5 Prospective Dynamics for Adaptive Logics

5.1 The problem

in general: no positive and no negative test for $\mathbf{AL}$-derivability

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to keep the discussion within bounds, I shall only consider the Reliability strategy for adaptive logics in standard format

5.2 Derivability at a Stage

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To derive A at a stage on some condition Δ we need:

– the prospective instructions for the lower limit logic

– some ‘Basic Schema’ instruction for the adaptive logic

General Basic Schema:

$$\frac{A \vee Dab(\Theta) \qquad\qquad \Delta}{A \qquad\qquad\qquad\qquad \Delta \cup \Theta}$$

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Basic Schema for $\mathbf{ACLuN}^r$ and $\mathbf{ACLuN}^m$:

$$\frac{\exists{\sim}A \vee B \qquad\qquad \Delta}{\exists\neg A \vee B \qquad\qquad \Delta \cup \{\exists(A \wedge {\sim}A)}$$

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Basic Schema for $\mathbf{IL}^r$ and $\mathbf{IL}^m$:

where $A \in \mathcal{F}^\circ$:

$$\frac{\exists A \vee B \qquad\qquad \Delta}{\forall A \vee B \qquad\qquad \Delta \cup \{\exists A \wedge \exists{\sim}A)}$$

These rules, together with the standard instructions for phase 1 and phase 2 (here called phase 1.1 and phase 1.2), lead to $\boldsymbol{A}$ being (or not being) derived on some condition $\boldsymbol{\Delta}$.

5.3 Implementation for ACLuN^r and ACLuN^m (propositionally)

Structural rules:

Prem	If $A \in \Gamma$, introduce $[\emptyset]\, A$ on the condition $\emptyset$.
Goal	Introduce $[G]\, G$ on the condition $\emptyset$.
EFQ	If $A \in \Gamma$, introduce $[\neg A]\, G$ on the condition $\emptyset$.

$\neg A$ is the *classical* negation of A; $*A$ is the *classical* complement of A

$\mathfrak{a}$	$\mathfrak{a}_1$	$\mathfrak{a}_2$		$\mathfrak{b}$	$\mathfrak{b}_1$	$\mathfrak{b}_2$
$A \wedge B$	A	B		$\neg(A \wedge B)$	$*A$	$*B$
$A \equiv B$	$A \supset B$	$B \supset A$		$\neg(A \equiv B)$	$\neg(A \supset B)$	$\neg(B \supset A)$
$\neg(A \vee B)$	$*A$	$*B$		$A \vee B$	A	B
$\neg(A \supset B)$	A	$*B$		$A \supset B$	$*A$	B
$\neg\neg A$	A	A				

pp($\neg A, \sim A$) and pp($\sim A, \neg A$)

Formula analysing rules:

$$\frac{[\Delta]\ \mathfrak{a} \quad \Theta}{[\Delta]\ \mathfrak{a}_1 \quad \Theta} \qquad \frac{[\Delta]\ \mathfrak{a} \quad \Theta}{[\Delta]\ \mathfrak{a}_2 \quad \Theta}$$

$$\frac{[\Delta]\ \mathfrak{b} \quad \Theta}{[\Delta \cup \{*\mathfrak{b}_2\}]\ \mathfrak{b}_1 \quad \Theta} \qquad \frac{[\Delta]\ \mathfrak{b} \quad \Theta}{[\Delta \cup \{*\mathfrak{b}_1\}]\ \mathfrak{b}_2 \quad \Theta}$$

Plus:

$$\sim\text{E} \quad \frac{[\Delta] \sim A \quad \Theta}{[\Delta]\ \neg A \quad \Theta \cup \{A \wedge \sim A\}} \qquad \neg\sim\text{E} \quad \frac{[\Delta]\ \neg\sim A \quad \Theta}{[\Delta]\ A \quad \Theta}$$

Condition analysing rules:

$$\frac{[\boldsymbol{\Delta} \cup \{\mathfrak{a}\}]\ A \qquad\qquad \boldsymbol{\Theta}}{[\boldsymbol{\Delta} \cup \{\mathfrak{a}_1, \mathfrak{a}_2\}]\ A \quad \boldsymbol{\Theta}}$$

$$\frac{[\boldsymbol{\Delta} \cup \{\mathfrak{b}\}]\ A \qquad \boldsymbol{\Theta}}{[\boldsymbol{\Delta} \cup \{\mathfrak{b}_1\}]\ A \quad \boldsymbol{\Theta}} \qquad \frac{[\boldsymbol{\Delta} \cup \{\mathfrak{b}\}]\ A \qquad \boldsymbol{\Theta}}{[\boldsymbol{\Delta} \cup \{\mathfrak{b}_2\}]\ A \quad \boldsymbol{\Theta}}$$

Plus:

$$\text{C}{\sim}\text{E} \quad \frac{[\boldsymbol{\Delta} \cup \{{\sim}B\}]\ A \quad \boldsymbol{\Theta}}{[\boldsymbol{\Delta} \cup \{\neg B\}]\ A \quad \boldsymbol{\Theta}} \quad \text{C}\neg{\sim}\text{E} \quad \frac{[\boldsymbol{\Delta} \cup \{\neg{\sim}B\}]\ A \quad \boldsymbol{\Theta}}{[\boldsymbol{\Delta} \cup \{B\}]\ A \qquad \boldsymbol{\Theta} \cup \{B \wedge {\sim}B\}}$$

Further rules:

Trans

$$\frac{\begin{array}{ll}[\Delta \cup \{B\}]\, A & \Theta \\ [\Delta']\, B & \Theta'\end{array}}{\begin{array}{ll}[\Delta \cup \Delta']\, A & \Theta \cup \Theta'\end{array}}$$

EM

$$\frac{\begin{array}{ll}[\Delta \cup \{B\}]\, A & \Theta \\ [\Delta' \cup \{\neg B\}]\, A & \Theta'\end{array}}{\begin{array}{ll}[\Delta \cup \Delta']\, A & \Theta \cup \Theta'\end{array}}$$

EM0

$$\frac{\begin{array}{ll}[\Delta \cup \{\neg A\}]\, A & \Theta\end{array}}{\begin{array}{ll}[\Delta]\, A & \Theta\end{array}}$$

IC

$$\frac{\begin{array}{ll}[\Delta]\, Dab(\Lambda \cup \Lambda') & \Theta \cup \Lambda'\end{array}}{\begin{array}{ll}[\Delta]\, Dab(\Lambda \cup \Lambda') & \Theta\end{array}}$$

5.4 Implementation for $\mathbf{IL}^r$ and $\mathbf{IL}^m$

replace $\neg$ by the standard negation $\sim$

— in the table for $\mathfrak{a}$-formulas and $\mathfrak{b}$-formulas

— in the standard formula analysing rules and condition analysing rules (without the "plus")

restore pp as for $\mathbf{CL}$

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add pp$(\forall \boldsymbol{A}, \exists \boldsymbol{A})$

Add to the formula analysing rules:

$$\exists\mathsf{E}' \quad \frac{[\boldsymbol{\Delta}]\ \exists \boldsymbol{A} \vee \boldsymbol{B} \quad \boldsymbol{\Theta}}{[\boldsymbol{\Delta}]\ \forall \boldsymbol{A} \vee \boldsymbol{B} \quad \boldsymbol{\Theta} \cup \{\exists \boldsymbol{A} \wedge \exists{\sim}\boldsymbol{A}\}}$$

Add to the condition analysing rules:

$$\exists\mathsf{E}' \quad \frac{[\boldsymbol{\Delta} \cup \{\forall \boldsymbol{B}\}]\ \boldsymbol{A} \quad \boldsymbol{\Theta}}{[\boldsymbol{\Delta} \cup \{\exists \boldsymbol{B}\}]\ \boldsymbol{A} \quad \boldsymbol{\Theta} \cup \{\exists \boldsymbol{B} \wedge \exists{\sim}\boldsymbol{B}\}}$$

Prem, Goal, [EFQ], Trans, EM, EM0, IC

5.5 Procedural Criterion for Final Derivability

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$\boldsymbol{A}$ is finally derivable iff $\boldsymbol{\Delta} \cap \boldsymbol{U}(\boldsymbol{\Gamma}) = \emptyset$

How can we find out whether $\boldsymbol{\Delta} \cap \boldsymbol{U}(\boldsymbol{\Gamma}) = \emptyset$?

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In this section, keep reading $\sim$ as *classical* negation.

We obtained A on the condition Δ.

A is finally derivable iff $\Delta \cap U(\Gamma) = \emptyset$

How can we find out whether $\Delta \cap U(\Gamma) = \emptyset$?

For *all* Θ, if $Dab(\Delta)$ is derivable on a condition Θ with $\Delta \cap \Theta = \emptyset$,

i.e. $\Gamma \vdash_{\mathbf{LLL}} Dab(\Delta \cup \Theta)$ and $\Delta \cap \Theta = \emptyset$,

then $Dab(\Theta)$ must be derivable on the condition $\emptyset$.

In other words:

After obtaining $\boldsymbol{A}$ on the condition $\boldsymbol{\Delta}$,

we introduce the A-Goal $[\boldsymbol{Dab}(\boldsymbol{\Delta})]\, \boldsymbol{Dab}(\boldsymbol{\Delta})$

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After obtaining $\boldsymbol{A}$ on the condition $\boldsymbol{\Delta}$,

we introduce the A-Goal $[\boldsymbol{Dab}(\boldsymbol{\Delta})]\,\boldsymbol{Dab}(\boldsymbol{\Delta})$

Next, if $\boldsymbol{Dab}(\boldsymbol{\Delta})$ is derived on the condition $\boldsymbol{\Theta}$ (with $\boldsymbol{\Delta} \cap \boldsymbol{\Theta} = \emptyset$),

we introduce the X-Goal $[\boldsymbol{Dab}(\boldsymbol{\Theta})]\,\boldsymbol{Dab}(\boldsymbol{\Theta})$,

which should be reached on the condition $\emptyset$.

Phase 1: try to derive A on a condition Δ

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no success: $\boldsymbol{\Gamma} \nvdash_{\mathbf{AL}} \boldsymbol{A}$
success: try to show that all members of $\boldsymbol{\Delta}$ are reliable, viz. move to

Phase 2: try to derive $\boldsymbol{Dab}(\boldsymbol{\Delta})$ on a condition $\boldsymbol{\Theta}$ (with $\boldsymbol{\Delta} \cap \boldsymbol{\Theta} = \emptyset$)

no success: no member of $\boldsymbol{\Delta}$ is unreliable and hence $\boldsymbol{\Gamma} \vdash_{\mathbf{AL}} \boldsymbol{A}$

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Phase 3: try to derive $\boldsymbol{Dab}(\boldsymbol{\Theta})$ on the condition $\emptyset$

no success: some member of $\boldsymbol{\Delta}$ is unreliable
go back to phase 1 and try to derive $\boldsymbol{A}$ on a new condition $\boldsymbol{\Delta}'$

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no success: some member of $\boldsymbol{\Delta}$ is unreliable
go back to phase 1 and try to derive $\boldsymbol{A}$ on a new condition $\boldsymbol{\Delta}'$
success: $\boldsymbol{Dab}(\boldsymbol{\Theta})$ is a minimal $\boldsymbol{Dab}$-consequence of $\boldsymbol{\Gamma}$
go back to phase 2 and try to derive $\boldsymbol{Dab}(\boldsymbol{\Delta})$ on a new condition $\boldsymbol{\Theta}'$
$(\boldsymbol{\Delta} \cap \boldsymbol{\Theta} = \emptyset)$

5.6 Phrase the Criterion in Problem Solving Terms

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(PM)

This leads to problems that contain questions of which one direct answer only is sensible.

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Complete insight in the premises may be reached with respect to a specific A.