5 Prospective Dynamics for Adaptive Logics

5.1 The problem

in general: no positive and no negative test for $\mathbf{AL}$-derivability

What is the aim of the procedure?

1. Procedure for deriving a desired consequence from Γ at a stage (and on some condition).

2. Can the procedure be continued in such a way that it forms a criterion for deciding that the formula is finally derived at a line in a proof from Γ?

3. What if no criterion applies?

to keep the discussion within bounds, I shall only consider the Reliability strategy for adaptive logics in standard format

5.2 Derivability at a Stage

Aim: to find out whether $\Gamma \vdash_{\mathbf{AL}} A$

Alternatively: given Γ and $\mathbf{AL}$, to answer the question $?\{A, \sim A\}$

To derive A at a stage on some condition Δ we need:

– the prospective instructions for the lower limit logic

– some 'Basic Schema' instruction for the adaptive logic

General Basic Schema:

$$\frac{\boldsymbol{A} \vee \boldsymbol{Dab}(\boldsymbol{\Theta}) \qquad \boldsymbol{\Delta}}{\boldsymbol{A} \qquad \boldsymbol{\Delta} \cup \boldsymbol{\Theta}}$$

Basic Schema for $\mathbf{ACLuN}^r$ and $\mathbf{ACLuN}^m$:

$$\frac{\exists{\sim}\boldsymbol{A} \vee \boldsymbol{B} \qquad \boldsymbol{\Delta}}{\exists\neg\boldsymbol{A} \vee \boldsymbol{B} \qquad \boldsymbol{\Delta} \cup \{\exists(\boldsymbol{A} \wedge {\sim}\boldsymbol{A})}$$

Basic Schema for $\mathbf{IL}^r$ and $\mathbf{IL}^m$:

where $\boldsymbol{A} \in \mathcal{F}^{\circ}$:

$$\frac{\exists\boldsymbol{A} \vee \boldsymbol{B} \qquad \boldsymbol{\Delta}}{\forall\boldsymbol{A} \vee \boldsymbol{B} \qquad \boldsymbol{\Delta} \cup \{\exists\boldsymbol{A} \wedge \exists{\sim}\boldsymbol{A})}$$

These rules, together with the standard instructions for phase 1 and phase 2 (here called phase 1.1 and phase 1.2), lead to $\boldsymbol{A}$ being (or not being) derived on some condition $\boldsymbol{\Delta}$.

5.3 Implementation for ACLuN^r and ACLuN^m (propositionally)

Structural rules:

Prem	If $A \in \Gamma$, introduce $[\emptyset]\, A$ on the condition $\emptyset$.
Goal	Introduce $[G]\, G$ on the condition $\emptyset$.
EFQ	If $A \in \Gamma$, introduce $[\neg A]\, G$ on the condition $\emptyset$.

$\neg A$ is the *classical* negation of A; $*A$ is the *classical* complement of A

$\mathfrak{a}$	$\mathfrak{a}_1$	$\mathfrak{a}_2$		$\mathfrak{b}$	$\mathfrak{b}_1$	$\mathfrak{b}_2$
$A \wedge B$	A	B		$\neg(A \wedge B)$	$*A$	$*B$
$A \equiv B$	$A \supset B$	$B \supset A$		$\neg(A \equiv B)$	$\neg(A \supset B)$	$\neg(B \supset A)$
$\neg(A \vee B)$	$*A$	$*B$		$A \vee B$	A	B
$\neg(A \supset B)$	A	$*B$		$A \supset B$	$*A$	B
$\neg\neg A$	A	A				

pp$(\neg A, \sim A)$ and pp$(\sim A, \neg A)$

Formula analysing rules:

$$\frac{[\Delta]\ \mathfrak{a} \quad \Theta}{[\Delta]\ \mathfrak{a}_1 \quad \Theta} \qquad \frac{[\Delta]\ \mathfrak{a} \quad \Theta}{[\Delta]\ \mathfrak{a}_2 \quad \Theta}$$

$$\frac{[\Delta]\ \mathfrak{b} \quad \Theta}{[\Delta \cup \{*\mathfrak{b}_2\}]\ \mathfrak{b}_1 \quad \Theta} \qquad \frac{[\Delta]\ \mathfrak{b} \quad \Theta}{[\Delta \cup \{*\mathfrak{b}_1\}]\ \mathfrak{b}_2 \quad \Theta}$$

Plus:

$$\sim\mathsf{E}\ \frac{[\Delta]\ {\sim}A \quad \Theta}{[\Delta]\ \neg A \quad \Theta \cup \{A \wedge {\sim}A\}} \qquad \neg{\sim}\mathsf{E}\ \frac{[\Delta]\ \neg{\sim}A \quad \Theta}{[\Delta]\ A \quad \Theta}$$

Condition analysing rules:

$$\frac{[\boldsymbol{\Delta} \cup \{\mathfrak{a}\}]\, A \qquad\qquad \boldsymbol{\Theta}}{[\boldsymbol{\Delta} \cup \{\mathfrak{a}_1, \mathfrak{a}_2\}]\, A \quad \boldsymbol{\Theta}}$$

$$\frac{[\boldsymbol{\Delta} \cup \{\mathfrak{b}\}]\, A \qquad \boldsymbol{\Theta}}{[\boldsymbol{\Delta} \cup \{\mathfrak{b}_1\}]\, A \quad \boldsymbol{\Theta}} \qquad \frac{[\boldsymbol{\Delta} \cup \{\mathfrak{b}\}]\, A \qquad \boldsymbol{\Theta}}{[\boldsymbol{\Delta} \cup \{\mathfrak{b}_2\}]\, A \quad \boldsymbol{\Theta}}$$

Plus:

$$\text{C}{\sim}\text{E} \quad \frac{[\boldsymbol{\Delta} \cup \{{\sim}B\}]\, A \quad \boldsymbol{\Theta}}{[\boldsymbol{\Delta} \cup \{\neg B\}]\, A \quad \boldsymbol{\Theta}} \qquad \text{C}\neg{\sim}\text{E} \quad \frac{[\boldsymbol{\Delta} \cup \{\neg{\sim}B\}]\, A \quad \boldsymbol{\Theta}}{[\boldsymbol{\Delta} \cup \{B\}]\, A \qquad \boldsymbol{\Theta} \cup \{B \wedge {\sim}B\}}$$

Further rules:

Trans
$$\frac{\begin{array}{ll}[\Delta \cup \{B\}]\, A & \Theta \\ [\Delta']\, B & \Theta'\end{array}}{\begin{array}{ll}[\Delta \cup \Delta']\, A & \Theta \cup \Theta'\end{array}}$$

EM
$$\frac{\begin{array}{ll}[\Delta \cup \{B\}]\, A & \Theta \\ [\Delta' \cup \{\neg B\}]\, A & \Theta'\end{array}}{\begin{array}{ll}[\Delta \cup \Delta']\, A & \Theta \cup \Theta'\end{array}}$$

EM0
$$\frac{[\Delta \cup \{\neg A\}]\, A \quad \Theta}{[\Delta]\, A \quad \Theta}$$

IC
$$\frac{[\Delta]\, Dab(\Lambda \cup \Lambda') \quad \Theta \cup \Lambda'}{[\Delta]\, Dab(\Lambda \cup \Lambda') \quad \Theta}$$

5.4 Implementation for $\mathbf{IL}^r$ and $\mathbf{IL}^m$

replace $\neg$ by the standard negation $\sim$

— in the table for $\mathfrak{a}$-formulas and $\mathfrak{b}$-formulas

— in the standard formula analysing rules and condition analysing rules (without the "plus")

restore pp as for $\mathbf{CL}$

add CAR and FAR for the quantifiers and for identity

add pp($\forall \boldsymbol{A}, \exists \boldsymbol{A}$)

Add to the formula analysing rules:

$$\exists\mathrm{E}' \quad \frac{[\boldsymbol{\Delta}]\ \exists \boldsymbol{A} \vee \boldsymbol{B} \quad \boldsymbol{\Theta}}{[\boldsymbol{\Delta}]\ \forall \boldsymbol{A} \vee \boldsymbol{B} \quad \boldsymbol{\Theta} \cup \{\exists \boldsymbol{A} \wedge \exists{\sim}\boldsymbol{A}\}}$$

Add to the condition analysing rules:

$$\exists\mathrm{E}' \quad \frac{[\boldsymbol{\Delta} \cup \{\forall \boldsymbol{B}\}]\ \boldsymbol{A} \quad \boldsymbol{\Theta}}{[\boldsymbol{\Delta} \cup \{\exists \boldsymbol{B}\}]\ \boldsymbol{A} \quad \boldsymbol{\Theta} \cup \{\exists \boldsymbol{B} \wedge \exists{\sim}\boldsymbol{B}\}}$$

Prem, Goal, [EFQ], Trans, EM, EM0, IC

5.5 Procedural Criterion for Final Derivability

In this section, keep reading $\sim$ as *classical* negation.

We obtained A on the condition Δ.

A is finally derivable iff $\Delta \cap U(\Gamma) = \emptyset$

How can we find out whether $\Delta \cap U(\Gamma) = \emptyset$?

For *all* Θ, if $Dab(\Delta)$ is derivable on a condition Θ with $\Delta \cap \Theta = \emptyset$,

i.e. $\Gamma \vdash_{\mathbf{LLL}} Dab(\Delta \cup \Theta)$ and $\Delta \cap \Theta = \emptyset$,

then $Dab(\Theta)$ must be derivable on the condition $\emptyset$.

In other words:

After obtaining $\boldsymbol{A}$ on the condition $\boldsymbol{\Delta}$,

we introduce the A-Goal $[\boldsymbol{Dab}(\boldsymbol{\Delta})]\,\boldsymbol{Dab}(\boldsymbol{\Delta})$

Next, if $\boldsymbol{Dab}(\boldsymbol{\Delta})$ is derived on the condition $\boldsymbol{\Theta}$ (with $\boldsymbol{\Delta} \cap \boldsymbol{\Theta} = \emptyset$),

we introduce the X-Goal $[\boldsymbol{Dab}(\boldsymbol{\Theta})]\,\boldsymbol{Dab}(\boldsymbol{\Theta})$,

which should be reached on the condition $\emptyset$.

Phase 1: try to derive $\boldsymbol{A}$ on a condition $\boldsymbol{\Delta}$

no success: $\boldsymbol{\Gamma} \nvdash_{\mathbf{AL}} \boldsymbol{A}$
success: try to show that all members of $\boldsymbol{\Delta}$ are reliable, viz. move to

Phase 2: try to derive $\boldsymbol{Dab}(\boldsymbol{\Delta})$ on a condition $\boldsymbol{\Theta}$ (with $\boldsymbol{\Delta} \cap \boldsymbol{\Theta} = \emptyset$)

no success: no member of $\boldsymbol{\Delta}$ is unreliable and hence $\boldsymbol{\Gamma} \vdash_{\mathbf{AL}} \boldsymbol{A}$
success: move to

Phase 3: try to derive $\boldsymbol{Dab}(\boldsymbol{\Theta})$ on the condition $\emptyset$

no success: some member of $\boldsymbol{\Delta}$ is unreliable
go back to phase 1 and try to derive $\boldsymbol{A}$ on a new condition $\boldsymbol{\Delta}'$
success: $\boldsymbol{Dab}(\boldsymbol{\Theta})$ is a minimal $\boldsymbol{Dab}$-consequence of $\boldsymbol{\Gamma}$
go back to phase 2 and try to derive $\boldsymbol{Dab}(\boldsymbol{\Delta})$ on a new condition $\boldsymbol{\Theta}'$
($\boldsymbol{\Delta} \cap \boldsymbol{\Theta} = \emptyset$)

5.6 Phrase the Criterion in Problem Solving Terms

(PM)

This leads to problems that contain questions of which one direct answer only is sensible.

5.7 What If No Criterion Applies

Given the presupposition that abnormalities are false until and unless proven otherwise, the derivability of A on a condition Δ of which no member is shown to be unreliable is a good reason to consider A as derivable.

The more so as the block analysis shows that, as the proof proceeds, one may obtain more insights in the premises (and cannot loose insight in the premises).

If complete insight in the premises is reached, the proof becomes stable. (Derivability at this stage = final derivability.)

Complete insight in the premises may be reached with respect to a specific A.