3 Problem-solving processes

3.1 Aim and introductory remarks

backbone of formal approach to problem solving

- aim: *explication* of problem solving processes (psps)
- backbone: solve $\{?\{A, \sim A\}\}$ by deriving A or $\sim A$ from Γ by $\mathbf{CL}$
- empirical means (observation and experiment)
- $+$ new available information (not originally seen as relevant) (easy extension)
- $+$ corrective and ampliative logics, handling inconsistency, ...
  includes forming new hypotheses
  adaptive logics: control by conditions and marking definition
  (easy extension)
- $+$ devise new empirical means: future research (seems within reach)
- $+$ model-based reasoning, ...: future research

Plan

given a logic (or a set of logics) $\mathbf{L}$, we can handle the heuristics
(see previous lecture)

viz. define the procedure
for solving $\{?\{A, \sim A\}\}$ by deriving A or $\sim A$ from Γ by $\mathbf{L}$

adaptive logics enable us to explicate the reasoning behind many psps
(see next lecture)

Background

- philosophy of science: Nickles, Meheus, Batens
- erotetic logic (varying on Wiśniewski)
- logic
  - adaptive logics
  - prospective dynamics
  - procedures

problem determined by (changing) constraints

- conditions on the solution
- methodological instructions / heuristics / examples
- certainties (conceptual system . . .)

Formal approach

formal but not *logic* with the usual connotations

- proofs – success
- arbitrary sequence of applications of rules
- infinite consequence set
- *useless* subsequences
- deductive
- **CL**

- psp (problem solving process) – success?
- goal directed
- unique aim (possibly unspecified at outset)
- *unsuccessful* subsequences
- also other forms of reasoning
- multiplicity of logics

differences partly rely on confusion

- proof search is goal-directed process (and is a psp)
- proof search is not always successful
- no arbitrary sequences result of proof search
- proof search for *one* formula from given premises
  (but set of problems solvable by certain means)
- unsuccessful subsequences in proof search
  no 'useless' subsequences in goal-directed proofs
- that all logic is deductive (or is $\mathbf{CL}$) is a plain prejudice

3.2 Problem-solving processes: first elements

terminology: psp refers to *explicandum* and to *explicatum*

- psps contain unsuccessful subsequences
  - justified at some point in the psp
  - not justified any more at later point
  - *and* vice versa

'unsuccessful' is a *dynamic* property

- psps require prospective dynamics + derived problems

· prospective dynamics (previous lecture)
now breath first (better w.r.t. problems)

· derived problems:

$\{?\{A, \sim A\}\}$	(problem)
...	
$[B_1, \ldots, B_n]\, A$	(if $B_1, \ldots, B_n$ true, then also A)
$\{?\{B_1, \sim B_1\}, \ldots, ?\{B_n, \sim B_n\}\}$	derived problem

Lines occurring in a psp

problem lines: problem = non-empty set of questions

declarative lines

- conditional: $[\boldsymbol{B_1, \ldots, B_n}]\, \boldsymbol{A}$
- unconditional: $[\emptyset]\, \boldsymbol{A}$, viz. $\boldsymbol{A}$

a stage of a psp: sequence of lines

a psp: chain of stages

next stage obtained by adding new line
marks may change (governed by marking definitions)

relation between stages governed by *procedure*

rehearsal

- the complement of a formula
- $\mathfrak{a}$-formulas and $\mathfrak{b}$-formulas ($\mathfrak{a}$ and $\mathfrak{b}$)
- formula analysing rules and condition analysing rules
- pp$(\boldsymbol{A}, \boldsymbol{B})$ ($\boldsymbol{A}$ is a positive part of $\boldsymbol{B}$)
- the Prem rule
- EM, EM0 and Trans
- direct answer to a question / problem

Where $\{?\{M, \sim M\}\}$ is the main (or original) problem:

Main Start a psp with the line:

1 $\{?\{M, \sim M\}\}$ Main

Target rule (to choose a target that one tries to obtain)

Target If P is the problem of an unmarked problem line, and A is a direct answer of a member of P, then one may add:

k $[A]\, A$ Target

Derive problems:

DP If A is an unmarked target from problem line i and $[B_1, \ldots, B_n]\, A$ is the formula of an unmarked line j, then one may add:

k $\{?\{B_1, \sim B_1\}, \ldots, ?\{B_n, \sim B_n\}\}$ i, j; DP

3.3 An example

main problem: $?\{p \vee q, \sim(p \vee q)\}$

premise set: $\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$ ▼

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
2	$[\sim(p \vee q)]\ \sim(p \vee q)$	Target	D^3
3	$[\sim p, \sim q]\ \sim(p \vee q)$	2; C∼∨E	D^3
4	$[p \vee q]\ p \vee q$	Target	
5	$[p]\ p \vee q$	4; C∨E	D^5
6	$[q]\ p \vee q$	4; C∨E	

▼

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	
4	$[p \vee q]\ p \vee q$	Target	
6	$[q]\ p \vee q$	4; C∨E	
7	$\{?\{q, \sim q\}\}$	4, 6; DP	
8	$[q]\ q$	Target	
9	$(q \vee u) \supset (\sim t \vee q)$	Prem	
10	$[q \vee u]\ \sim t \vee q$	9; ⊃E	
11	$[q]\ \sim t \vee q$	10; C∨E	D^{12}
12	$[q, t]\ q$	11; ∨E	I^{12}
13	$[u]\ \sim t \vee q$	10; C∨E	
14	$[u, t]\ q$	13; ∨E	
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	
16	$[t]\ t$	Target	

cleaning up for lack of space

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$

8	$[q]\, q$	Target	
. . .	. . .	. . .	
14	$[u, t]\, q$	13; $\vee$E	S^{23} R^{24}
15	$\{?\{u, \sim u\}, ?\{t, \sim t\}\}$	8, 14; DP	R^{23}
16	$[t]\, t$	Target	R^{23}
17	$(r \wedge t) \vee s$	Prem	
18	$[\sim s]\, r \wedge t$	17; $\vee$E	
19	$[\sim s]\, t$	18; $\wedge$E	S^{22} R^{23}
20	$\{?\{s, \sim s\}\}$	16, 19; DP	R^{22}
21	$[\sim s]\, \sim s$	Target	R^{22}
22	$\sim s$	Prem	
23	t	19, 22; Trans	
24	$[u]\, q$	14, 23; Trans	
25	$\{?\{u, \sim u\}\}$	8, 24; DP	

cleaning up for lack of space

$\{\sim s, \sim u \vee r, (r \wedge t) \vee s, (q \vee u) \supset (\sim t \vee q), t \supset u\}$ ▼

1	$\{?\{p \vee q, \sim(p \vee q)\}\}$	Main	R^{31}
...			
6	$[q]\, p \vee q$	4; C∨E	S^{30} R^{31}
...			
23	t	19, 22; Trans	
24	$[u]\, q$	14, 23; Trans	S^{29} R^{30}
25	$\{?\{u, \sim u\}\}$	8, 24; DP	R^{29}
26	$[u]\, u$	Target	R^{29}
27	$t \supset u$	Prem	
28	$[t]\, u$	27; ⊃E	S^{28} R^{29}
29	u	23, 28; Trans	
30	q	24, 29; Trans	
31	$p \vee q$	6, 30; Trans	

3.4 The rules and the permissions and obligations

Prem, FAR, CAR, EM, EM0, Trans, Main, Target, DP

permissions + further comments on some rules

(marking definitions follow)

Main	Start a psp with the line:		
	1	$\{?\{\boldsymbol{M}, \sim\boldsymbol{M}\}\}$	Main
Target	If P is the problem of an unmarked problem line, and $\boldsymbol{A}$ is a direct answer of a member of P, then one may add:		
	$\boldsymbol{k}$	$[\boldsymbol{A}]\ \boldsymbol{A}$	Target
Prem	If $\boldsymbol{A}$ is an unmarked target, $\boldsymbol{B} \in \boldsymbol{\Gamma}$, and pp$(\boldsymbol{A}, \boldsymbol{B})$, then one may add:		
	$\boldsymbol{k}$	$\boldsymbol{B}$	Prem

Formula analysing rules (bring one closer to a target):

$$\frac{[\Delta]\,\mathfrak{a}}{[\Delta]\,\mathfrak{a}_1 \quad [\Delta]\,\mathfrak{a}_2} \qquad \frac{[\Delta]\,\mathfrak{b}}{[\Delta \cup \{*\mathfrak{b}_2\}]\,\mathfrak{b}_1 \quad [\Delta \cup \{*\mathfrak{b}_1\}]\,\mathfrak{b}_2}$$

FAR If C is an unmarked target,
$[\Delta]\,A$ is the formula of an unmarked line i,
$[\Delta]\,A$ / $[\Delta \cup \Delta']\,B$ is a formula analysing rule,
and pp(C, B),
then one may add:
k $[\Delta \cup \Delta']\,B$ i; R
in which R is the name of the formula analysing rule.

Condition analysing rules (reveal other means to reach a target):

$$\frac{[\Delta \cup \{\mathfrak{a}\}]\, A}{[\Delta \cup \{\mathfrak{a}_1, \mathfrak{a}_2\}]\, A} \qquad \frac{[\Delta \cup \{\mathfrak{b}\}]\, A}{[\Delta \cup \{\mathfrak{b}_1\}]\, A \quad [\Delta \cup \{\mathfrak{b}_2\}]\, A}$$

CAR If A is an unmarked target,
$[\Delta \cup \{B\}]\, A$ is the formula of an unmarked line i,
and $[\Delta \cup \{B\}]\, A$ / $[\Delta \cup \Delta']\, A$ is a condition analysing rule,
then one may add:

k $\quad [\Delta \cup \Delta']\, A \quad$ i; R

in which R is the name of the condition analysing rule.

Eliminate some problems without answering them:

EM0 If $[\Delta \cup \{*A\}]\, A$ is the formula of a line i that is neither R-marked nor I-marked, then one may add:

k $[\Delta]\, A$ i; EM0

EM If A is an unmarked target,
$[\Delta \cup \{B\}]\, A$ and $[\Delta' \cup \{\sim B\}]\, A$ are the respective formulas of the unmarked or only D-marked lines i and j,
and $\Delta \subseteq \Delta'$ or $\Delta' \subseteq \Delta$,
then one may add:

k $[\Delta \cup \Delta']\, A$ i, j; EM eliminate obtained elements from a condition (and solved questions from a problem)

and

summarize remaining problems (and paths):

Trans If A is an unmarked target,
and $[\Delta \cup \{B\}]\, A$ and $[\Delta']\, B$ are the respective formulas of the at most S-marked (not R-, I- or D-marked) lines i and j,
then one may add:
k $[\Delta \cup \Delta']\, A$ i, j; Trans

handle derived problems:

DP If $\boldsymbol{A}$ is an unmarked target from problem line $\boldsymbol{i}$
and $[\boldsymbol{B_1}, \ldots, \boldsymbol{B_n}]\,\boldsymbol{A}$ is the formula of an unmarked line $\boldsymbol{j}$,
then one may add:
$\boldsymbol{k}$ $\quad \{?\{\boldsymbol{B_1}, \sim\boldsymbol{B_1}\}, \ldots, ?\{\boldsymbol{B_n}, \sim\boldsymbol{B_n}\}\}$ $\quad \boldsymbol{i}$, $\boldsymbol{j}$; DP

no instruction for applying EFQ

in view of the intended applications

(deriving predictions, finding explanations, etc.)

the *only* exception seems to be: answering $\{?\{\Gamma \vdash_{\mathbf{CL}} A, \Gamma \nvdash_{\mathbf{CL}} A\}\}$

but every possible application seems to require $\mathbf{CL}^{-}$.

Marking definitions

redundant lines are R-marked: (unconditional A identified with $[\emptyset]\,A$)

Definition 1 *An at most S-marked declarative line* i *that has* $[\Delta]\,A$ *as its formula is R-marked at a stage iff, at that stage,* $[\Theta]\,A$ *is the formula of a line for some* $\Theta \subset \Delta$.

Definition 2 *An unmarked problem line* i *is R-marked at a stage iff, at that stage, a direct answer* A *of a question of line* i *is the formula of a line.*

the following definitions require:

- target from a problem line
- resolution line
- direct target from
- target sequence
- grounded target sequence

inoperative lines are I-marked (not useful for extant problem):

Definition 3 *An at most S-marked target line that has* $[\boldsymbol{A}]\,\boldsymbol{A}$ *as its formula is I-marked at a stage iff every problem line from which* $\boldsymbol{A}$ *is a target is marked at that stage.*

Definition 4 *An at most S-marked resolution line of which* $[\boldsymbol{\Delta}^1]\,\boldsymbol{A}^1$ *is the formula for some* $\boldsymbol{\Delta}^1 \neq \emptyset$ *is* I-marked *at a stage iff, at that stage, for every grounded target sequence* $\langle [\boldsymbol{\Delta}^n]\,\boldsymbol{A}^n, \ldots, [\boldsymbol{\Delta}^1]\,\boldsymbol{A}^1 \rangle$*,*

(i) *some target* $[\boldsymbol{A}^i]\,\boldsymbol{A}^i$ *(*$1 \leq \boldsymbol{i} \leq \boldsymbol{n}$*) is marked, or*
(ii) $\{\boldsymbol{A}^n, \ldots, \boldsymbol{A}^1\} \cap \boldsymbol{\Delta}^1 \neq \emptyset$*, or*
(iii) $\boldsymbol{\Delta}^1 \cup \ldots \cup \boldsymbol{\Delta}^n \cup \boldsymbol{\Gamma}^\circ_{\boldsymbol{s}}$ *is flatly inconsistent.*

Definition 5 *An unmarked problem line is I-marked iff no unmarked resolution line generates it.*

Dead end lines are D-marked (no further action from such line)

- $\boldsymbol{A}$ is a *dead end* ($\boldsymbol{A}$ is literal and not a positive part of a premise)
- CAR-descendant of $[\boldsymbol{\Delta} \cup \{\boldsymbol{A}\}]\, \boldsymbol{B}$

Definition 6 *An at most S-marked resolution line with formula* $[\boldsymbol{\Delta}]\, \boldsymbol{A}$ *is D-marked at a stage iff some* $\boldsymbol{B} \in \boldsymbol{\Delta}$ *is a dead end or, at that stage, all CAR-descendants of* $[\boldsymbol{\Delta}]\, \boldsymbol{A}$ *occur in the psp and are D-marked.*

Definition 7 *An at most S-marked target line with formula* $[\boldsymbol{A}]\, \boldsymbol{A}$ *is D-marked at a stage iff* $\boldsymbol{A}$ *is a dead end or no further action can be taken in view of target* $\boldsymbol{A}$*.*

it can be shown that, for all consistent Γ:

(i) the procedure applied to Γ and $\{?\{A, \sim A\}\}$ results in the answer A, iff $\Gamma \vdash_{\mathbf{CL}} A$

and

(ii) the procedure applied to Γ and $\{?\{A, \sim A\}\}$ stops without the main problem being answered, or results in the answer $\sim A$ iff $\Gamma \nvdash_{\mathbf{CL}} A$

for the predicative case:

if the procedure applied to Γ and $\{?\{A, \sim A\}\}$ stops without the main problem being answered, or results in the answer $\sim A$, then $\Gamma \nvdash_{\mathbf{CL}} A$

because

(ii′) the procedure applied to Γ and $\{?\{A, \sim A\}\}$ stops without the main problem being answered, or results in the answer $\sim A$, *or does not stop* iff $\Gamma \nvdash_{\mathbf{CL}} A$

Speed up the procedure by S-marks

- Γ_s°: union of Γ and of the set of conditionless formulas that occur at stage s of the psp

Definition 8 *A R-unmarked resolution line in which $[\Delta^1]\, A^1$ is derived is S-marked iff*

(i) $\Delta^1 \cap \Gamma_s^\circ \neq \emptyset$, *or*
(ii) for some target sequence $\langle [\Delta^n]\, A^n, \ldots, [\Delta^1]\, A^1 \rangle$, $\{A^n\} \cup \Delta^1$ *is flatly inconsistent whereas* Δ^1 *is not flatly inconsistent, or*
(iii) $\Delta_1 \subset \Delta^n \cup \ldots \cup \Delta^2$ *for some target sequence* $\langle [\Delta^n]\, A^n, \ldots, [\Delta^1]\, A^1 \rangle$.

3.5 Answerable questions

$\mathbb{A}$ is a set of questions that can be answered by standard means (for example: observation or experiment)

idea: whenever an unmarked target is a positive part of a direct answer of a member of $\mathbb{A}$, that question can be answered (outside the proof) and the answer can be introduced as a new premise

whether some $?\{A, \sim A\} \in \mathbb{A}$ is launched depends on pragmatic (economic) considerations “dead end” needs to be redefined: $\boldsymbol{A}$ is a dead end iff it is not a positive part of a premise or of a direct answer to a member of $\mathbb{A}$.

a target $\boldsymbol{A}$ justifies asking the answerable question
· $?\{\boldsymbol{A}, {\sim}\boldsymbol{A}\}$
· $?\{\boldsymbol{A} \wedge \boldsymbol{B}, {\sim}\boldsymbol{A} \wedge \boldsymbol{B}\}$
· $?\{\boldsymbol{A} \vee \boldsymbol{B}, {\sim}\boldsymbol{A} \vee \boldsymbol{B}\}$
· $?\{\boldsymbol{B} \supset \boldsymbol{A}, {\sim}(\boldsymbol{B} \supset \boldsymbol{A})\}$
etc.

so $\boldsymbol{A}$ need not be an unmarked target in order for $?\{\boldsymbol{A}, {\sim}\boldsymbol{A}\}$ to be launched a launched question is answered *outside* the psp

its answer is introduced as a new premise

this is awkward from a logical point of view

so better
redefine $\mathbb{A}$ as a set of couples $\langle ?\{\boldsymbol{A}, \sim\boldsymbol{A}\}, \boldsymbol{B}\rangle$ with $\boldsymbol{B} \in \{\boldsymbol{A}, \sim\boldsymbol{A}\}$

$\boldsymbol{B}$ is determined but unknown to the problem solver

that $\boldsymbol{A}$ is the target justifies launching $\langle ?\{\boldsymbol{A}, \sim\boldsymbol{A}\}, \boldsymbol{A}\rangle$ just as much as it justifies launching $\langle ?\{\boldsymbol{A}, \sim\boldsymbol{A}\}, \sim\boldsymbol{A}\rangle$ (etc.)

instruction:

New — If A is the target of the unmarked target line i, pp(A, B) or pp($A, \sim B$), and $\langle ?\{B, \sim B\}, C\rangle \in \mathbb{A}$ (where $C \in \{B, \sim B\}$), then one may add:

$k \quad C \qquad i$; New

obvious from New:

which member of $\mathbb{A}$ was launched in view of which target

$\Gamma = \{(q \wedge r) \supset p, \sim s \vee q, s, \ldots\}$ $\mathbb{A} = \{\langle ?\{q \supset r, \sim(q \supset r)\}, q \supset r\rangle, \ldots\}$.

1	$\{?\{p, \sim p\}\}$	Main	R^{19}
2	$[p]\, p$	Target	R^{19}
3	$(q \wedge r) \supset p$	Prem	
4	$[q \wedge r]\, p$	3; ⊃E	R^{19}
5	$[q, r]\, p$	4; C∧E	S^{9} R^{10}
6	$\{?\{q, \sim q\}, ?\{r, \sim r\}\}$	2, 5; DP	I^{10}
7	$[r]\, r$	Target	I^{10}
8	$q \supset r$	7; New	
9	$[q]\, r$	8; C⊃E	I^{10}
10	$[q]\, p$	5, 9; Trans	R^{19}
11	$\{?\{q, \sim q\}\}$	2, 10; DP	R^{18}
12	$[q]\, q$	Target	R^{18}
13	$\sim s \vee q$	Prem	
14	$[s]\, q$	13; ∨E	S^{17} R^{18}
15	$\{?\{s, \sim s\}\}$	12, 14; DP	R^{17}
16	$[s]\, s$	Target	R^{17}
17	s	Prem	
18	q	14, 17; Trans	
19	p	10, 18; Trans	

3.6 Variants, extensions and comments

- procedural variants
- extensions to other logics
  including adaptive logics to handle inconsistencies, abduction, inductive generalization, . . . (fifth lecture)

procedure was not intended to be maximally efficient in view of its aim:
(i) explicate actual problem-solving processes
(ii) avoid steps that are useless with respect to the main problem, the premises and the directly answerable questions
(iii) easily generalizable to other logics (devise prospective dynamics)

procedure is probably not maximally efficient in its kind
· this requires further research
· it does not undermine the main aim of the enterprise:
  to delineate sensible reasoning