3 Problem-solving processes

3.1 Aim and introductory remarks

backbone of formal approach to problem solving

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- $+$ corrective and ampliative logics, handling inconsistency, ...
  includes forming new hypotheses
  adaptive logics: control by conditions and marking definition
  (easy extension)

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  includes forming new hypotheses
  adaptive logics: control by conditions and marking definition
  (easy extension)

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- $+$ model-based reasoning, ...: future research

Plan

given a logic (or a set of logics) $\mathbf{L}$, we can handle the heuristics (see previous lecture)

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for solving $\{?\{A, \sim A\}\}$ by deriving A or $\sim A$ from Γ by $\mathbf{L}$

adaptive logics enable us to explicate the reasoning behind many psps
(see next lecture)

Background

- philosophy of science: Nickles, Meheus, Batens
- erotetic logic (varying on Wiśniewski)
- logic
  - adaptive logics
  - prospective dynamics
  - procedures

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problem determined by (changing) constraints

- conditions on the solution
- methodological instructions / heuristics / examples
- certainties (conceptual system . . .)

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formal but not *logic* with the usual connotations

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Formal approach

formal but not *logic* with the usual connotations

- proofs – success
- arbitrary sequence of applications of rules
- infinite consequence set
- *useless* subsequences
- deductive
- **CL**

- psp (problem solving process) – success?
- goal directed
- unique aim (possibly unspecified at outset)
- *unsuccessful* subsequences
- also other forms of reasoning
- multiplicity of logics

differences partly rely on confusion

- proof search is goal-directed process (and is a psp)
- proof search is not always successful
- no arbitrary sequences result of proof search
- proof search for *one* formula from given premises
  (but set of problems solvable by certain means)
- unsuccessful subsequences in proof search
  no ‘useless’ subsequences in goal-directed proofs
- that all logic is deductive (or is $\mathbf{CL}$) is a plain prejudice

3.2 Problem-solving processes: first elements

terminology: psp refers to *explicandum* and to *explicatum*

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terminology: psp refers to *explicandum* and to *explicatum*

- psps contain unsuccessful subsequences
  - justified at some point in the psp
  - not justified any more at later point
  - *and* vice versa

'unsuccessful' is a *dynamic* property

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· prospective dynamics (previous lecture)
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· derived problems:

$\{?\{A, \sim A\}\}$	(problem)
...	
$[B_1, \ldots, B_n]\, A$	(if $B_1, \ldots, B_n$ true, then also A)

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· derived problems:

$\{?\{A, \sim A\}\}$	(problem)
$\ldots$	
$[B_1, \ldots, B_n]\, A$	(if $B_1, \ldots, B_n$ true, then also A)
$\{?\{B_1, \sim B_1\}, \ldots, ?\{B_n, \sim B_n\}\}$	derived problem

Lines occurring in a psp

problem lines: problem = non-empty set of questions

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declarative lines

- conditional: $[\boldsymbol{B_1}, \ldots, \boldsymbol{B_n}]\, \boldsymbol{A}$
- unconditional: $[\emptyset]\, \boldsymbol{A}$, viz. $\boldsymbol{A}$

a stage of a psp: sequence of lines

a psp: chain of stages

next stage obtained by adding new line
marks may change (governed by marking definitions)

relation between stages governed by *procedure*

rehearsal

- the complement of a formula
- $\mathfrak{a}$-formulas and $\mathfrak{b}$-formulas ($\mathfrak{a}$ and $\mathfrak{b}$)
- formula analysing rules and condition analysing rules
- pp$(\boldsymbol{A}, \boldsymbol{B})$ ($\boldsymbol{A}$ is a positive part of $\boldsymbol{B}$)
- the Prem rule
- EM, EM0 and Trans
- direct answer to a question / problem

Specific rules

Where $\{?\{M, \sim M\}\}$ is the main (or original) problem:

Main Start a psp with the line:

1	$\{?\{M, \sim M\}\}$	Main

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Target rule (to choose a target that one tries to obtain)

Target If P is the problem of an unmarked problem line, and A is a direct answer of a member of P, then one may add:

k	$[A]\, A$	Target

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	1	$\{?\{M, \sim M\}\}$	Main

Target rule (to choose a target that one tries to obtain)

Target If P is the problem of an unmarked problem line, and A is a direct answer of a member of P, then one may add:

	k	$[A]\, A$	Target

Derive problems:

DP If A is an unmarked target from problem line i and $[B_1, \ldots, B_n]\, A$ is the formula of an unmarked line j, then one may add:

	k	$\{?\{B_1, \sim B_1\}, \ldots, ?\{B_n, \sim B_n\}\}$	i, j; DP

3.3 An example

main problem: $?\{p \vee q, \sim(p \vee q)\}$

premise set: $\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1 $\{?\{p \vee q, \sim(p \vee q)\}\}$ Main

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
2	$[\sim(p \vee q)]\ \sim(p \vee q)$	Target

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
2	$[\sim(p \vee q)] \sim(p \vee q)$	Target	D^3
3	$[\sim p, \sim q] \sim(p \vee q)$	2; C∼∨E	D^3

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
2	$[\sim(p \vee q)] \sim(p \vee q)$	Target	D^3
3	$[\sim p, \sim q] \sim(p \vee q)$	2; C∼∨E	D^3
4	$[p \vee q]\, p \vee q$	Target	

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
2	$[\sim(p \vee q)]\ \sim(p \vee q)$	Target	D^3
3	$[\sim p, \sim q]\ \sim(p \vee q)$	2; C$\sim\vee$E	D^3
4	$[p \vee q]\ p \vee q$	Target	
5	$[p]\ p \vee q$	4; C$\vee$E	D^5

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
2	$[\sim(p \vee q)] \sim(p \vee q)$	Target	D^3
3	$[\sim p, \sim q] \sim(p \vee q)$	2; C∼∨E	D^3
4	$[p \vee q]\ p \vee q$	Target	
5	$[p]\ p \vee q$	4; C∨E	D^5
6	$[q]\ p \vee q$	4; C∨E	

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
4	$[p \vee q]\ p \vee q$	Target
6	$[q]\ p \vee q$	4; C∨E

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$ ▼

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
4	$[p \vee q]\ p \vee q$	Target
6	$[q]\ p \vee q$	4; C∨E
7	$\{?\{q, \sim q\}\}$	4, 6; DP

pursued answer: q

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
4	$[p \vee q]\ p \vee q$	Target
6	$[q]\ p \vee q$	4; C∨E
7	$\{?\{q, \sim q\}\}$	4, 6; DP
8	$[q]\ q$	Target

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
4	$[p \vee q]\ p \vee q$	Target
6	$[q]\ p \vee q$	4; C∨E
7	$\{?\{q, \sim q\}\}$	4, 6; DP
8	$[q]\ q$	Target
9	$(q \vee u) \supset (\sim t \vee q)$	Prem

4 and 6 have no premise in their path

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$ ▼

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
4	$[p \vee q]\ p \vee q$	Target
6	$[q]\ p \vee q$	4; C∨E
7	$\{?\{q, \sim q\}\}$	4, 6; DP
8	$[q]\ q$	Target
9	$(q \vee u) \supset (\sim t \vee q)$	Prem
10	$[q \vee u]\ \sim t \vee q$	9; ⊃E

$\{\sim s, \sim u \lor r, (r \land t) \lor s, (q \lor u) \supset (\sim t \lor q), t \supset u\}$ ▼

1	$\{?\{p \lor q, \sim(p \lor q)\}\}$	Main
4	$[p \lor q]\ p \lor q$	Target
6	$[q]\ p \lor q$	4; C∨E
7	$\{?\{q, \sim q\}\}$	4, 6; DP
8	$[q]\ q$	Target
9	$(q \lor u) \supset (\sim t \lor q)$	Prem
10	$[q \lor u]\ \sim t \lor q$	9; ⊃E
11	$[q]\ \sim t \lor q$	10; C∨E

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
4	$[p \vee q]\ p \vee q$	Target	
6	$[q]\ p \vee q$	4; C∨E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q]\ q$	Target	
9	$(q \vee u) \supset (\sim t \vee q)$	Prem	
10	$[q \vee u]\ \sim t \vee q$	9; ⊃E	
11	$[q]\ \sim t \vee q$	10; C∨E	D^{12}
12	$[q, t]\ q$	11; ∨E	I^{12}

12: inoperative line

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
4	$[p \vee q]\ p \vee q$	Target	
6	$[q]\ p \vee q$	4; C∨E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q]\ q$	Target	
9	$(q \vee u) \supset (\sim t \vee q)$	Prem	
10	$[q \vee u]\ \sim t \vee q$	9; ⊃E	
11	$[q]\ \sim t \vee q$	10; C∨E	D^{12}
12	$[q, t]\ q$	11; ∨E	I^{12}
13	$[u]\ \sim t \vee q$	10; C∨E	

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$ ▼

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
4	$[p \vee q]\ p \vee q$	Target	
6	$[q]\ p \vee q$	4; C∨E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q]\ q$	Target	
9	$(q \vee u) \supset (\sim t \vee q)$	Prem	
10	$[q \vee u]\ \sim t \vee q$	9; ⊃E	
11	$[q]\ \sim t \vee q$	10; C∨E	D^{12}
12	$[q, t]\ q$	11; ∨E	I^{12}
13	$[u]\ \sim t \vee q$	10; C∨E	
14	$[u, t]\ q$	13; ∨E	

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$ ▼

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
4	$[p \vee q]\ p \vee q$	Target	
6	$[q]\ p \vee q$	4; C∨E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q]\ q$	Target	
9	$(q \vee u) \supset (\sim t \vee q)$	Prem	
10	$[q \vee u]\ \sim t \vee q$	9; ⊃E	
11	$[q]\ \sim t \vee q$	10; C∨E	D^{12}
12	$[q, t]\ q$	11; ∨E	I^{12}
13	$[u]\ \sim t \vee q$	10; C∨E	
14	$[u, t]\ q$	13; ∨E	
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
4	$[p \vee q]\ p \vee q$	Target	
6	$[q]\ p \vee q$	4; C∨E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q]\ q$	Target	
9	$(q \vee u) \supset (\sim t \vee q)$	Prem	
10	$[q \vee u]\ \sim t \vee q$	9; ⊃E	
11	$[q]\ \sim t \vee q$	10; C∨E	D^{12}
12	$[q, t]\ q$	11; ∨E	I^{12}
13	$[u]\ \sim t \vee q$	10; C∨E	
14	$[u, t]\ q$	13; ∨E	
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	
16	$[t]\ t$	Target	

cleaning up for lack of space

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

8	$[q]\, q$	Target
...	...	...
14	$[u, t]\, q$	13; ∨E
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP
16	$[t]\, t$	Target

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8	$[q]\, q$	Target
...	...	...
14	$[u, t]\, q$	13; ∨E
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP
16	$[t]\, t$	Target
17	$(r \wedge t) \vee s$	Prem

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

8	$[q]\, q$	Target
. . .	. . .	. . .
14	$[u, t]\, q$	13; ∨E
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP
16	$[t]\, t$	Target
17	$(r \wedge t) \vee s$	Prem
18	$[\sim s]\, r \wedge t$	17; ∨E

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

8	$[q]\ q$	Target
. . .	. . .	. . .
14	$[u, t]\ q$	13; ∨E
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP
16	$[t]\ t$	Target
17	$(r \wedge t) \vee s$	Prem
18	$[\sim s]\ r \wedge t$	17; ∨E
19	$[\sim s]\ t$	18; ∧E

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$ ▼

8	$[q]\, q$	Target
...	...	...
14	$[u, t]\, q$	13; ∨E
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP
16	$[t]\, t$	Target
17	$(r \wedge t) \vee s$	Prem
18	$[\sim s]\, r \wedge t$	17; ∨E
19	$[\sim s]\, t$	18; ∧E
20	$\{?\{s, \sim s\}\}$	16, 19; DP

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

8	$[q]\ q$	Target
. . .	. . .	. . .
14	$[u, t]\ q$	13; $\vee$E
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP
16	$[t]\ t$	Target
17	$(r \wedge t) \vee s$	Prem
18	$[\sim s]\ r \wedge t$	17; $\vee$E
19	$[\sim s]\ t$	18; $\wedge$E
20	$\{?\{s, \sim s\}\}$	16, 19; DP
21	$[\sim s]\ \sim s$	Target

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

8	$[q]\, q$	Target	
...	...	...	
14	$[u, t]\, q$	13; ∨E	
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	
16	$[t]\, t$	Target	
17	$(r \wedge t) \vee s$	Prem	
18	$[\sim s]\, r \wedge t$	17; ∨E	
19	$[\sim s]\, t$	18; ∧E	S^{22}
20	$\{?\{s, \sim s\}\}$	16, 19; DP	R^{22}
21	$[\sim s]\, \sim s$	Target	R^{22}
22	$\sim s$	Prem	

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$ ▼

8	$[q]\ q$	Target	
...	...	...	
14	$[u, t]\ q$	13; $\vee$E	S^{23}
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	R^{23}
16	$[t]\ t$	Target	R^{23}
17	$(r \wedge t) \vee s$	Prem	
18	$[\sim s]\ r \wedge t$	17; $\vee$E	
19	$[\sim s]\ t$	18; $\wedge$E	S^{22} R^{23}
20	$\{?\{s, \sim s\}\}$	16, 19; DP	R^{22}
21	$[\sim s]\ \sim s$	Target	R^{22}
22	$\sim s$	Prem	
23	t	19, 22; Trans	

$\{{\sim}s, {\sim}u \vee r, (r \wedge t) \vee s, (q \vee u) \supset ({\sim}t \vee q), t \supset u\}$

8	$[q]\ q$	Target	
...	...	...	
14	$[u, t]\ q$	13; ∨E	S^{23} R^{24}
15	$\{?\{u, {\sim}u\}, ?\{t, {\sim}t\}\}$	8, 14; DP	R^{23}
16	$[t]\ t$	Target	R^{23}
17	$(r \wedge t) \vee s$	Prem	
18	$[{\sim}s]\ r \wedge t$	17; ∨E	
19	$[{\sim}s]\ t$	18; ∧E	S^{22} R^{23}
20	$\{?\{s, {\sim}s\}\}$	16, 19; DP	R^{22}
21	$[{\sim}s]\ {\sim}s$	Target	R^{22}
22	${\sim}s$	Prem	
23	t	19, 22; Trans	
24	$[u]\ q$	14, 23; Trans	

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

8	$[q]\, q$	Target	
. . .	. . .	. . .	
14	$[u, t]\, q$	13; $\vee$E	S^{23} R^{24}
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	R^{23}
16	$[t]\, t$	Target	R^{23}
17	$(r \wedge t) \vee s$	Prem	
18	$[\sim s]\, r \wedge t$	17; $\vee$E	
19	$[\sim s]\, t$	18; $\wedge$E	S^{22} R^{23}
20	$\{?\{s, \sim s\}\}$	16, 19; DP	R^{22}
21	$[\sim s] \sim s$	Target	R^{22}
22	$\sim s$	Prem	
23	t	19, 22; Trans	
24	$[u]\, q$	14, 23; Trans	
25	$\{?\{u, \sim u\}\}$	8, 24; DP	

cleaning up for lack of space

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$ ▼

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
...		
6	$[q]\, p \vee q$	4; C∨E
...		
23	t	19, 22; Trans
24	$[u]\, q$	14, 23; Trans
25	$\{?\{u, \sim u\}\}$	8, 24; DP

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$ ▼

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
...		
6	$[q]\, p \vee q$	4; C∨E
...		
23	t	19, 22; Trans
24	$[u]\, q$	14, 23; Trans
25	$\{?\{u, \sim u\}\}$	8, 24; DP
26	$[u]\, u$	Target

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main
...		
6	$[q]\ p \vee q$	4; C∨E
...		
23	t	19, 22; Trans
24	$[u]\ q$	14, 23; Trans
25	$\{?\{u, \sim u\}\}$	8, 24; DP
26	$[u]\ u$	Target
27	$t \supset u$	Prem

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
. . .			
6	$[q]\, p \vee q$	4; C∨E	
. . .			
23	t	19, 22; Trans	
24	$[u]\, q$	14, 23; Trans	
25	$\{?\{u, \sim u\}\}$	8, 24; DP	
26	$[u]\, u$	Target	
27	$t \supset u$	Prem	
28	$[t]\, u$	27; ⊃E	S^{28}

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
. . .			
6	$[q]\, p \vee q$	4; C∨E	
. . .			
23	t	19, 22; Trans	
24	$[u]\, q$	14, 23; Trans	S^{29}
25	$\{?\{u, \sim u\}\}$	8, 24; DP	R^{29}
26	$[u]\, u$	Target	R^{29}
27	$t \supset u$	Prem	
28	$[t]\, u$	27; ⊃E	S^{28} R^{29}
29	u	23, 28; Trans	

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$ ▼

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
. . .			
6	$[q]\, p \vee q$	4; C∨E	S^{30}
. . .			
23	t	19, 22; Trans	
24	$[u]\, q$	14, 23; Trans	S^{29} R^{30}
25	$\{?\{u, \sim u\}\}$	8, 24; DP	R^{29}
26	$[u]\, u$	Target	R^{29}
27	$t \supset u$	Prem	
28	$[t]\, u$	27; ⊃E	S^{28} R^{29}
29	u	23, 28; Trans	
30	q	24, 29; Trans	

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$ ▼

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	R^{31}
...			
6	$[q]\, p \vee q$	4; C∨E	S^{30} R^{31}
...			
23	t	19, 22; Trans	
24	$[u]\, q$	14, 23; Trans	S^{29} R^{30}
25	$\{?\{u, \sim u\}\}$	8, 24; DP	R^{29}
26	$[u]\, u$	Target	R^{29}
27	$t \supset u$	Prem	
28	$[t]\, u$	27; ⊃E	S^{28} R^{29}
29	u	23, 28; Trans	
30	q	24, 29; Trans	
31	$p \vee q$	6, 30; Trans	

3.4 The rules and the permissions and obligations

Prem, FAR, CAR, EM, EM0, Trans, Main, Target, DP

permissions + further comments on some rules

(marking definitions follow)

Main Start a psp with the line:

1	$\{?\{M, {\sim}M\}\}$	Main

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	1	$\{?\{M, \sim M\}\}$	Main
Target	If P is the problem of an unmarked problem line, and A is a direct answer of a member of P, then one may add:		
	k	$[A]\, A$	Target

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	1	$\{?\{M, \sim M\}\}$	Main
Target	If P is the problem of an unmarked problem line, and A is a direct answer of a member of P, then one may add:		
	k	$[A]\, A$	Target
Prem	If A is an unmarked target, $B \in \Gamma$, and pp(A, B), then one may add:		
	k	B	Prem

Formula analysing rules (bring one closer to a target):

$$\frac{[\Delta]\, \mathfrak{a}}{[\Delta]\, \mathfrak{a}_1 \quad [\Delta]\, \mathfrak{a}_2} \qquad \frac{[\Delta]\, \mathfrak{b}}{[\Delta \cup \{*\mathfrak{b}_2\}]\, \mathfrak{b}_1 \quad [\Delta \cup \{*\mathfrak{b}_1\}]\, \mathfrak{b}_2}$$

FAR If C is an unmarked target,
$[\Delta]\, A$ is the formula of an unmarked line i,
$[\Delta]\, A$ / $[\Delta \cup \Delta']\, B$ is a formula analysing rule,
and pp(C, B),
then one may add:
k $[\Delta \cup \Delta']\, B$ i; R
in which R is the name of the formula analysing rule.

Condition analysing rules (reveal other means to reach a target):

$$\frac{[\boldsymbol{\Delta} \cup \{\mathfrak{a}\}]\, \boldsymbol{A}}{[\boldsymbol{\Delta} \cup \{\mathfrak{a}_1, \mathfrak{a}_2\}]\, \boldsymbol{A}} \qquad \frac{[\boldsymbol{\Delta} \cup \{\mathfrak{b}\}]\, \boldsymbol{A}}{[\boldsymbol{\Delta} \cup \{\mathfrak{b}_1\}]\, \boldsymbol{A} \quad [\boldsymbol{\Delta} \cup \{\mathfrak{b}_2\}]\, \boldsymbol{A}}$$

CAR If $\boldsymbol{A}$ is an unmarked target,
$[\boldsymbol{\Delta} \cup \{\boldsymbol{B}\}]\, \boldsymbol{A}$ is the formula of an unmarked line $\boldsymbol{i}$,
and $[\boldsymbol{\Delta} \cup \{\boldsymbol{B}\}]\, \boldsymbol{A}$ / $[\boldsymbol{\Delta} \cup \boldsymbol{\Delta}']\, \boldsymbol{A}$ is a condition analysing rule,
then one may add:

$\boldsymbol{k}$ $[\boldsymbol{\Delta} \cup \boldsymbol{\Delta}']\, \boldsymbol{A}$ $\boldsymbol{i}$; R

in which R is the name of the condition analysing rule.

Eliminate some problems without answering them:

EM0 If $[\boldsymbol{\Delta} \cup \{*\boldsymbol{A}\}]\, \boldsymbol{A}$ is the formula of a line $\boldsymbol{i}$ that is neither R-marked nor I-marked, then one may add:

$\boldsymbol{k}$ $[\boldsymbol{\Delta}]\, \boldsymbol{A}$ $\boldsymbol{i}$; EM0

EM If $\boldsymbol{A}$ is an unmarked target,
$[\boldsymbol{\Delta} \cup \{\boldsymbol{B}\}]\, \boldsymbol{A}$ and $[\boldsymbol{\Delta}' \cup \{\sim\boldsymbol{B}\}]\, \boldsymbol{A}$ are the respective formulas of the unmarked or only D-marked lines $\boldsymbol{i}$ and $\boldsymbol{j}$,
and $\boldsymbol{\Delta} \subseteq \boldsymbol{\Delta}'$ or $\boldsymbol{\Delta}' \subseteq \boldsymbol{\Delta}$,
then one may add:

$\boldsymbol{k}$ $[\boldsymbol{\Delta} \cup \boldsymbol{\Delta}']\, \boldsymbol{A}$ $\boldsymbol{i}$, $\boldsymbol{j}$; EM eliminate obtained elements from a condition (and solved questions from a problem)

and

summarize remaining problems (and paths):

Trans If $\boldsymbol{A}$ is an unmarked target,
and $[\boldsymbol{\Delta} \cup \{\boldsymbol{B}\}]\, \boldsymbol{A}$ and $[\boldsymbol{\Delta}']\, \boldsymbol{B}$ are the respective formulas of the at most S-marked (not R-, I- or D-marked) lines $\boldsymbol{i}$ and $\boldsymbol{j}$, then one may add:

$\boldsymbol{k}$	$[\boldsymbol{\Delta} \cup \boldsymbol{\Delta}']\, \boldsymbol{A}$	$\boldsymbol{i}$, $\boldsymbol{j}$; Trans

handle derived problems:

DP If A is an unmarked target from problem line i
and $[B_1, \ldots, B_n]\, A$ is the formula of an unmarked line j,
then one may add:
k $\{?\{B_1, {\sim}B_1\}, \ldots, ?\{B_n, {\sim}B_n\}\}$ i, j; DP

no instruction for applying EFQ

in view of the intended applications

(deriving predictions, finding explanations, etc.)

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the *only* exception seems to be: answering $\{?\{\Gamma \vdash_{\mathbf{CL}} A, \Gamma \nvdash_{\mathbf{CL}} A\}\}$

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the *only* exception seems to be: answering $\{?\{\Gamma \vdash_{\mathbf{CL}} A, \Gamma \nvdash_{\mathbf{CL}} A\}\}$

but every possible application seems to require $\mathbf{CL}^{-}$.

Marking definitions

redundant lines are R-marked: (unconditional A identified with $[\emptyset]\, A$)

Definition 1 *An at most S-marked declarative line i that has $[\Delta]\, A$ as its formula is R-marked at a stage iff, at that stage, $[\Theta]\, A$ is the formula of a line for some $\Theta \subset \Delta$.*

Definition 2 *An unmarked problem line i is R-marked at a stage iff, at that stage, a direct answer A of a question of line i is the formula of a line.*

the following definitions require:

- target from a problem line
- resolution line
- direct target from
- target sequence
- grounded target sequence

inoperative lines are I-marked (not useful for extant problem):

Definition 3 *An at most S-marked target line that has $[A]\,A$ as its formula is I-marked at a stage iff every problem line from which A is a target is marked at that stage.*

Definition 4 *An at most S-marked resolution line of which $[\Delta^1]\,A^1$ is the formula for some $\Delta^1 \neq \emptyset$ is* I-marked *at a stage iff, at that stage, for every grounded target sequence $\langle [\Delta^n]\,A^n, \ldots, [\Delta^1]\,A^1 \rangle$,*

(i) some target $[A^i]\,A^i$ $(1 \leq i \leq n)$ is marked, or
(ii) $\{A^n, \ldots, A^1\} \cap \Delta^1 \neq \emptyset$, or
(iii) $\Delta^1 \cup \ldots \cup \Delta^n \cup \Gamma^\circ_s$ is flatly inconsistent.

Definition 5 *An unmarked problem line is I-marked iff no unmarked resolution line generates it.*

Dead end lines are D-marked (no further action from such line)

- A is a *dead end* (A is literal and not a positive part of a premise)
- CAR-descendant of $[\Delta \cup \{A\}]\, B$

Definition 6 *An at most S-marked resolution line with formula* $[\Delta]\, A$ *is D-marked at a stage iff some* $B \in \Delta$ *is a dead end or, at that stage, all CAR-descendants of* $[\Delta]\, A$ *occur in the psp and are D-marked.*

Definition 7 *An at most S-marked target line with formula* $[A]\, A$ *is D-marked at a stage iff* A *is a dead end or no further action can be taken in view of target* A.

it can be shown that, for all consistent Γ:

(i) the procedure applied to Γ and $\{?\{A, \sim A\}\}$ results in the answer A, iff $\Gamma \vdash_{\mathbf{CL}} A$

and

(ii) the procedure applied to Γ and $\{?\{A, \sim A\}\}$ stops without the main problem being answered, or results in the answer $\sim A$ iff $\Gamma \nvdash_{\mathbf{CL}} A$

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for the predicative case:

if the procedure applied to Γ and $\{?\{A, \sim A\}\}$ stops without the main problem being answered, or results in the answer $\sim A$, then $\Gamma \nvdash_{\mathbf{CL}} A$

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because

(ii′) the procedure applied to Γ and $\{?\{A, \sim A\}\}$ stops without the main problem being answered, or results in the answer $\sim A$, *or does not stop* iff $\Gamma \nvdash_{\mathbf{CL}} A$

Speed up the procedure by S-marks

- Γ_s°: union of Γ and of the set of conditionless formulas that occur at stage s of the psp

Definition 8 *A R-unmarked resolution line in which $[\Delta^1]\, A^1$ is derived is S-marked iff*

(i) $\Delta^1 \cap \Gamma_s^\circ \neq \emptyset$, *or*

(ii) for some target sequence $\langle [\Delta^n]\, A^n, \ldots, [\Delta^1]\, A^1 \rangle$, $\{A^n\} \cup \Delta^1$ *is flatly inconsistent whereas* Δ^1 *is not flatly inconsistent, or*

(iii) $\Delta_1 \subset \Delta^n \cup \ldots \cup \Delta^2$ *for some target sequence* $\langle [\Delta^n]\, A^n, \ldots, [\Delta^1]\, A^1 \rangle$.

3.5 Answerable questions

$\mathbb{A}$ is a set of questions that can be answered by standard means (for example: observation or experiment)

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whether some $?\{A, \sim A\} \in \mathbb{A}$ is launched depends on pragmatic (economic) considerations

“dead end” needs to be redefined: $\boldsymbol{A}$ is a dead end iff it is not a positive part of a premise or of a direct answer to a member of $\mathbb{A}$.

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- $?\{\boldsymbol{A} \vee \boldsymbol{B}, \sim\boldsymbol{A} \vee \boldsymbol{B}\}$
- $?\{\boldsymbol{B} \supset \boldsymbol{A}, \sim(\boldsymbol{B} \supset \boldsymbol{A})\}$

etc.

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- $?\{\boldsymbol{B} \supset \boldsymbol{A}, \sim(\boldsymbol{B} \supset \boldsymbol{A})\}$

etc.

so $\boldsymbol{A}$ need not be an unmarked target in order for $?\{\boldsymbol{A}, \sim\boldsymbol{A}\}$ to be launched

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so better
redefine $\mathbb{A}$ as a set of couples $\langle ?\{\boldsymbol{A}, \sim\boldsymbol{A}\}, \boldsymbol{B}\rangle$ with $\boldsymbol{B} \in \{\boldsymbol{A}, \sim\boldsymbol{A}\}$

$\boldsymbol{B}$ is determined but unknown to the problem solver

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$\boldsymbol{B}$ is determined but unknown to the problem solver

that $\boldsymbol{A}$ is the target justifies launching $\langle ?\{\boldsymbol{A}, \sim\boldsymbol{A}\}, \boldsymbol{A}\rangle$ just as much as it justifies launching $\langle ?\{\boldsymbol{A}, \sim\boldsymbol{A}\}, \sim\boldsymbol{A}\rangle$ (etc.)

instruction:

New If $\boldsymbol{A}$ is the target of the unmarked target line $\boldsymbol{i}$, pp$(\boldsymbol{A}, \boldsymbol{B})$ or pp$(\boldsymbol{A}, {\sim}\boldsymbol{B})$, and $\langle ?\{\boldsymbol{B}, {\sim}\boldsymbol{B}\}, \boldsymbol{C}\rangle \in \mathbb{A}$ (where $\boldsymbol{C} \in \{\boldsymbol{B}, {\sim}\boldsymbol{B}\}$), then one may add:

$\boldsymbol{k}$ $\quad$ $\boldsymbol{C}$ $\quad$ $\boldsymbol{i}$; New

obvious from New:

which member of $\mathbb{A}$ was launched in view of which target

$\Gamma = \{(q \wedge r) \supset p, \sim s \vee q, s, \ldots\}$ $\mathbb{A} = \{\langle ?\{q \supset r, \sim(q \supset r)\}, q \supset r\rangle, \ldots\}$.

1	$\{?\{p, \sim p\}\}$	Main	R^{19}
2	$[p]\, p$	Target	R^{19}
3	$(q \wedge r) \supset p$	Prem	
4	$[q \wedge r]\, p$	3; ⊃E	R^{19}
5	$[q, r]\, p$	4; C∧E	S^{9} R^{10}
6	$\{?\{q, \sim q\}, ?\{r, \sim r\}\}$	2, 5; DP	I^{10}
7	$[r]\, r$	Target	I^{10}
8	$q \supset r$	7; New	
9	$[q]\, r$	8; C⊃E	I^{10}
10	$[q]\, p$	5, 9; Trans	R^{19}
11	$\{?\{q, \sim q\}\}$	2, 10; DP	R^{18}
12	$[q]\, q$	Target	R^{18}
13	$\sim s \vee q$	Prem	
14	$[s]\, q$	13; ∨E	S^{17} R^{18}
15	$\{?\{s, \sim s\}\}$	12, 14; DP	R^{17}
16	$[s]\, s$	Target	R^{17}
17	s	Prem	
18	q	14, 17; Trans	
19	p	10, 18; Trans	

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- extensions to other logics
  including adaptive logics to handle inconsistencies, abduction, inductive generalization, . . . (fifth lecture)

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procedure is probably not maximally efficient in its kind

· this requires further research

· it does not undermine the main aim of the enterprise:
  to delineate sensible reasoning