B_1, \ldots, B_n]\, A$ by FAR only if target is pp of A
– next, introduce a new premise A iff target is pp of A
– apply CAR only to target A after Prem and FAR are exhausted
– apply Trans only if Δ' is empty

Phase 2:

– only: new $[\Delta]\, G$ by EM, Trans or CAR from R-unmarked lines
next return to phase 1

2.5 Where went Ex Falso Quodlibet?

Let the logic defined by the procedure be $\mathbf{pCL}^-$

$$p, \sim p \nvdash_{\mathbf{pCL}^-} q$$

2.5 Where went Ex Falso Quodlibet?

Let the logic defined by the procedure be $\mathbf{pCL}^{-}$

$\boldsymbol{p}, \ {\sim}\boldsymbol{p} \ \nvdash_{\mathbf{pCL}^{-}} \ \boldsymbol{q}$

EFQ requires, for example:

EFQ To introduce $[{\sim}\boldsymbol{A}]\,\boldsymbol{G}$ for an $\boldsymbol{A} \in \boldsymbol{\Gamma}$.

This rule may be applied to every $\boldsymbol{A} \in \boldsymbol{\Gamma}$.

2.5 Where went Ex Falso Quodlibet?

Let the logic defined by the procedure be $\mathbf{pCL}^{-}$

$$p,\ {\sim}p \nvdash_{\mathbf{pCL}^{-}} q$$

EFQ requires, for example:

EFQ To introduce $[{\sim}A]\, G$ for an $A \in \Gamma$.

This rule may be applied to every $A \in \Gamma$.

EFQ is an isolated, unnatural and *ad hoc* rule.

Where $\mathbf{pCL}$ is (propositional) $\mathbf{pCL}^{-}$ + EFQ:

$\boldsymbol{\Gamma} \vdash_{\mathbf{pCL}} \boldsymbol{A}$ iff $\boldsymbol{\Gamma} \vdash_{\mathbf{CL}} \boldsymbol{A}$

Where $\mathbf{pCL}$ is (propositional) $\mathbf{pCL}^-$ + EFQ:

$\boldsymbol{\Gamma} \vdash_{\mathbf{pCL}} \boldsymbol{A}$ iff $\boldsymbol{\Gamma} \vdash_{\mathbf{CL}} \boldsymbol{A}$

That is:

If $\boldsymbol{\Gamma} \vdash_{\mathbf{CL}} \boldsymbol{A}$, the procedure will lead to a proof of $\boldsymbol{A}$ from $\boldsymbol{\Gamma}$.

If $\boldsymbol{\Gamma} \nvdash_{\mathbf{CL}} \boldsymbol{A}$, the procedure will stop without $\boldsymbol{A}$ being derived.

2.6 Some properties of CL^{-}

— natural explication of all sensible classical proofs

2.6 Some properties of CL^-

– natural explication of all sensible classical proofs
– EFQ is absent, whence isolated, and unnatural

2.6 Some properties of $\mathbf{CL}^-$

- natural explication of all sensible classical proofs
- EFQ is absent, whence isolated, and unnatural
- assigns same consequences as $\mathbf{CL}$ to consistent Γ

2.6 Some properties of $\mathbf{CL}^{-}$

– natural explication of all sensible classical proofs
– EFQ is absent, whence isolated, and unnatural
– assigns same consequences as $\mathbf{CL}$ to consistent $\mathbf{\Gamma}$
(the intended domain of application of $\mathbf{CL}$)

2.6 Some properties of CL^{-}

– natural explication of all sensible classical proofs
– EFQ is absent, whence isolated, and unnatural
– assigns same consequences as $\mathbf{CL}$ to consistent $\mathbf{\Gamma}$
(the intended domain of application of $\mathbf{CL}$)
– derives a contradiction from all inconsistent $\mathbf{\Gamma}$,
but not triviality (except in border cases)

2.6 Some properties of $\mathbf{CL}^{-}$

– natural explication of all sensible classical proofs

– EFQ is absent, whence isolated, and unnatural

– assigns same consequences as $\mathbf{CL}$ to consistent Γ (the intended domain of application of $\mathbf{CL}$)

– derives a contradiction from all inconsistent Γ, but not triviality (except in border cases)
$\Rightarrow$ assigns sensible consequence set to inconsistent Γ

2.6 Some properties of CL^{-}

▼

– natural explication of all sensible classical proofs
– EFQ is absent, whence isolated, and unnatural
– assigns same consequences as $\mathbf{CL}$ to consistent $\mathbf{\Gamma}$
(the intended domain of application of $\mathbf{CL}$)
– derives a contradiction from all inconsistent $\mathbf{\Gamma}$,
but not triviality (except in border cases)
$\Rightarrow$ assigns sensible consequence set to inconsistent $\mathbf{\Gamma}$
– resulting consequence relation:
$\star$ *characterized* by a semantics (and tableau method)

2.6 Some properties of CL^-

▼

— natural explication of all sensible classical proofs
— EFQ is absent, whence isolated, and unnatural
— assigns same consequences as $\mathbf{CL}$ to consistent $\mathbf{\Gamma}$
(the intended domain of application of $\mathbf{CL}$)
— derives a contradiction from all inconsistent $\mathbf{\Gamma}$,
but not triviality (except in border cases)
$\Rightarrow$ assigns sensible consequence set to inconsistent $\mathbf{\Gamma}$
— resulting consequence relation:
$\star$ *characterized* by a semantics (and tableau method)
$\star$ reflexive and monotonic

2.6 Some properties of $\mathbf{CL}^{-}$

▼

— natural explication of all sensible classical proofs
— EFQ is absent, whence isolated, and unnatural
— assigns same consequences as $\mathbf{CL}$ to consistent Γ
(the intended domain of application of $\mathbf{CL}$)
— derives a contradiction from all inconsistent Γ,
but not triviality (except in border cases)
$\Rightarrow$ assigns sensible consequence set to inconsistent Γ
— resulting consequence relation:
⋆ *characterized* by a semantics (and tableau method)
⋆ reflexive and monotonic
⋆ not transitive (even weak cut does not hold)

2.6 Some properties of $\mathbf{CL}^{-}$

— natural explication of all sensible classical proofs
— EFQ is absent, whence isolated, and unnatural
— assigns same consequences as $\mathbf{CL}$ to consistent Γ
(the intended domain of application of $\mathbf{CL}$)
— derives a contradiction from all inconsistent Γ,
but not triviality (except in border cases)
$\Rightarrow$ assigns sensible consequence set to inconsistent Γ
— resulting consequence relation:
⋆ *characterized* by a semantics (and tableau method)
⋆ reflexive and monotonic
⋆ not transitive (even weak cut does not hold)
but transitive if restricted to consistent premise sets

2.6 Some properties of CL^-

– natural explication of all sensible classical proofs

– EFQ is absent, whence isolated, and unnatural

– assigns same consequences as $\mathbf{CL}$ to consistent Γ
(the intended domain of application of $\mathbf{CL}$)

– derives a contradiction from all inconsistent Γ,
but not triviality (except in border cases)
$\Rightarrow$ assigns sensible consequence set to inconsistent Γ

– resulting consequence relation:

⋆ *characterized* by a semantics (and tableau method)

⋆ reflexive and monotonic

⋆ not transitive (even weak cut does not hold)
but transitive if restricted to consistent premise sets

⋆ in an interesting (specific) sense relevant

2.6 Some properties of $\mathbf{CL}^-$

— natural explication of all sensible classical proofs
— EFQ is absent, whence isolated, and unnatural
— assigns same consequences as $\mathbf{CL}$ to consistent Γ
(the intended domain of application of $\mathbf{CL}$)
— derives a contradiction from all inconsistent Γ,
but not triviality (except in border cases)
$\Rightarrow$ assigns sensible consequence set to inconsistent Γ
— resulting consequence relation:
⋆ *characterized* by a semantics (and tableau method)
⋆ reflexive and monotonic
⋆ not transitive (even weak cut does not hold)
but transitive if restricted to consistent premise sets
⋆ in an interesting (specific) sense relevant
⋆ exactly the same theorems as $\mathbf{CL}$

2.6 Some properties of $\mathbf{CL}^-$

– natural explication of all sensible classical proofs
– EFQ is absent, whence isolated, and unnatural
– assigns same consequences as $\mathbf{CL}$ to consistent Γ
(the intended domain of application of $\mathbf{CL}$)
– derives a contradiction from all inconsistent Γ,
but not triviality (except in border cases)
$\Rightarrow$ assigns sensible consequence set to inconsistent Γ
– resulting consequence relation:
⋆ *characterized* by a semantics (and tableau method)
⋆ reflexive and monotonic
⋆ not transitive (even weak cut does not hold)
but transitive if restricted to consistent premise sets
⋆ in an interesting (specific) sense relevant
⋆ exactly the same theorems as $\mathbf{CL}$
⋆ adequate w.r.t. $\mathbf{CL}$-semantics if restricted to consistent Γ

Sensible

$$p \vee q,\ \sim p,\ \sim q\ \vdash\ p \wedge \sim p$$

$$p \vee q,\ \sim p,\ \sim q\ \vdash\ q \wedge \sim q$$

To derive Russell's paradox from Frege's set theory.

Sensible

$$p \vee q,\ \sim p,\ \sim q \ \vdash\ p \wedge \sim p$$

$$p \vee q,\ \sim p,\ \sim q \ \vdash\ q \wedge \sim q$$

To derive Russell's paradox from Frege's set theory.

not sensible

$$p \vee q,\ \sim p,\ \sim q \ \vdash\ r \wedge \sim r$$

To derive from Frege's set theory that the moon is and is not a blue cheese (or that $\wp(\emptyset) = \emptyset \wedge \wp(\emptyset) \neq \emptyset$).

Sensible

$$\boldsymbol{p \vee q,\ \sim p,\ \sim q\ \vdash\ p \wedge \sim p}$$

$$\boldsymbol{p \vee q,\ \sim p,\ \sim q\ \vdash\ q \wedge \sim q}$$

To derive Russell's paradox from Frege's set theory.

not sensible

$$\boldsymbol{p \vee q,\ \sim p,\ \sim q\ \vdash\ r \wedge \sim r}$$

To derive from Frege's set theory that the moon is and is not a blue cheese (or that $\wp(\emptyset) = \emptyset \wedge \wp(\emptyset) \neq \emptyset$).

In problem-solving processes, $\mathbf{CL}^-$ need to be applied.

A semantics (Suszko: every logic has a 2-valued semantics)

$v : \mathcal{W} \mapsto \{0,1\}$ is a partial function

1. if $v(A) \in \{0,1\}$ and sub(B, A), then $v(B), v(*B) \in \{0,1\}$
2. if $v(A \wedge B) = 1$ then $v(A) = 1$ and $v(B) = 1$.
3. if $v(A \wedge B) = 0$ then $v(A) = 0$ or $v(B) = 0$.
4. if $v(A \equiv B) = 1$ then $v(A \supset B) = 1$ and $v(B \supset A) = 1$.
5. if $v(A \equiv B) = 0$ then $v(A \supset B) = 0$ or $v(B \supset A) = 0$.
6. if $v(\sim(A \vee B)) = 1$ then $v(*A) = 1$ and $v(*B) = 1$.
7. if $v(\sim(A \vee B)) = 0$ then $v(*A) = 0$ or $v(*B) = 0$.
8. if $v(\sim(A \supset B)) = 1$ then $v(A) = 1$ and $v(*B) = 1$.
9. if $v(\sim(A \supset B)) = 0$ then $v(A) = 0$ or $v(*B) = 0$.
10. if $v(\sim\sim A) = 1$ then $v(A) = 1$.
11. if $v(\sim\sim A) = 0$ then $v(A) = 0$.
12. if $v(A \vee B) = 1$ then $v(*A) = 0$ or $v(B) = 1$.
13. if $v(A \vee B) = 1$ then $v(A) = 1$ or $v(*B) = 0$.
14. if $v(A \vee B) = 0$ then $v(A) = 0$ and $v(B) = 0$.
15. if $v(A \supset B) = 1$ then $v(A) = 0$ or $v(B) = 1$.
16. if $v(A \supset B) = 1$ then $v(*A) = 1$ or $v(*B) = 0$.
17. if $v(A \supset B) = 0$ then $v(*A) = 0$ and $v(B) = 0$.
18. if $v(\sim(A \wedge B)) = 1$ then $v(A) = 0$ or $v(*B) = 1$.
19. if $v(\sim(A \wedge B)) = 1$ then $v(*A) = 1$ or $v(B) = 0$.
20. if $v(\sim(A \wedge B)) = 0$ then $v(*A) = 0$ and $v(*B) = 0$.
21. if $v(\sim(A \equiv B)) = 1$ then $v((A \supset B)) = 0$ or $v(\sim(B \supset A)) = 1$.
22. if $v(\sim(A \equiv B)) = 1$ then $v(\sim(A \supset B)) = 1$ or $v((B \supset A)) = 0$.
23. if $v(\sim(A \equiv B)) = 0$ then $v(\sim(A \supset B)) = v(\sim(B \supset A)) = 0$.
24. if $v(A) = 0$ then $v(*A) = 1$.

Definition

$A_1, \ldots, A_n \models B$ (B is a semantic consequence of $A_1, \ldots, A_n$)
iff
all valuations that verify $A_1, \ldots, A_n$
and for which B is determined,
verify B.

Definition
$A_1, \ldots, A_n \vDash B$ (B is a semantic consequence of $A_1, \ldots, A_n$)
iff
all valuations that verify $A_1, \ldots, A_n$
and for which B is determined,
verify B.

That is:

Definition
$A_1, \ldots, A_n \vDash B$ (B is a semantic consequence of $A_1, \ldots, A_n$)
iff no valuation that verifies $A_1, \ldots, A_n$ falsifies B.

Definition

$A_1, \ldots, A_n \vDash B$ (B is a semantic consequence of $A_1, \ldots, A_n$)
iff
all valuations that verify $A_1, \ldots, A_n$
and for which B is determined,
verify B.

That is:

Definition

$A_1, \ldots, A_n \vDash B$ (B is a semantic consequence of $A_1, \ldots, A_n$)
iff no valuation that verifies $A_1, \ldots, A_n$ falsifies B.

PM: three valued truth-functional semantics

Theorem

If $[A_1, \ldots, A_n]B$ is derived in a $\mathbf{pCL}^{-}$-proof for $\Gamma \vdash G$,
then $v(B) = 1$
whenever $v(A_1) = \ldots = v(A_n) = 1$ and $v(B) \in \{0, 1\}$.

Corollary

If G is derived in a $\mathbf{pCL}^{-}$-proof for $\Gamma \vdash G$,
then $\Gamma \vDash_{\mathbf{CL}^{-}} G$. (Soundness)

Theorem

If a prospective proof for $\Gamma \vdash G$ halts without G begin derived,
then $\Gamma \nvDash_{\mathbf{CL}^{-}} G$. (Completeness)

Note: Tableau method

$$\frac{\boldsymbol{T A \wedge B}}{\begin{array}{c}\boldsymbol{TA}\\ \boldsymbol{TB}\end{array}} \qquad \frac{\boldsymbol{T A \vee B}}{\begin{array}{c|c|c}\boldsymbol{F{*}A} & \boldsymbol{TA} & \boldsymbol{F{*}B}\\ & \boldsymbol{TB} & \end{array}}$$

etc. (read off from semantic clauses)

Non-logicians sometimes apply **CL** to inconsistent premises.

They consider EFQ (explosion) as a logicians trick.

Non-logicians sometimes apply **CL** to inconsistent premises.

They consider EFQ (explosion) as a logicians trick.

Logicians know: **EFQ cannot be isolated in CL**

Non-logicians sometimes apply **CL** to inconsistent premises.

They consider EFQ (explosion) as a logicians trick.

Logicians know: **EFQ cannot be isolated in CL**

avoiding EFQ requires avoiding:
– Addition or Disjunctive Syllogism
– $A \ / \ B \supset A$ or $\sim A \supset (B \wedge \sim B) \ / \ A$
– $A \ / \ B \supset A$ or $A \supset (B \wedge \sim B) \ / \ \sim A$ or $\sim\sim A \ / \ A$
– etc.

Non-logicians sometimes apply **CL** to inconsistent premises.

They consider EFQ (explosion) as a logicians trick.

Logicians know: **EFQ cannot be isolated in CL**

avoiding EFQ requires avoiding:
– Addition or Disjunctive Syllogism
– $A \;/\; B \supset A$ or $\sim A \supset (B \wedge \sim B) \;/\; A$
– $A \;/\; B \supset A$ or $A \supset (B \wedge \sim B) \;/\; \sim A$ or $\sim\sim A \;/\; A$
– etc.

However:
That EFQ cannot be isolated in CL
depends on our view on logic (mere rules vs. procedures).

The upgrade to predicate logic (minus EFQ):

is straightforward

if the procedure stops (not for all $\boldsymbol{\Gamma}$ and $\boldsymbol{A}$),

with $\boldsymbol{A}$ derived: then $\boldsymbol{\Gamma} \vdash_{\mathbf{CL}^-} \boldsymbol{A}$

with $\boldsymbol{A}$ not derived: then $\boldsymbol{\Gamma} \nvdash_{\mathbf{CL}^-} \boldsymbol{A}$

2.7 Afterthought

Hintikka:

distinction between rules and heuristics

comparison with game of chess

2.7 Afterthought

Hintikka:

distinction between rules and heuristics

comparison with game of chess

This is a mistake:

- heuristic reasoning leads to sensible proofs
- part of this reasoning can be pushed into the (object-language) proofs

Moreover: truth-in-a-model is a touchy matter:

• given $\mathbf{CL}$-models one can distinguish valid consequences from sensible consequences

Moreover: truth-in-a-model is a touchy matter:

- given **CL**-models one can distinguish valid consequences from sensible consequences

BUT:

- there is a semantics that is adequate for *sensible* reasoning in **CL**

Moreover: truth-in-a-model is a touchy matter:

- given $\mathbf{CL}$-models one can distinguish valid consequences from sensible consequences

BUT:

- there is a semantics that is adequate for *sensible* reasoning in $\mathbf{CL}$

$\boldsymbol{A}$ is a $\mathbf{CL}^{-}$-consequence of $\boldsymbol{\Gamma}$ (no $\mathbf{CL}^{-}$-model of $\boldsymbol{\Gamma}$ falsifies $\boldsymbol{A}$)
iff
$\boldsymbol{A}$ is a *sensible* $\mathbf{CL}$-consequence of $\boldsymbol{\Gamma}$

Moreover: truth-in-a-model is a touchy matter: ▼

• given $\mathbf{CL}$-models one can distinguish valid consequences from sensible consequences

BUT:

• there is a semantics that is adequate for *sensible* reasoning in $\mathbf{CL}$

A is a $\mathbf{CL}^-$-consequence of Γ (no $\mathbf{CL}^-$-model of Γ falsifies A)
iff
A is a *sensible* $\mathbf{CL}$-consequence of Γ

in other words:

sensibility can be incorporated into truth