Recent Results by the Inconsistency-Adaptive Labourers*

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Abstract

This paper offers an incomplete survey of recent results on inconsistency-adaptive logics (disregarding results on other adaptive logics). Much attention is paid to the so-called standard format, because it provides most of the required metatheory for adaptive logics that are phrased in this format. Combined adaptive logics are also briefly discussed. Other results briefly reported on concern (i) rethinking some philosophical theories from a paraconsistent viewpoint, (ii) the characterization of some (further) inconsistency-handling mechanisms in terms of adaptive logics, and (iii) the problem of defining criteria for final derivability. The final section concerns a further step towards eliminating (undesired) inconsistencies from a theory.

1 Introduction

As the title suggests, this paper briefly reports on some recent results in inconsistency-adaptive logics. Completeness was not sought. The aim is rather to report on some technical results, and on the way in which they shape the programme from a philosophical point of view.

Adaptive logics are meant to characterize consequence relations for which there is no positive test (that are not even partially recursive)—see [30, 29]. All but not only non-monotonic consequence relations lack a positive test. Adaptive logics require dynamic proofs, viz. proofs in which formulas that are considered as derived at some stage are considered as not derived at a later stage, and *vice versa*. During the last five years, most results on adaptive logics concern ampliative adaptive logics—logics that extend **CL** (Classical Logic). These will not be reported on in the present paper. However, we shall briefly mention some ampliative results that apply generally, including in inconsistent situations—in principle all ampliative logics have variants that apply in inconsistent situations, but unfortunately not all of these have been studied.

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For readers that are not familiar with inconsistency-adaptive logics, it may be useful to recall their effect in intuitive terms. Consider the premise set $\Gamma = \{p, {\sim}p, p \vee q, {\sim}r, r \vee s\}$. On nearly all (monotonic) paraconsistent logics,[1] Disjunctive Syllogism is invalid and neither q nor s is a consequence of Γ. An inconsistency-adaptive logic **AL** interprets a premise set *as consistently as possible.* While Γ requires $p \wedge {\sim}p$ to be true, it does not require $r \wedge {\sim}r$ to be true. So, on a minimally inconsistent interpretation of Γ, r comes out false and hence s comes out true. Put differently, s is an **AL**-consequence of Γ, whereas q is not. Monotonic paraconsistent logics invalidate certain **CL**-rules, for example Disjunctive Syllogism, whereas inconsistency-adaptive logics invalidate certain *applications* of some **CL**-rules, and it depends on the premises which applications are invalidated.

The adaptive programme is neutral with respect to the question whether there are true inconsistencies. Dialetheists, who answer the question in the positive, will argue that most true statements are consistent, and hence that consistency can be presupposed unless and until proven otherwise—this is precisely what inconsistency-adaptive logics do. Graham Priest has spelled this out in [54]. People who deny that there are true inconsistencies, for example classical logicians and intuitionists, cannot get around the fact that inconsistencies occurred in the history of the sciences—see [31, 36, 37, 41, 49, 50, 51, 59] for some case studies—and that, in the presence of inconsistent theories, one should *reason from them* in order to find consistent replacements for them. As was argued already in [3], the first step to be taken in such circumstances is to interpret the inconsistent theory as consistent as possible, in other words to apply an inconsistency-adaptive logic. The two aforementioned positions do not exhaust all possibilities. Thus our own position, which is outlined and argued for in [9], is that there is no warrant that there is a true consistent description of the world (in a conceptual system that humans can handle), but that there are good reasons to adopt the methodological maxim that one should try to eliminate inconsistencies from our knowledge—this entails that inconsistencies are seen as a problem, although not necessarily as the most urgent one.

2 The Standard Format

The growing multiplicity of adaptive logics called for systematization. The idea was not to restrict the variety of applications that the logics capture, but rather to find a common formal characterization. This characterization is provided by the so-called standard format. The basic mechanism behind all adaptive logics is the same. So the standard format, if well designed, should do most if not all of the work. We shall see that it does.

In many situations one needs to combine adaptive logics. In this respect, the aim was to rely on the standard format in order to design general stratagems for combining adaptive logics. Again, the combination stratagems, rather than specific properties of the combined logics, should warrant that the combination does the desired job. Combined adaptive logics are discussed in Section 3.

The standard format, first presented in [8], is both simple and perspicuous. An adaptive logic **AL** is in standard format if it is characterized as a triple

[1] Meheus' **AN**$\emptyset$ from [39] is an exception, but we disregard it for the present intuitive discussion.

consisting of the following elements:

(i) **LLL**, a lower limit logic,
(ii) Ω, a set of abnormalities that all have the same logical form,
(iii) an adaptive strategy.

The lower limit logic **LLL** determines the part of the adaptive logic that is not subject to adaptation. From a proof theoretic point of view, the lower limit logic delineates the rules of inference that hold unexceptionally. From a semantic point of view, the adaptive models of Γ are a selection of the lower limit models of Γ. It follows that $Cn_{\mathbf{LLL}}(\Gamma) \subseteq Cn_{\mathbf{AL}}(\Gamma)$. In principle, the lower limit logic is a monotonic and compact logic—see Section 3 on combined adaptive logics.

Abnormalities are formulas that are presupposed to be false, unless and until proven otherwise. Ω comprises all formulas of a certain logical form, which may be restricted—see below.

For many inconsistency-adaptive logics, Ω is the set of formulas of the form $\exists(A \wedge \sim A)$, in which $\exists A$ abbreviates the existential closure of A. For other inconsistency-adaptive logics, the set is restricted, for example, to formulas in which A is a primitive formula—a formula that contains no logical symbols except for identity.[2] Where introduced, the restriction is justifiable or even desirable, as we shall explain in the paragraph on flip-flop logics. Examples of such inconsistency-adaptive logics are those that have as their lower limit logic, for example, Schütte's $\mathbf{\Phi}_v$ from [58],[3] Priest's **LP** from [53], or Meheus' $\mathbf{AN}\emptyset$ from [39], and modal adaptive logics that characterize paraconsistent inference relations under a translation—several examples follow in Section 4. Incidentally, in all these logics, every formula $\exists(A \wedge \sim A)$ in which A is not primitive entails a disjunction of formulas of the form $\exists(B \wedge \sim B)$ in which B is primitive.

If the lower limit logic is extended with the requirement that no abnormality is logically possible, one obtains the *upper limit logic* **ULL**. Syntactically, **ULL** is obtained by extending **LLL** with an axiom stating that members of Ω entail triviality. The lower limit logic of inconsistency-adaptive logics is a paraconsistent logic, and the axiom schema $(A \wedge \sim A) \supset B$ is the most popular candidate for obtaining the upper limit logic. Even if Ω is characterized by a restricted logical form, there is no need to impose a restriction on the axiom schema if, for any C of the unrestricted form, there is a $\Delta \subseteq \Omega$ for which $C \vdash_{\mathbf{LLL}} \bigvee(\Delta)$. There also is no need to restate the axiom schema as $\exists(A \wedge \sim A) \supset B$, provided quantification behaves normally. Semantically, the upper limit logic is characterized by the lower limit models that verify no abnormality. **ULL** requires premise sets to be normal, and 'explodes' abnormal premise sets (assigns the trivial consequence set to them).

If, as is the case for many inconsistency-adaptive logics, the lower limit logic is a paraconsistent logic that contains full positive **CL** as well as excluded middle, for example expressed as $(\sim A \supset A) \supset A$,[4] and the set of abnormalities

[2]Similar restrictions are imposed on many ampliative adaptive logics. See, for example, [19] and [18].

[3]The Ghent name for the predicative extension of this logic is **CLuNs**—see [17]. Schütte, who was not a paraconsistent logician, introduced $\mathbf{\Phi}_v$ for a special purpose. Many paraconsistent logicians rediscovered this logic or its predicative extension.

[4]The weakest such logic is **CLuN**, **CL** allowing for gl*u*ts with respect to *n*egation, which consists exactly of full positive **CL** together with $(\sim A \supset A) \supset A$—see [35] for a proof of the

comprises the formulas of the form $\exists(A \wedge \sim A)$, possibly restricted as described before, then the upper limit logic is **CL**.[5]

If the premise set does not require any abnormality to obtain, the adaptive logic will deliver the same consequences as the upper limit logic. If the premise set requires some abnormalities to obtain, the adaptive logic will still deliver more consequences than the lower limit logic, viz. all upper limit consequences that are not 'blocked' by those abnormalities. In sum, the adaptive logic interprets the set of premises 'as normally as possible'; it takes abnormalities to be false 'in as far as' the premises permit.

The lower limit logic and the upper limit logic do not determine the set of abnormalities Ω. A few paragraphs ago, we considered the case where Ω contains all formulas of the form $\exists(A \wedge \sim A)$ in which A is a primitive formula. If the restriction is removed, the upper limit logic is still **CL**, but the adaptive logic is different—see the paragraph on flip-flop logics.

An *adaptive strategy* is required because many premise sets **LLL**-entail a disjunction of abnormalities (members of Ω) without entailing any of its disjuncts. Disjunctions of abnormalities will be called *Dab-formulas*. In the sequel, any expression of the form $Dab(\Delta)$ will refer to the (classical) disjunction of the members of a finite $\Delta \subseteq \Omega$. *Dab*-formulas that are derivable by the lower limit logic from the premise set Γ will be called *Dab-consequences* of Γ. If $Dab(\Delta)$ is a *Dab*-consequence of Γ, then so is $Dab(\Delta \cup \Theta)$ for any finite $\Theta \subset \Omega$. For this reason, it is important to concentrate on the minimal *Dab*-consequences of the premise set: $Dab(\Delta)$ is a *minimal* *Dab*-consequence of Γ iff $\Gamma \vdash_{\mathbf{LLL}} Dab(\Delta)$ and there is no $\Theta \subset \Delta$ such that $\Gamma \vdash_{\mathbf{LLL}} Dab(\Theta)$. If $Dab(\Delta)$ is a minimal *Dab*-consequence of Γ, then Γ determines that some member of Δ behaves abnormally, but fails to determine which member of Δ behaves abnormally. We have seen that adaptive logics interpret a premise set 'as normally as possible'. As some minimal *Dab*-consequences of Γ may contain more than one disjunct, this phrase is not unambiguous. It is disambiguated by choosing a specific adaptive strategy.

The oldest known strategy is *Reliability* from [3]. The minimal abnormality strategy was first presented in [2]. It delivers at least the same consequences as the Reliability strategy, and for some premise sets it delivers more consequences.

Some lower limit logics and sets of abnormalities are such that Δ is a singleton whenever $Dab(\Delta)$ is a minimal *Dab*-consequence of a premise set. If this is the case, the Reliability strategy and the Minimal Abnormality strategy lead to the same result and coincide with what is called the *Simple* strategy: a formula behaves abnormally just in case the abnormality is derivable from the premise set—see [19] , [39] and [40] for examples. Several other strategies have been studied. Most of them are needed to characterize an existing consequence relation by an adaptive logic. Examples may be found in [7], [11], [22], [32] and [64].

propositional case. Remark that Replacement of Identicals does not hold in **CLuN**, viz. does not hold in the scope of a negation. From a technical point of view, it is easier to handle **CLuN** if it is extended with a classical negation $\neg$ (or with $\bot$, characterized by $\bot \supset A$). This greatly simplifies the metatheory. In our preferred application contexts, classical negation does not occur in the premises.

[5]Priest's **LP** does not contain a detachable implication, and hence the requirement that all abnormalities are false should be introduced by a rule, viz. $A \wedge \sim A/B$, in which case the upper limit logic is **CL**.

Every line of an annotated dynamic proof consists of a line number, a formula, a justification, and a *condition*. The proofs are governed by three (generic) rules and a marking definition. Let

$$A \qquad \Delta$$

abbreviate that A occurs in the proof on the condition Δ, the rules may then be phrased as follows:

PREM If $A \in \Gamma$:

$$\begin{array}{ll} \dots & \dots \\ \hline A & \emptyset \end{array}$$

RU If $A_1, \dots, A_n \vdash_{\mathbf{LLL}} B$:

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \dots \cup \Delta_n \end{array}$$

RC If $A_1, \dots, A_n \vdash_{\mathbf{LLL}} B \vee Dab(\Theta)$

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \dots \cup \Delta_n \cup \Theta \end{array}$$

Given a dynamic proof, we shall say that $Dab(\Delta)$ is a *minimal Dab-formula* at stage s of the proof if, at stage s, $Dab(\Delta)$ occurs in the proof on the empty condition and, for any $\Delta' \subset \Delta$, $Dab(\Delta')$ does not occur in the proof on the empty condition. Where $Dab(\Delta_1)$, ..., $Dab(\Delta_n)$ are the minimal Dab-formulas at stage s of the proof, $U_s(\Gamma) = \Delta_1 \cup \dots \cup \Delta_n$ is the set of unreliable formulas at stage s.

Definition 1 *Marking for Reliability: Line i is marked at stage s iff, where Θ is the condition of line i, $\Theta \cap U_s(\Gamma) \neq \emptyset$.*

We refer to [8] (and elsewhere) for the marking definition of the Minimal Abnormality strategy and for that of the Simple strategy.

Remark that the rules depend on the lower limit logic and on the set of abnormalities, whereas the marking definition depends on the strategy.

A formula is *derived* from Γ *at a stage* of the proof iff it is the formula of a line that is unmarked at that stage. As the proof proceeds, unmarked lines may be marked and vice versa. So, it is important that one defines a different, stable, kind of derivability:

Definition 2 *A is* finally derived *from Γ on line i of a proof at stage s iff (i) A is the formula of line i, (ii) line i is not marked at stage s, and (iii) any extension of the proof in which line i is marked may be further extended in such a way that line i is unmarked.*

This means that there is a (possibly infinite) proof in which line i is unmarked and that is *stable* with respect to line i (line i is unmarked in all extensions of the proof). The previous definition is more appealing. The only way to establish the existence of an infinite proof is by a metalinguistic reasoning anyway. Moreover, the definition has a nice game-theoretic interpretation: whenever an opponent is able to extend the proof in such a way that line i is marked, the proponent is able to extend it further in such a way that line i is unmarked.

Definition 3 $\Gamma \vdash_{\mathbf{AL}} A$ *(A is* finally **AL**-derivable *from Γ) iff A is finally derived on a line of a proof from Γ.*

The *semantics* of all adaptive logics is defined in the same way. $M \models A$ will denote that M assigns a designated value to A, in other words that M verifies A. If the semantics is two-valued—and it is shown in [60] that any semantic system may be rephrased in two-valued terms—then $M \models A$ comes to $v_M(A) = 1$. $M \models \Gamma$ will denote that M verifies all members of Γ.

The abnormal part of a **LLL**-model M is defined as follows:

Definition 4 $Ab(M) = \{A \in \Omega \mid M \models A\}$

Where $Dab(\Delta_1),\ Dab(\Delta_2),\ \ldots$ are the minimal Dab-consequences of a premise set Γ,[6] $U(\Gamma) = \Delta_1 \cup \Delta_2 \cup \ldots$ is the set of formulas that are unreliable with respect to Γ. Let $\mathbf{AL}^r$ and $\mathbf{AL}^m$ be the adaptive logics defined from **LLL** and Ω by the Reliability strategy and the Minimal Abnormality strategy respectively.

Definition 5 *A* **LLL**-*model M of Γ is* reliable *iff $Ab(M) \subseteq U(\Gamma)$.*

In other words, M is a reliable model of Γ iff it verifies no abnormalities outside of members of $U(\Gamma)$, the set of formulas that are unreliable with respect to Γ.

Definition 6 $\Gamma \models_{\mathbf{AL}^r} A$ *iff A is verified by all reliable models of Γ.*

Definition 7 *A* **LLL**-*model M of Γ is* minimally abnormal *iff there is no* **LLL**-*model M' of Γ such that $Ab(M') \subset Ab(M)$.*

Definition 8 $\Gamma \models_{\mathbf{AL}^m} A$ *iff A is verified by all minimally abnormal models of Γ.*

So the proof theory and the semantics of an adaptive logic are fixed by the standard format. But there is more. Many metatheoretic properties of adaptive logics can be proved from the format itself, rather than from the specific properties of the logic. We mention only some examples. First and foremost, there are the *Soundness* and *Completeness* proof (given that the lower limit logic is sound and complete with respect to its semantics). The motor for the dynamic proofs is the *Derivability Adjustment* Theorem: $\Gamma \vdash_{\mathbf{ULL}} A$ iff there is a finite $\Delta \subset \Omega$ such that $\Gamma \vdash_{\mathbf{LLL}} A \vee Dab(\Delta)$. An important further property is *Proof Invariance*, which states that it does not depend on the way in which a proof sets out whether a conclusion is finally derivable from a premise set: If $\Gamma \vdash_{\mathbf{AL}} A$, then any proof from Γ can be extended into a proof in which A is finally derived from Γ. A semantically essential property is *Strong Reassurance*: If M is a **LLL**-model of Γ but not an **AL**-model of Γ, then there is an **AL**-model M' such that $Ab(M') \subset Ab(M)$. For proofs of these and more, see [12] and [16].

[6]The minimal Dab-consequences of Γ may be semantically defined in view of the soundness and completeness of **LLL** with respect to its semantics.

Nearly all known inconsistency-adaptive logics have a characterization in standard format.[7] In some cases, forging a consequence relation into standard format may require a translation, for example to a modal language—see Section 5 for an illustration. Even where it is useful to provide 'direct dynamic proofs' in untranslated terms—see [25] or [45]—the formulation in standard format has the advantage to provide the proof theory, semantics and metatheoretic properties, and to warrant (by an easy demonstration) that the direct proof theory is correct.

Flip-flops Some adaptive logics are called flip-flops because they have the following weird property: if Γ is normal (has upper limit models), then $Cn_{\mathbf{AL}}(\Gamma) = Cn_{\mathbf{ULL}}(\Gamma)$, if Γ is abnormal, then $Cn_{\mathbf{AL}}(\Gamma) = Cn_{\mathbf{LLL}}(\Gamma)$. Flip-flops typically result from an unsuitable choice of the set of abnormalities Ω. If, for some adaptive logic, this set is characterized by a restricted logical form, for example $\exists(A \wedge {\sim}A)$ in which A is a primitive formula, and is replaced by the set characterized by the unrestricted form, in the example all formulas of the form $\exists(A \wedge {\sim}A)$, a flip-flop results.[8]

Formula-preferential systems Some adaptive logics were characterized by formula-preferential systems in [35]—see also [1]. The idea is close to that behind adaptive logics: the consequence set is determined by the premises, a regular logic (this actually is the lower limit logic **LLL**) and a set of preferred formulas. A subset of the preferred formulas is added to the premises, and the subset should be maximal with respect to the premises and **LLL**. It is not clear whether all adaptive logics can be characterized by a formula-preferential system, but it is easily provable that all formula-preferential systems are characterized by an adaptive logic; the characterization requires that the preferences are expressed in the object language—see Section 3 for means to do so.

The best-studied (and very simple) inconsistency-adaptive logic is **ACLuN1**—see [3] for its propositional proof theory and [5] for both the syntax and semantics at the predicative level. Its lower limit logic is **CLuN**, full positive **CL** plus Excluded Middle—see footnote 4. The set of abnormalities, Ω, is the set of all formulas of the form $\exists(A \wedge {\sim}A)$, and the strategy is Reliability. The logic has many nice properties, studied in [5] but also (in a more general framework) in [8] and [12]. Other well-studied inconsistency-adaptive logics have another strategy or another lower limit logic.

Some new inconsistency-adaptive logics were developed recently, mainly as a result of a Ghent–Torun cooperation. A first example is the discussive logic $\mathbf{DL}^r$ that constitutes a (non-monotonic) alternative for Jaśkowski's paraconsistent system $\mathbf{D}_2$ (see [44]). Like $\mathbf{D}_2$, $\mathbf{DL}^r$ validates all single-premise rules of **CL**. However, for formulas that behave consistently, $\mathbf{DL}^r$ moreover validates all multiple-premise rules of **CL**—this is realized without the introduction of the discussive connectives. It is stipulated that A is a $\mathbf{DL}^r$-consequence of Γ iff $\Diamond A$ is an $\mathbf{AJ}^r$-consequence of $\Gamma^\Diamond = \{\Diamond A \mid A \in \Gamma\}$. $\mathbf{AJ}^r$ is in standard format, its

[7] An exception is Priest's $\mathbf{LP}^m$ from [54] and emended in [55]. This adaptive logic proceeds in terms of properties of the model, rather than in terms of the formulas verified by the model.

[8] Flip-flops were considered as utterly uninteresting, until some interesting prioritized adaptive logics turned out to be flip-flops—see [18].

elements being **S5**, $\Omega = \{\exists(\Diamond A \wedge \Diamond{\sim}A) \mid A \in \mathcal{F}^p\}$ ($\mathcal{F}^p$ are the sentential letters and primitive predicative formulas), and Reliability.

A second example is the logic $\mathbf{AD}_2$, developed by Marek Nasieniewski in [47] and partly presented in [48]. $\mathbf{AD}_2$ is an adaptive extension of $\mathbf{D}_2$, including the discussive connectives. In [48], several important properties of $\mathbf{AD}_2$ are studied and $\mathbf{AD}_2$ is compared to $\mathbf{DL}^r$.

3 Combining Adaptive Logics

Several ways to combine adaptive logics have been studied. Basically three methods were obtained, but there is no reason why these should be exhaustive—we shall not discuss all variants of the combination methods.

If two adaptive logics have the same lower limit logic and the same strategy, the obvious way to combine them is by taking the union of their sets of abnormalities as the set of abnormalities of the combined logic. The premise set is then interpreted as normally as possible with respect to both kinds of abnormalities.

If two or more adaptive logics share their upper limit logic and their strategy, a combined adaptive logic is obtained by taking as the lower limit logic the intersection of the lower limit logics and as the set of abnormalities the union of the sets of abnormalities (and retaining the strategy). The specific example we want to present deserves a philosophical comment.

Paraconsistent logics allow for negation gluts—they have models that verify formulas of the form $\exists(A \wedge {\sim}A)$. However, some inconsistent premise sets also have a non-trivial consequence set if they are handled by a logic that allows for conjunction gluts—these have models that verify formulas of the form $\exists((A \wedge B) \wedge {\sim}A)$ and of the form $\exists((A \wedge B) \wedge {\sim}B)$. The same is true for logics that have gluts with respect to some other logical symbol, including the quantifiers and identity. The same even holds for logics that allow for gaps with respect to a logical symbol. Consider, for example, the case of negation gaps. If a model falsifies both p and ${\sim}p$, it verifies the premise set $\{p \supset q, p \supset {\sim}q, {\sim}p \supset q, {\sim}p \supset {\sim}q\}$. If this premise set is closed by **CL**, its consequence set is trivial. It follows that paraconsistency need not be the best road to handling an inconsistent premise set. In specific cases, a logic allowing for other gluts, or allowing for gaps, may be more suitable. Let us call such logics paralogics.

Every paralogic gives rise to adaptive logics that interpret the premises as normally as possible. In other words, the adaptive logics presuppose that formulas describing an allowed gap or glut are false unless and until proven otherwise.[9]

All such logics may be combined by the method discussed two paragraphs ago. In [6] they are even combined with (a variant of) the ambiguity-adaptive logic from [61]. The resulting lower limit logic is zero-logic, according to which no formula (not even a premise) is derivable from a premise set. The adaptive logic still interprets the premises as normally as possible. In the special case where the premise set has **CL**-models, the adaptive logic delivers all **CL**-consequences of the premises.[10]

[9]Formulas describing gaps may require that the language is extended by the classical logical symbol that corresponds to the symbol displaying gluts or gaps. This is an easily solvable technical problem.

[10]The basic idea behind the adaptive logic is obviously that all properties of all logical

A last way to combine adaptive logics is by superimposing adaptive logics on top of each other. From a definitional point of view, the consequence set of the combined logic is $Cn_{\mathbf{AL2}}(Cn_{\mathbf{AL1}}(\Gamma))$. So, Γ is first interpreted as normally as possible with respect to the abnormalities of **AL1**, and the resulting consequence set is then interpreted as normally as possible with respect to the abnormalities of **AL2**. Two problems seem to arise here. First, it appears that the lower limit logic of **AL2** is the adaptive logic **AL1**, which puts **AL2** outside of the standard format. This is rather easily solved. One can take as the lower limit logic of **AL2** any monotonic logic that does not trivialize $Cn_{\mathbf{AL1}}(\Gamma)$—the lower limit logic of **AL1** is a good candidate, as $Cn_{\mathbf{AL1}}(\Gamma)$ is closed under this logic, but there are other suitable candidates as well.

The second problem is that the above construction seems to require that all **AL1**-consequences from the premises are obtained before the result is closed by **AL2**, which makes the construction problematic from a computational point of view. Fortunately, the combined adaptive logic avoids this problem in a way that seems impressively elegant to us. Let us restrict attention to the special case where the lower logics of **AL1** and **AL2** are the same, which makes their unconditional rules identical. One then simply applies the unconditional rule and both conditional rules in the usual way. The only difference is that the marking definition has to be changed into a two-step definition. It comes to this: first lines are marked with a mark of type 1 according to the marking definition of **AL1**, next lines are marked with a mark of type 2 according to the marking definition of **AL2**, except that the later definition takes only into account the lines that are type 1 unmarked. We refer to [8] for details.

A nice example enables one to build combined adaptive logics that handle preferred consequence sets. Let $\mathcal{W}$ be the set of closed formulas of the usual (non-modal) predicative language, and consider a modal language in which the diamond is interpreted as "it is plausible that", more diamonds indicating a lower plausibility. Moreover, let $\Diamond^n$ abbreviate a sequence of n diamonds. We now define an infinity of adaptive logics, $\mathbf{AML}_n$ having as their lower limit logic some standard modal logic **ML**, $\{\Diamond^n A \wedge {\sim} A \mid A \in \mathcal{W}\}$ (in which the n is the same as the one occurring in the name of the logic) as their set of abnormalities, Ω, and either Reliability or minimal Abnormality as their common strategy.[11] The upper limit logic of each of these adaptive logics is **Triv**—the system in which A is logically equivalent to $\Box A$ as well as to $\Diamond A$. All these adaptive logics can be easily combined by the method under discussion in order to handle preferred statements. The premises will be expressed by a single set of formulas of the form $\Diamond^n A$, in which $n = 0$ for statements that are taken to be certain, $n = 1$ for statements of the next highest priority, etc. The deductive closure of a premise set under the combined adaptive logic will contain the following members of $\mathcal{W}$: (i) all **CL**-consequences of premises that belong to $\mathcal{W}$, (ii) all **CL**-consequences of the previous set together with as many as can be consistently added from the following set: the $A \in \mathcal{W}$ for which $\Diamond^1 A$ is a **ML**-consequence of Γ, etc.

This construction has been applied (for specific purposes) in [18], [23], [62] and other papers. As it is described here, it functions only as a way to prevent inconsistencies from arising. However, it can also be applied to stepwise extend

symbols are defeasible in the sense that one can interpret sensibly a 'text' that requires gluts or gaps with respect to any logical symbol.

[11] If the purpose is different, the set of abnormalities will be taken to be $\{\Box^n A \mid A \in \mathcal{W};\ \nvdash_{\mathbf{ML}} \Box A\}$—see [19] for an example.

an inconsistent set with (consequences of) statements that have (a higher or lower) priority and are compatible with the inconsistent premise set or with its previous extension—this problem was solved by the results from [43].

4 Rethinking in Terms of Paraconsistency

There is a kind of consistency-laziness in philosophy. Even after being convinced that many theories are inconsistent, either in themselves or with respect to data or to other theories, many philosophers will stay content with solving a problem for the consistent case. Some will argue that it only is the task of philosophers to offer an explication for or idealization of actual thinking, and then invoke some fallacious reasoning to conclude that the idealized situation is consistent. Of course they are free to define whatever they like as the idealized situation. The point is that they should spell out the relation between their ideal (their theory) and reality (actual reasoning), and precisely this they usually fail to do. Others might admit that their theory should handle the inconsistent case to be correct, but may still argue that they solved an important problem and that it is now someone else's task to consider the inconsistent case. Sometimes they even think the remaining problem to be easily solvable by some paraconsistent trick. Given this situation, we consider it essential that paraconsistent logicians put the rethinking of philosophical concepts and theories very high on their agenda. In this section we consider a few examples where the challenge was met (in terms of the Ghent approach).

Compatibility is a concept that is invoked in numerous situations. To give just one example, any extension (not revision) of our knowledge should be compatible with our present knowledge. So, if the task is to extend our knowledge with a new hypothesis or theory, one has to check whether it is compatible with our present knowledge. There obviously is no positive test for compatibility (at the predicative level), whence, even in the consistent case, compatibility is characterized by a adaptive logic, which was described in [19].

As our present knowledge may be inconsistent, this does not fully solve the problem. Moreover, even defining compatibility in inconsistent environments raises an interesting problem in that most definitions for consistent compatibility classify either nothing or everything as compatible with an inconsistent set of statements. A proposal (possibly the only sensible one) was put forward in [43]. There A is defined as compatible with Γ iff $\Gamma \cup \{A\}$ is not more inconsistent than Γ—this vague criterion obtains an exact articulation. Relying on it, the adaptive logic of compatibility for the inconsistent case is spelled out. This logic is a true generalization of the logic for the consistent case in the sense that both lead to the same consequence set if Γ is consistent.

In [14] a similar job was done for a very different theory, viz. Hintikka's theory of the process of explanation from [34] and elsewhere. Hintikka's criteria for an explanation are first adjusted in such a way that they lead to exactly the same result as the original criteria in the consistent case, but allow for explanation from inconsistent theories. Next, it is shown that these criteria can be met by proceeding exactly as Hintikka, except that **CL** is replaced by the inconsistency-adaptive **ACLuN1**. It is shown that the result is a proper generalization of Hintikka's theory in that it leads to precisely the same results in the consistent case, and adjusts this theory in a natural and minimal way in the inconsistent case.

The criteria from [14] are applied in [46] to design an adaptive logic for abduction that enables one to generate explanations for novel facts as well as for anomalous facts (facts not consistent with one's background assumptions), and that moreover can handle possibly inconsistent theories.

An interesting theory on question evocation (by a set of declarative premises) and question implication (by a set of declarative premises together with a question) was proposed by Andrzej Wiśniewski, for example in [65] and [66]. Although Wiśniewski intends his theory to be general and relatively independent of the underlying logic (for the declarative premises), it cannot be applied as it stands in inconsistent situations. This situation was repaired for question evocation by Kristof De Clercq in a section in [33] and by Joke Meheus in [38]. Both contributions contain a system that allows for sensible question evocation in the presence of inconsistent declarative premises and in both cases answers to questions will repair the undesired incompleteness of the premises—this triggered the questions. However, the first approach (relying on **ACLuN1**) is more suitable if the premises are considered as true, whereas the second is more suitable if one sets out to obtain a consistent replacement (on the basis of new evidence) for the premises—it presupposes that the answers obtained have a higher preference than the original premises. Both approaches generalize Wiśniewski's theory in the sense that they lead to the same results as this theory in the consistent case.

It is well-known that there has hardly been any attention for inconsistent beliefs in the literature on belief revision. A major contribution to repair this is contained in [33]. In the second part of this book, Kristof De Clercq presents a full-fledged replacement for most of the standard theory. Taking inconsistent belief sets into account, he stepwise adjusts the postulates, the (tiresome) definitions, and the construction. He goes on to present and prove an impressive set of metatheorems, often developing new proof techniques, to show that the whole framework is adequate. He also shows that his framework leads to the same changes as the standard one if it is applied to consistent belief sets.

5 Further Work on Integration

Some of the Rescher–Manor mechanisms from [57]—for an overview see [26] for the flat ones and [27] for the prioritized ones—were among the first inconsistency-handling mechanisms. The flat ones were characterized by an adaptive logic in [7] (in terms of inconsistency-adaptive logics close to **ACLuN1**). Insights provided by [44] led to new results in [11]: the flat mechanisms are characterized in terms of modal adaptive logics, and sensible extensions of them are presented (mainly with discussive application contexts in mind—see also [63]). The prioritized mechanisms were characterized by adaptive logics in [64].

Another approach to paraconsistent reasoning that has been gaining popularity are the signed systems from [28] and other papers. These systems are characterized by adaptive logics in [22].

6 Criteria for Final Derivability

The proofs of adaptive logics are dynamic: a line that was introduced may later be marked, still later unmarked, etc. This is not a disadvantage of adaptive logics, but a result of the properties of the consequence relations that adaptive logics explicate. As there is no positive test for these consequence relations (at the predicative level), a non-dynamic proof theory cannot possibly be provided.

Even in the absence of a positive test, there may be criteria that enable one to decide that a finite proof warrants that one of its lines will not be marked in any extension of the proof—in other words that the extensions of the proof are stable with respect to this line. The search for criteria led, first to [4], which provided interesting but weak criteria, and next to [20] and [21], which provided criteria in terms of tableau methods. The disadvantage of the latter is that they are rather remote from proofs and moreover require many steps that seem intuitively useless.

Work on a completely different problem provided a way out. In [24], a special proof format was devised (for propositional **CL**) in which part of the proof heuristics is pushed into the proof—in [56] the result is extended to full (predicative) **CL**. In [10] and [13]—the latter paper is in the most recent format and contains the metatheory—the approach is applied to propositional **CLuN** and next is turned into a procedural criterion for final derivability in **ACLuN1**.[12] It can easily be seen that the approach can be generalized to the predicative case (where it really pays), and next can be generalized to any adaptive logic—although further work is clearly required to incorporate other strategies. The advantage of the procedural criterion is that it sets off from a clear goal (the formula one tries to derive), that every step in the prospective proof is demonstrably sensible (in view of the stage of the proof) for reaching this goal, and that, if the procedure stops, which obviously cannot be warranted at the predicative level, one knows whether the goal is or is not derivable from the premises. If desired, the prospective proof can be algorithmically turned into a standard adaptive logic proof.

7 Moving towards Consistency

We saved a philosophical problem for the last section. Suppose that a theory was intended as consistent but turns out to be inconsistent, which is the preferred application context of inconsistency-adaptive logics. The adaptive logic locates the inconsistencies and isolates them (prevents them from spreading). As we have seen, the **CL**-consequences that cannot be drawn from the theory (or premise set) will be those that are prevented by the minimal disjunctions of abnormalities that are derivable from the premises by the lower limit logic. Suppose that the following formula is a minimal disjunction:

$$(p \wedge \sim p) \vee (q \wedge \sim q) \vee (r \wedge \sim r)\,.$$

So the theory states that either p or q or r behaves inconsistently, but fails to specify which of the three does.

[12] A computer program that implements the procedure can be downloaded from `http://logica.ugent.be/centrum/writings/programs.php`.

One clearly will like to go beyond isolating the inconsistencies: one will want to obtain a consistent replacement for the inconsistent theory, provided the latter is an improvement of the former. Given this qualification the task is not one for logic—it may require gathering new evidence, forging a new conceptual system, etc. However, the task will require guidance by logic.

In agreement with Wiśniewski's erotetic logic, the minimal *Dab*-consequence of the premises will evoke the question $?\{p \wedge \sim p, q \wedge \sim q, r \wedge \sim r\}$. Depending on the epistemic situation, it may be possible to obtain a (full or partial) answer to this question. Nearly every answer will free one of the three formulas from the suspicion of inconsistency. As an effect, more **CL**-consequences of the premise set will be finally derivable and the remaining problems are better located. The explication of such processes proceeds in terms of the method from [38].

In some cases, no answer to the aforementioned question can be obtained. Even then, a researcher may discover reasons to narrow down the suspicion. For example, q may be well entrenched in other theories that are considered unproblematic, whence the researcher may decide to consider q as behaving consistently. Or the researcher might have reasons to suspect p, and decide that the inconsistency lies there. Clearly such decisions should themselves be defeasible. New information may put the blame elsewhere.

These problems are considered in [15] and the required adaptive logics are spelled out and shown to be adequate. Actually two approaches turn out sensible for expressing suspicion or freedom of suspicion in a defeasible way, one relying on the modal approach mentioned at the end of Section 3, the other in terms of a hierarchy of negations of inconsistencies.

There are many open problems, even for logic, concerning the road from inconsistent theories to consistent replacements for them (in cases where there are such replacements). Still, the previous results clarify a further bit of that road. We'll get there, step by step.[13]

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