The Standard Format for Adaptive Logics as a Step towards Universal Logic

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1 Introductory Remarks

adaptive logics interpret a premise set “as normally as possible” with respect to some standard of normality

they explicate reasoning processes that display an internal (and possibly an external) dynamics

- external dynamics: non-monotonicity
- internal dynamics: revise conclusions as insights in premises grow

1 Introductory Remarks

adaptive logics interpret a premise set "as normally as possible" with respect to some standard of normality

they explicate reasoning processes that display an internal (and possibly an external) dynamics

external dynamics: non-monotonicity

internal dynamics: revise conclusions as insights in premises grow

technical reason for dynamics:
absence of positive test for derivability (at predicative level)

- many reasoning patterns explicated by an adaptive logic surv
- number of known inference relations characterized by an adaptive logic

many (not all) adaptive logics seem to have a common structure

some can be given this structure under a translation

the structure is central for
proof theory, semantics, soundness and completeness,
proofs of further properties, computational aspects, . . .

whence the plan:
- describe the structure: the SF (standard format)
- define the proof theory and semantics from the SF
- prove as many properties as possible by relying on the SF only

the results are provisional (as everything):
- not all adaptive logics have been phrased in SF
- a more general characterization may be possible
(with sets of properties depending on specifications)

2 The Standard Format

- *lower limit logic*
  standard (monotonic, compact, . . .) logic
- *set of abnormalities* Ω
  characterized by a (possibly restricted) logical form
- *strategy*
  Reliability, Minimal Abnormality, Simple strategy, . . .

upper limit logic:

$\mathbf{ULL} = \mathbf{LLL} +$ axiom/rule that trivializes abnormalities

semantically: the $\mathbf{LLL}$-models that verify no abnormality

general idea behind adaptive logics:

$Cn_{\mathbf{AL}}(\Gamma)$: $Cn_{\mathbf{LLL}}(\Gamma)$ + what follows if as many members of Ω are false as the premises permit

Example: the inconsistency-adaptive $\mathbf{ACLuN}^m$

- *lower limit logic*: $\mathbf{CLuN}$

- *set of abnormalities*: $\Omega = \{\exists(A \wedge {\sim}A) \mid A \in \mathcal{F}\}$

- *strategy*: Minimal Abnormality

upper limit logic:

$\mathbf{CL} = \mathbf{CLuN} + (A \wedge {\sim}A) \supset B$

semantically: the $\mathbf{CLuN}$-models that verify no inconsistency

corrective adaptive logic (if $\mathbf{CL}$ is the standard)

Example: logic of inductive generalization: $\mathbf{IL}^m$

- *lower limit logic*: $\mathbf{CL}$

- *set of abnormalities*: $\Omega = \{\exists A \wedge \exists{\sim}A \mid A \in \mathcal{F}^{\circ}\}$

- *strategy*: Minimal Abnormality

upper limit logic:

$\mathbf{UCL} = \mathbf{CL} + \exists\alpha A(\alpha) \supset \forall\alpha A(\alpha)$

semantically: the uniform $\mathbf{CL}$-models ($v(\pi^r) \in \{\emptyset, D^{(r)}\}$)

ampliative adaptive logic (if $\mathbf{CL}$ is the standard)

Example: Strong Consequence Relation (Rescher–Manor)

consider $\mathbf{ACLuN}^m$ with classical negation ($\neg$) occurring in the language

let $\mathcal{W}^{\not\sim}$ be the closed formulas that do not contain $\sim$

the theorems in $\mathcal{W}^{\not\sim}$ are those of $\mathbf{CL}$ (with $\neg$ the standard negation)

let $\Gamma^{\sim\neg} = \{\sim\neg A \mid A \in \Gamma\}$

where $\Gamma \cup \{A\} \subseteq \mathcal{W}^{\not\sim}$: $\quad \Gamma \vdash_{Strong} A$ iff $\Gamma^{\sim\neg} \models_{\mathbf{ACLuN}^m} A$

corrective consequence relation characterized by an adaptive logic (under a translation)

Conventions

- to simplify the metatheoretic proofs, we add (where necessary) all logical symbols of $\mathbf{CL}$ to the $\mathbf{LLL}$
  - harmless
  - these symbols need not occur in the premises or conclusion
  - notation: $\neg$, $\supset$, $\sqcap$, $\sqcup$, $(\sqcap\alpha)$, $(\sqcup\alpha)$, and $==$

so $\mathbf{LLL}$ contains $\mathbf{CL}$ (in one sense, even if it may be weaker in another)

- $\boldsymbol{Dab}$-formula: classical disjunction of the members of a finite $\boldsymbol{\Delta} \subset \boldsymbol{\Omega}$
  notation: $\boldsymbol{Dab}(\boldsymbol{\Delta})$

3 Proofs

- rules of inference (determined by $\mathbf{LLL}$ and $\boldsymbol{\Omega}$)

- a marking definition (determined by $\boldsymbol{\Omega}$ and the stategy)

dynamics of the proofs controlled by attaching conditions (finite subsets of $\boldsymbol{\Omega}$) to derived formulas

line of annotated proof: number, formula, justification, condition

the rules govern the conditions

marking definition: determines for every line i at every stage s of a proof

whether i is IN or OUT in view of $\left\{ \begin{array}{l} \text{the condition of } i \\ \text{the } Dab\text{-formulas derived} \end{array} \right.$

Rules of inference (depend on $\mathbf{LLL}$ and $\mathbf{\Omega}$, *not* on the strategy)

PREM If $A \in \Gamma$:

$$\frac{\ldots \quad \ldots}{A \quad \emptyset}$$

RU If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B$:

$$\begin{array}{ll} A_1 & \Delta_1 \\ \ldots & \ldots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \end{array}$$

RC If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B \sqcup Dab(\Theta)$

$$\begin{array}{ll} A_1 & \Delta_1 \\ \ldots & \ldots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \cup \Theta \end{array}$$

for example:

$p,\ p \supset q \vdash_{\mathbf{CLuN}} q$

$\sim p,\ p \vee q \vdash_{\mathbf{CLuN}} q \vee (p \wedge \sim p)$

Marking definition

proceeds in terms of the minimal Dab-formulas that are derived at the stage of the proof

$Dab(\Delta)$ is a minimal Dab-formula at stage s:
- $Dab(\Delta)$ derived on line with condition $\emptyset$
- no $Dab(\Delta')$ with $\Delta' \subset \Delta$ derived on line with condition $\emptyset$

where $Dab(\Delta_1)$, ..., $Dab(\Delta_n)$ are the minimal Dab-formulas derived on condition $\emptyset$ at stage s,

$$U_s(\Gamma) = \Delta_1 \cup \ldots \cup \Delta_n$$

Definition

where Δ is the condition of line i,

line i is marked at stage s iff $\Delta \cap U_s(\Gamma) \neq \emptyset$

Marking Definition for Minimal Abnormality

where $Dab(\Delta_1), \ldots, Dab(\Delta_n)$ are the minimal Dab-formulas derived on condition $\emptyset$ at stage s,

$\Phi^{\circ}_s(\Gamma)$: set of all sets that contain one member of each Δ_i

$\Phi^{\star}_s(\Gamma)$: contains, for any $\varphi \in \Phi^{\circ}_s(\Gamma)$, $Cn_{\mathbf{LLL}}(\varphi) \cap \Omega$

$\Phi_s(\Gamma)$: $\varphi \in \Phi^{\star}_s(\Gamma)$ that are not proper supersets of a $\varphi' \in \Phi^{\star}_s(\Gamma)$

minimal sets of abnormalities that should be true
in order for all Dab-formulas derived at stage s to be true

Definition

where A is the formula and Δ is the condition of line i,
line i is marked at stage s iff,

(i) there is no $\varphi \in \Phi_s(\Gamma)$ such that $\varphi \cap \Delta = \emptyset$, or

(ii) for some $\varphi \in \Phi_s(\Gamma)$, there is no line at which A is derived on a condition Θ for which $\varphi \cap \Theta = \emptyset$

Definition
where Δ is the condition of line i,
line i is marked at stage s iff some $A \in \Delta$ is derived on condition $\emptyset$

only suitable iff, for all Γ,

$$\Gamma \vdash_{\mathbf{LLL}} Dab(\Delta) \quad \text{iff} \quad \text{for some } A \in \Delta,\ \Gamma \vdash_{\mathbf{LLL}} A.$$

in other words: if $Dab(\Delta)$ is derived on condition $\emptyset$,

then, for some $A \in \Delta$, A is derivable on condition $\emptyset$

in this case, Reliability and Minimal Abnormality both coincide with the Simple Strategy

Derivability at a stage vs. **final derivability**

idea: A derived on an unmarked line i
and the proof is stable with respect to i

stability concerns a specific line

Definition

A is *finally derived* from Γ at line i of a proof at stage s iff

(i) A is the second element of line i,

(ii) line i is unmarked at stage s, and

(iii) any extension of the proof may be further extended in such a way that line i is unmarked.

Definition

$\Gamma \vdash_{\mathbf{AL}} A$ (A is *finally* $\mathbf{AL}$-*derivable* from Γ) iff A is finally derived at a line of a proof from Γ.

Two remarks:

even at the predicative level, there are criteria for final derivability

- **ULL** extends **LLL** by validating some further rules
- **AL** extends **LLL** by validating some applications of those **ULL**-rules

Extremely simple propositional example for $\mathbf{ACLuN}^r$ (and $\mathbf{ACLuN}^m$)

1	$(p \wedge q) \wedge t$	PREM	$\emptyset$
2	$\sim p \vee r$	PREM	$\emptyset$
3	$\sim q \vee s$	PREM	$\emptyset$
4	$\sim p \vee \sim q$	PREM	$\emptyset$
5	$t \supset \sim p$	PREM	$\emptyset$

Extremely simple propositional example for $\mathbf{ACLuN}^r$ (and $\mathbf{ACLuN}^m$)

1	$(p \wedge q) \wedge t$	PREM	$\emptyset$
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4	$\sim p \vee \sim q$	PREM	$\emptyset$
5	$t \supset \sim p$	PREM	$\emptyset$
6	r	1, 2; RC	$\{p \wedge \sim p\}$

Extremely simple propositional example for $\mathbf{ACLuN}^r$ (and $\mathbf{ACLuN}^m$)

1	$(p \wedge q) \wedge t$	PREM	$\emptyset$
2	$\sim p \vee r$	PREM	$\emptyset$
3	$\sim q \vee s$	PREM	$\emptyset$
4	$\sim p \vee \sim q$	PREM	$\emptyset$
5	$t \supset \sim p$	PREM	$\emptyset$
6	r	1, 2; RC	$\{p \wedge \sim p\}$
7	s	1, 3; RC	$\{q \wedge \sim q\}$

Extremely simple propositional example for $\mathbf{ACLuN}^r$ (and $\mathbf{ACLuN}^m$)

1	$(p \wedge q) \wedge t$	PREM	$\emptyset$	
2	$\sim p \vee r$	PREM	$\emptyset$	
3	$\sim q \vee s$	PREM	$\emptyset$	
4	$\sim p \vee \sim q$	PREM	$\emptyset$	
5	$t \supset \sim p$	PREM	$\emptyset$	
6	r	1, 2; RC	$\{p \wedge \sim p\}$	$\checkmark$
7	s	1, 3; RC	$\{q \wedge \sim q\}$	$\checkmark$
8	$(p \wedge \sim p) \vee (q \wedge \sim q)$	1, 4; RU	$\emptyset$	

Extremely simple propositional example for $\mathbf{ACLuN}^r$ (and $\mathbf{ACLuN}^m$)

1	$(p \wedge q) \wedge t$	PREM	$\emptyset$	
2	$\sim p \vee r$	PREM	$\emptyset$	
3	$\sim q \vee s$	PREM	$\emptyset$	
4	$\sim p \vee \sim q$	PREM	$\emptyset$	
5	$t \supset \sim p$	PREM	$\emptyset$	
6	r	1, 2; RC	$\{p \wedge \sim p\}$	$\surd$
7	s	1, 3; RC	$\{q \wedge \sim q\}$	
8	$(p \wedge \sim p) \vee (q \wedge \sim q)$	1, 4; RU	$\emptyset$	
9	$p \wedge \sim p$	1, 5; RU	$\emptyset$	

nothing interesting happens when the proof is continued

no mark will be removed or added

4 Semantics

$Dab(\Delta)$ is a minimal Dab-consequence of Γ:

$\Gamma \vDash_{\mathbf{LLL}} Dab(\Delta)$ and, for all $\Delta' \subset \Delta$, $\Gamma \nvDash_{\mathbf{LLL}} Dab(\Delta')$

where $Dab(\Delta_1)$, $Dab(\Delta_2)$, $\ldots$ are the minimal Dab-consequences of Γ,

$U(\Gamma) = \Delta_1 \cup \Delta_2 \cup \ldots$

where M is a $\mathbf{LLL}$-model: $Ab(M) = \{A \in \Omega \mid M \models A\}$

the $\mathbf{AL}$-semantics selects some $\mathbf{LLL}$-models of Γ as $\mathbf{AL}$-models of Γ

the selection depends on Ω and on the strategy

Reliability

a $\mathbf{LLL}$-model M of Γ is reliable iff $Ab(M) \subseteq U(\Gamma)$

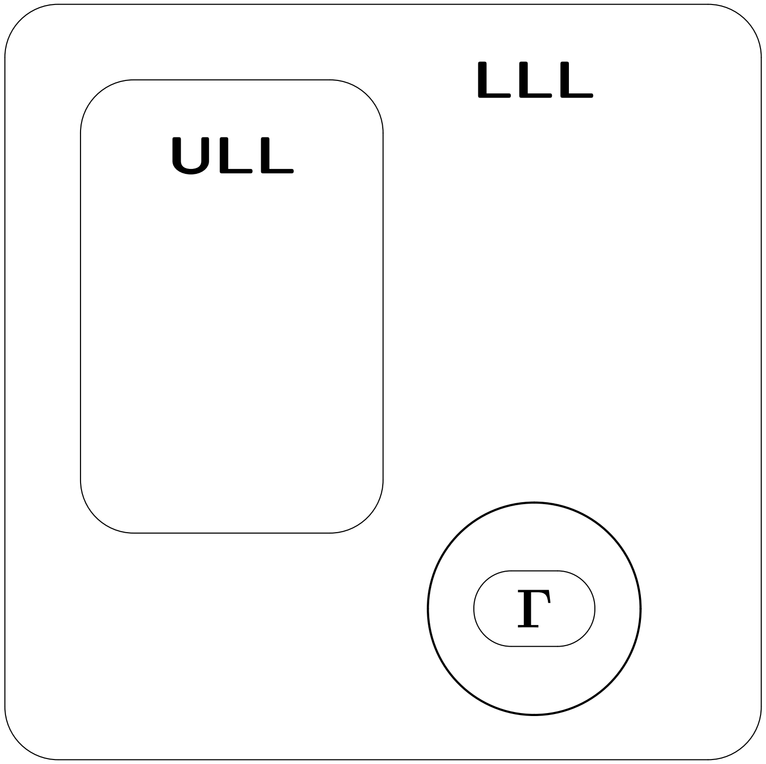
$\Gamma \vDash_{\mathbf{AL}^r} A$ iff all reliable models of Γ verify A

Minimal Abnormality

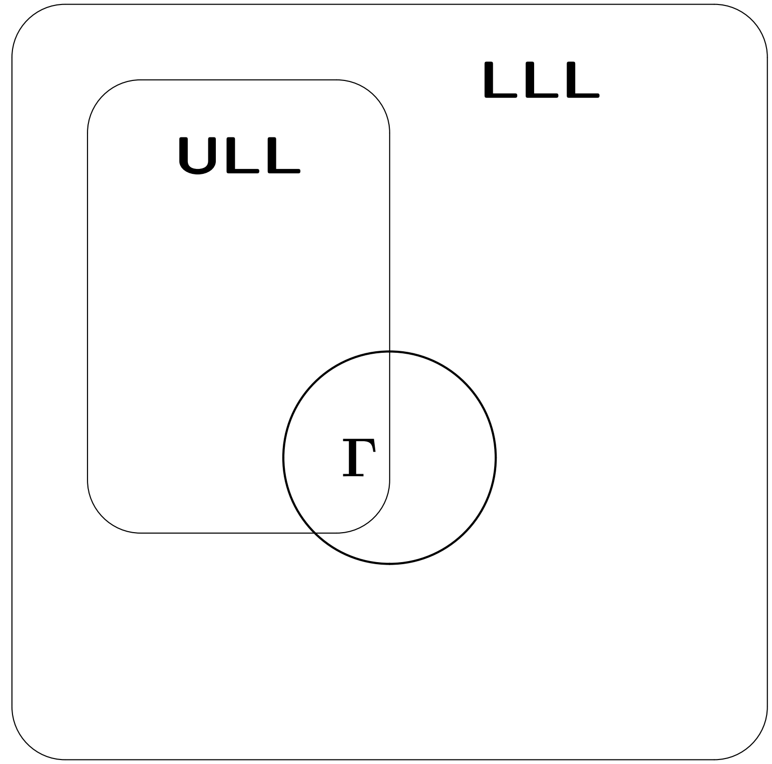
a $\mathbf{LLL}$-model M of Γ is minimally abnormal
iff
there is no $\mathbf{LLL}$-model M' of Γ for which $Ab(M') \subset Ab(M)$

$\Gamma \vDash_{\mathbf{AL}^m} A$ iff all minimally abnormal models of Γ verify A

Simple strategy: either of the above if the Simple strategy is suitable



Abnormal Γ



Normal Γ

flip-flop (if Ω not suitably restricted or because of strategy)

there are no $\mathbf{AL}$-models, but only $\mathbf{AL}$-models of some Γ

5 Some Metatheory

5.1 Preliminaries

LLL is reflexive, transitive, monotonic, compact, contains **CL** (see before) and has a characteristic semantics

Ω: all formulas of a (possibly restricted) logical form **F**

provisos:
- if A has the form **F**, then $A \vdash_{\mathbf{LLL}} Dab(\Delta)$ for some (finite) $\Delta \in \Omega$
- every $A \in \Omega$ is falsified by a **LLL**-model

the provisos are only required for obtaining a standard **ULL** by a standard procedure, *not* for the rest of the metatheory

strategy: we shall consider only Reliability and Minimal Abnormality (the Simple strategy reduces to these where it is sensible)

5.2 On the ULL

Definition $\Gamma \vdash_{\mathbf{ULL}} A$ iff $\Gamma \cup \Omega^{\neg} \vdash_{\mathbf{LLL}} A$

viz. **ULL**: exactly as **LLL**, except that it trivializes abnormalities

Theorem 1
Where Ω is characterized by the logical form **F** + a (possibly empty) restriction, **ULL** is **LLL** + the axiom schema ¬**F**.

Proof.
(1) **LLL** + ¬**F** contains **ULL**: obvious
(2) **ULL** contains **LLL** + ¬**F**:
suppose: B has the form **F**
there is a finite $\Delta \in \Omega$ such that $B \vdash_{\mathbf{LLL}} Dab(\Delta)$
for every $C \in \Delta$, $\vdash_{\mathbf{ULL}} \neg C$
so $\vdash_{\mathbf{ULL}} \neg Dab(\Delta)$ and also $\vdash_{\mathbf{ULL}} \neg B$

$\mathbf{ULL} = \mathbf{LLL} + \neg\textsf{F}$

$\mathbf{ULL}$-semantics: the $\mathbf{LLL}$-models that verify no member of $\mathbf{\Omega}$

Theorem 2
$\mathbf{LLL}$ + the axiom schema $\neg\textsf{F}$ is sound and complete w.r.t. the $\mathbf{ULL}$-semantics.

Obvious in view of the proof of Theorem 1.

Theorem 3

$\Gamma \vdash_{\mathbf{ULL}} A$ iff there is a finite $\Delta \subseteq \Omega$ such that $\Gamma \vdash_{\mathbf{LLL}} A \sqcup Dab(\Delta)$. (Derivability Adjustment Theorem)

Proof.

The following six statements are equivalent:

$\Gamma \vdash_{\mathbf{ULL}} A$

$\Gamma \cup \Omega^{\neg} \vdash_{\mathbf{LLL}} A$ — Def. $\mathbf{ULL}$

$\Gamma' \cup \Delta^{\neg} \vdash_{\mathbf{LLL}} A$ for a finite $\Gamma' \subseteq \Gamma$ and a finite $\Delta \subseteq \Omega$ — $\mathbf{LLL}$ compact

$\Gamma' \vdash_{\mathbf{LLL}} A \sqcup Dab(\Delta)$ for those Γ' and Δ — $\mathbf{LLL}$ contains $\mathbf{CL}$

$\Gamma \vdash_{\mathbf{LLL}} A \sqcup Dab(\Delta)$ for a finite $\Delta \subseteq \Omega$ — $\mathbf{LLL}$ monotonic

'motor' for the adaptive logic: one tries to get as close to $\mathbf{ULL}$ as possible by considering $Dab(\Delta)$ as false whenever Γ permits so obvious:

Theorem 4

ULL contains **CL**

ULL is reflexive, transitive, monotonic, and uniform

ULL is compact

5.3 Strong Reassurance

Stopperedness, Smoothness

if a model of the premisses is not selected, this is justified by the fact that a selected model of the premisses is less abnormal

$\mathcal{M}_{\Gamma}^{\mathbf{LLL}}$: the $\mathbf{LLL}$-models of Γ

$\mathcal{M}_{\Gamma}^{m}$: the $\mathbf{AL}^{m}$-models of Γ

$\mathcal{M}_{\Gamma}^{r}$: the $\mathbf{AL}^{r}$-models of Γ

Theorem 5

If $M \in \mathcal{M}_\Gamma^{\mathbf{LLL}} - \mathcal{M}_\Gamma^m$, then there is a $M' \in \mathcal{M}_\Gamma^m$ such that $Ab(M') \subset Ab(M)$. (Strong Reassurance for Minimal Abnormality.)

Proof. (holds vacuously if $\mathcal{M}_\Gamma^m = \mathcal{M}_\Gamma^{\mathbf{LLL}}$)

Consider $M \in \mathcal{M}_\Gamma^{\mathbf{LLL}} - \mathcal{M}_\Gamma^m$; $D_1, D_2, \ldots$ list of all members of Ω

$\Delta_0 = \emptyset$

$\Delta_{i+1} = \Delta_i \cup \{\neg D_{i+1}\}$

if $Ab(M') \subseteq Ab(M)$ for some M' of $\Gamma \cup \Delta_i \cup \{\neg D_{i+1}\}$, otherwise

$\Delta_{i+1} = \Delta_i$

$\Delta = \Delta_0 \cup \Delta_1 \cup \Delta_2 \cup \ldots$

$\Gamma \cup \Delta$ has $\mathbf{LLL}$-models (compactness of $\mathbf{LLL}$ + construction)

$\Rightarrow$

Theorem 5

If $M \in \mathcal{M}_\Gamma^{\mathbf{LLL}} - \mathcal{M}_\Gamma^m$, then there is a $M' \in \mathcal{M}_\Gamma^m$ such that $Ab(M') \subset Ab(M)$. (Strong Reassurance for Minimal Abnormality.)

Proof. (holds vacuously if $\mathcal{M}_\Gamma^m = \mathcal{M}_\Gamma^{\mathbf{LLL}}$)
Consider $M \in \mathcal{M}_\Gamma^{\mathbf{LLL}} - \mathcal{M}_\Gamma^m$. $\Gamma \cup \Delta$ has $\mathbf{LLL}$-models

Step 1. If M' is a model of $\Gamma \cup \Delta$, then $Ab(M') \subset Ab(M)$.

Suppose that there is a $D_j \in \Omega$ such that $D_j \in Ab(M') - Ab(M)$. Let M'' be a model of $\Gamma \cup \Delta_{j-1}$ for which $Ab(M'') \subseteq Ab(M)$. As $D_j \notin Ab(M)$, $D_j \notin Ab(M'')$. Hence M'' is a model of $\Gamma \cup \Delta_{j-1} \cup \{\neg D_j\}$ and $Ab(M'') \subseteq Ab(M)$. So $\neg D_j \in \Delta_j \subseteq \Delta$. As M' is a model of $\Gamma \cup \Delta$, $D_j \notin Ab(M')$. But this contradicts the supposition.

$\Rightarrow$

Theorem 5
If $M \in \mathcal{M}^{\mathbf{LLL}}_{\Gamma} - \mathcal{M}^{m}_{\Gamma}$, then there is a $M' \in \mathcal{M}^{m}_{\Gamma}$ such that $Ab(M') \subset Ab(M)$. (Strong Reassurance for Minimal Abnormality.)

Proof. (holds vacuously if $\mathcal{M}^{m}_{\Gamma} = \mathcal{M}^{\mathbf{LLL}}_{\Gamma}$)
Consider $M \in \mathcal{M}^{\mathbf{LLL}}_{\Gamma} - \mathcal{M}^{m}_{\Gamma}$. $\Gamma \cup \Delta$ has **LLL**-models

Step 1. If M' is a model of $\Gamma \cup \Delta$, then $Ab(M') \subset Ab(M)$.

Step 2. Every model of $\Gamma \cup \Delta$ is a minimal abnormal model of Γ.
Suppose that M' is a model of $\Gamma \cup \Delta$, but is not a minimal abnormal model of Γ. Hence [...] there is a model M'' of Γ for which $Ab(M'') \subset Ab(M')$.

It follows that M'' is a model of $\Gamma \cup \Delta$. If it were not, then, as M'' is a model of Γ, there is a $\neg D_j \in \Delta$ such that M' verifies $\neg D_j$ and M'' falsifies $\neg D_j$. But then M' falsifies D_j and M'' verifies D_j, which is impossible in view of $Ab(M'') \subset Ab(M')$.

Consider any $D_j \in Ab(M') - Ab(M'') \neq \emptyset$. As M'' is a model of $\Gamma \cup \Delta_{j-1}$ that falsifies D_j, it is a model of $\Gamma \cup \Delta_{j-1} \cup \{\neg D_j\}$. As $Ab(M'') \subset Ab(M')$ and $Ab(M') \subseteq Ab(M)$, $Ab(M'') \subset Ab(M)$. It follows that $\Delta_j = \Delta_{j-1} \cup \{\neg D_j\}$ and hence that $\neg D_j \in \Delta$. But then $D_j \notin Ab(M')$. Hence, $Ab(M'') = Ab(M')$. So the supposition leads to a contradiction.

Theorem 5
If $M \in \mathcal{M}_{\Gamma}^{\mathbf{LLL}} - \mathcal{M}_{\Gamma}^{m}$, then there is a $M' \in \mathcal{M}_{\Gamma}^{m}$ such that $Ab(M') \subset Ab(M)$. (Strong Reassurance for Minimal Abnormality.)

Proof. (holds vacuously if $\mathcal{M}_{\Gamma}^{m} = \mathcal{M}_{\Gamma}^{\mathbf{LLL}}$)
Consider $M \in \mathcal{M}_{\Gamma}^{\mathbf{LLL}} - \mathcal{M}_{\Gamma}^{m}$. $\Gamma \cup \Delta$ has $\mathbf{LLL}$-models

Step 1. If M' is a model of $\Gamma \cup \Delta$, then $Ab(M') \subset Ab(M)$.

Step 2. Every model of $\Gamma \cup \Delta$ is a minimal abnormal model of Γ.

Lemma

$\mathcal{M}_\Gamma^m \subseteq \mathcal{M}_\Gamma^r$ (all Minimal Abnormal models are Reliable models)

Theorem 6

If $M \in \mathcal{M}_\Gamma^{\mathrm{LLL}} - \mathcal{M}_\Gamma^r$, then there is a $M' \in \mathcal{M}_\Gamma^r$ such that $Ab(M') \subset Ab(M)$. (Strong Reassurance for Reliability.)

Theorem 7
$\Gamma \vdash_{\mathbf{AL}^r} A$ iff $\Gamma \vdash_{\mathbf{LLL}} A \sqcup Dab(\Delta)$ and $\Delta \cap U(\Gamma) = \emptyset$ for a finite $\Delta \subset \Omega$.

Proof.
Both directions obvious in view of previous Lemma and the definition of $\Gamma \vdash_{\mathbf{AL}^r} A$.

Theorem 7

$\Gamma \vdash_{\mathbf{AL}^r} A$ iff $\Gamma \vdash_{\mathbf{LLL}} A \sqcup Dab(\Delta)$ and $\Delta \cap U(\Gamma) = \emptyset$ for a finite $\Delta \subset \Omega$.

Theorem 8

$\Gamma \vDash_{\mathbf{AL}^r} A$ iff $\Gamma \vDash_{\mathbf{LLL}} A \sqcup Dab(\Delta)$ and $\Delta \cap U(\Gamma) = \emptyset$ for a finite $\Delta \subset \Omega$.

Proof.

$\Rightarrow$

all models in $\mathcal{M}^r_\Gamma$ verify A

so $\Gamma \cup (\Omega - U(\Gamma))^\neg \vDash_{\mathbf{LLL}} A$

so $\Gamma' \cup \Delta^\neg \vDash_{\mathbf{LLL}} A$ for finite $\Gamma' \subset \Gamma$ and $\Delta \subset \Omega$ — compact

so $\Gamma' \vDash_{\mathbf{LLL}} A \sqcup Dab(\Delta)$ — $\mathbf{CL}$

so $\Gamma \vDash_{\mathbf{LLL}} A \sqcup Dab(\Delta)$ — monotonic

$\Leftarrow$

suppose there are $\mathbf{LLL}$-models of Γ and they all verify $A \sqcup Dab(\Delta)$

so there are $\mathbf{AL}^r$-models of Γ — Strong Reassurance

all $\mathbf{AL}^r$-models of Γ falsify $Dab(\Delta)$ — $\Delta \cap U(\Gamma) = \emptyset$

so all $\mathbf{AL}^r$-models of Γ verify A

Theorem 7
$\Gamma \vdash_{\mathbf{AL}^r} \boldsymbol{A}$ iff $\Gamma \vdash_{\mathbf{LLL}} \boldsymbol{A} \sqcup \boldsymbol{Dab}(\boldsymbol{\Delta})$ and $\boldsymbol{\Delta} \cap \boldsymbol{U}(\Gamma) = \emptyset$ for a finite $\boldsymbol{\Delta} \subset \boldsymbol{\Omega}$.

Theorem 8
$\Gamma \vDash_{\mathbf{AL}^r} \boldsymbol{A}$ iff $\Gamma \vDash_{\mathbf{LLL}} \boldsymbol{A} \sqcup \boldsymbol{Dab}(\boldsymbol{\Delta})$ and $\boldsymbol{\Delta} \cap \boldsymbol{U}(\Gamma) = \emptyset$ for a finite $\boldsymbol{\Delta} \subset \boldsymbol{\Omega}$.

Lemma
$\Gamma \vdash_{\mathbf{LLL}} \boldsymbol{A} \sqcup \boldsymbol{Dab}(\boldsymbol{\Delta})$ iff $\Gamma \vDash_{\mathbf{LLL}} \boldsymbol{A} \sqcup \boldsymbol{Dab}(\boldsymbol{\Delta})$.

Proof. By the soundness and completeness of **LLL**.

Theorem 7

$\Gamma \vdash_{\mathbf{AL}^r} A$ iff $\Gamma \vdash_{\mathbf{LLL}} A \sqcup Dab(\Delta)$ and $\Delta \cap U(\Gamma) = \emptyset$ for a finite $\Delta \subset \Omega$.

Theorem 8

$\Gamma \vDash_{\mathbf{AL}^r} A$ iff $\Gamma \vDash_{\mathbf{LLL}} A \sqcup Dab(\Delta)$ and $\Delta \cap U(\Gamma) = \emptyset$ for a finite $\Delta \subset \Omega$.

Lemma

$\Gamma \vdash_{\mathbf{LLL}} A \sqcup Dab(\Delta)$ iff $\Gamma \vDash_{\mathbf{LLL}} A \sqcup Dab(\Delta)$.

Corollary

$\Gamma \vdash_{\mathbf{AL}^r} A$ iff $\Gamma \vDash_{\mathbf{AL}^r} A$.

5.5 Some Further properties

terminology

minimal $\boldsymbol{Dab}$-consequence of Γ

$\Phi^\circ(\Gamma)$:	set of all sets that contain one member of each minimal $\boldsymbol{Dab}$-consequence of Γ
$\Phi^\star(\Gamma)$:	contains, for every $\varphi \in \Phi^\circ(\Gamma)$, $\boldsymbol{Cn}_{\mathbf{LLL}}(\varphi) \cap \Omega$
$\Phi(\Gamma)$:	$\varphi \in \Phi^\star(\Gamma)$ that are not proper supersets of a $\varphi' \in \Phi^\star(\Gamma)$

Lemma

$M \in \mathcal{M}^m_\Gamma$ iff $M \in \mathcal{M}^{\mathbf{LLL}}_\Gamma$ and $\boldsymbol{Ab}(M) \in \Phi_\Gamma$.

Proof: long but perspicuous.

immediate or almost immediate consequences of the Lemma:

Theorem 9 each of the following obtains:

1. $\mathcal{M}^m_\Gamma \subseteq \mathcal{M}^r_\Gamma$. Hence $Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma)$.
2. If $A \in \Omega - U(\Gamma)$, then $\neg A \in Cn_{\mathbf{AL}^r}(\Gamma)$.
3. If $Dab(\Delta)$ is a minimal Dab-consequence of Γ and $A \in \Delta$, then some $M \in \mathcal{M}^m_\Gamma$ verifies A and falsifies all members (if any) of $\Delta - \{A\}$.
4. $\mathcal{M}^m_\Gamma = \mathcal{M}^m_{Cn_{\mathbf{AL}^m}(\Gamma)}$ whence $Cn_{\mathbf{AL}^m}(\Gamma) = Cn_{\mathbf{AL}^m}(Cn_{\mathbf{AL}^m}(\Gamma))$. (Fixed Point.)
5. $\mathcal{M}^r_\Gamma = \mathcal{M}^r_{Cn_{\mathbf{AL}^r}(\Gamma)}$ whence $Cn_{\mathbf{AL}^r}(\Gamma) = Cn_{\mathbf{AL}^r}(Cn_{\mathbf{AL}^r}(\Gamma))$. (Fixed Point.)
6. For all $\Delta \subseteq \Omega$, $Dab(\Delta) \in Cn_{\mathbf{AL}}(\Gamma)$ iff $Dab(\Delta) \in Cn_{\mathbf{LLL}}(\Gamma)$. (Immunity.)
7. If $\Gamma \vDash_{\mathbf{AL}} A$ for every $A \in \Gamma'$, and $\Gamma \cup \Gamma' \vDash_{\mathbf{AL}} B$, then $\Gamma \vDash_{\mathbf{AL}} B$. (Cautious Cut.)
8. If $\Gamma \vDash_{\mathbf{AL}} A$ for every $A \in \Gamma'$, and $\Gamma \vDash_{\mathbf{AL}} B$, then $\Gamma \cup \Gamma' \vDash_{\mathbf{AL}} B$. (Cautious Monotonicity.)

Theorem 10 each of the following obtains:

1. If Γ is normal, then $\mathcal{M}_{\Gamma}^{\mathbf{ULL}} = \mathcal{M}_{\Gamma}^{m} = \mathcal{M}_{\Gamma}^{r}$ whence $Cn_{\mathbf{AL}^r}(\Gamma) = Cn_{\mathbf{AL}^m}(\Gamma) = Cn_{\mathbf{ULL}}(\Gamma)$.

If Γ is normal, then $U(\Gamma) = \emptyset$ and only $\mathbf{ULL}$-models of Γ are minimally abnormal.

Theorem 10 each of the following obtains:

1. If Γ is normal, then $\mathcal{M}^{\mathbf{ULL}}_{\Gamma} = \mathcal{M}^{m}_{\Gamma} = \mathcal{M}^{r}_{\Gamma}$ whence $Cn_{\mathbf{AL}^r}(\Gamma) = Cn_{\mathbf{AL}^m}(\Gamma) = Cn_{\mathbf{ULL}}(\Gamma)$.
2. If Γ is abnormal and $\mathcal{M}^{\mathbf{LLL}}_{\Gamma} \neq \emptyset$, then $\mathcal{M}^{\mathbf{ULL}}_{\Gamma} \subset \mathcal{M}^{m}_{\Gamma}$ and hence $Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma) \subset Cn_{\mathbf{ULL}}(\Gamma)$.

If Γ is abnormal, then $\mathcal{M}^{\mathbf{ULL}}_{\Gamma} = \emptyset$.

Theorem 10 each of the following obtains:

1. If Γ is normal, then $\mathcal{M}_{\Gamma}^{\mathbf{ULL}} = \mathcal{M}_{\Gamma}^{m} = \mathcal{M}_{\Gamma}^{r}$ whence $Cn_{\mathbf{AL}^r}(\Gamma) = Cn_{\mathbf{AL}^m}(\Gamma) = Cn_{\mathbf{ULL}}(\Gamma)$.
2. If Γ is abnormal and $\mathcal{M}_{\Gamma}^{\mathbf{LLL}} \neq \emptyset$, then $\mathcal{M}_{\Gamma}^{\mathbf{ULL}} \subset \mathcal{M}_{\Gamma}^{m}$ and hence $Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma) \subset Cn_{\mathbf{ULL}}(\Gamma)$.
3. $\mathcal{M}_{\Gamma}^{\mathbf{ULL}} \subseteq \mathcal{M}_{\Gamma}^{m} \subseteq \mathcal{M}_{\Gamma}^{r} \subseteq \mathcal{M}_{\Gamma}^{\mathbf{LLL}}$ whence $Cn_{\mathbf{LLL}}(\Gamma) \subseteq Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma) \subseteq Cn_{\mathbf{ULL}}(\Gamma)$.

$\mathcal{M}_{\Gamma}^{\mathbf{ULL}} \subseteq \mathcal{M}_{\Gamma}^{m}$: from 1 and 2. $\mathcal{M}_{\Gamma}^{r} \subseteq \mathcal{M}_{\Gamma}^{\mathbf{LLL}}$ is immediate in view of the definition of a reliable model of Γ. $\mathcal{M}_{\Gamma}^{m} \subseteq \mathcal{M}_{\Gamma}^{r}$ is item 1 of the previous Theorem.

Theorem 10 each of the following obtains:

1. If Γ is normal, then $\mathcal{M}_\Gamma^{\mathbf{ULL}} = \mathcal{M}_\Gamma^m = \mathcal{M}_\Gamma^r$ whence $Cn_{\mathbf{AL}^r}(\Gamma) = Cn_{\mathbf{AL}^m}(\Gamma) = Cn_{\mathbf{ULL}}(\Gamma)$.
2. If Γ is abnormal and $\mathcal{M}_\Gamma^{\mathbf{LLL}} \neq \emptyset$, then $\mathcal{M}_\Gamma^{\mathbf{ULL}} \subset \mathcal{M}_\Gamma^m$ and hence $Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma) \subset Cn_{\mathbf{ULL}}(\Gamma)$.
3. $\mathcal{M}_\Gamma^{\mathbf{ULL}} \subseteq \mathcal{M}_\Gamma^m \subseteq \mathcal{M}_\Gamma^r \subseteq \mathcal{M}_\Gamma^{\mathbf{LLL}}$ whence $Cn_{\mathbf{LLL}}(\Gamma) \subseteq Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma) \subseteq Cn_{\mathbf{ULL}}(\Gamma)$.
4. $\mathcal{M}_\Gamma^r \subset \mathcal{M}_\Gamma^{\mathbf{LLL}}$ iff $\Gamma \cup \{A\}$ is $\mathbf{LLL}$-satisfiable for some $A \in \Omega - U(\Gamma)$.

Immediate in view of the definitions of a reliable model and $\Gamma \vDash_{\mathbf{AL}^r} A$.

Theorem 10 each of the following obtains:

1. If Γ is normal, then $\mathcal{M}_\Gamma^{\mathbf{ULL}} = \mathcal{M}_\Gamma^m = \mathcal{M}_\Gamma^r$ whence $Cn_{\mathbf{AL}^r}(\Gamma) = Cn_{\mathbf{AL}^m}(\Gamma) = Cn_{\mathbf{ULL}}(\Gamma)$.
2. If Γ is abnormal and $\mathcal{M}_\Gamma^{\mathbf{LLL}} \neq \emptyset$, then $\mathcal{M}_\Gamma^{\mathbf{ULL}} \subset \mathcal{M}_\Gamma^m$ and hence $Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma) \subset Cn_{\mathbf{ULL}}(\Gamma)$.
3. $\mathcal{M}_\Gamma^{\mathbf{ULL}} \subseteq \mathcal{M}_\Gamma^m \subseteq \mathcal{M}_\Gamma^r \subseteq \mathcal{M}_\Gamma^{\mathbf{LLL}}$ whence $Cn_{\mathbf{LLL}}(\Gamma) \subseteq Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma) \subseteq Cn_{\mathbf{ULL}}(\Gamma)$.
4. $\mathcal{M}_\Gamma^r \subset \mathcal{M}_\Gamma^{\mathbf{LLL}}$ iff $\Gamma \cup \{A\}$ is $\mathbf{LLL}$-satisfiable for some $A \in \Omega - U(\Gamma)$.
5. $Cn_{\mathbf{LLL}}(\Gamma) \subset Cn_{\mathbf{AL}^r}(\Gamma)$ iff $\mathcal{M}_\Gamma^r \subset \mathcal{M}_\Gamma^{\mathbf{LLL}}$.

$\Rightarrow$ Suppose $A \in Cn_{\mathbf{LLL}}(\Gamma) - Cn_{\mathbf{AL}^r}(\Gamma)$. So, for some $A \in \Omega - U(\Gamma)$, all $M \in \mathcal{M}_\Gamma^r$ falsify A whereas some $M \in \mathcal{M}_\Gamma^{\mathbf{LLL}} - \mathcal{M}_\Gamma^r$ verifies A.
$\Leftarrow$ obvious.

Theorem 10 each of the following obtains:

1. If $\mathbf{\Gamma}$ is normal, then $\mathcal{M}_{\Gamma}^{\mathbf{ULL}} = \mathcal{M}_{\Gamma}^{m} = \mathcal{M}_{\Gamma}^{r}$ whence $Cn_{\mathbf{AL}^r}(\Gamma) = Cn_{\mathbf{AL}^m}(\Gamma) = Cn_{\mathbf{ULL}}(\Gamma)$.
2. If $\mathbf{\Gamma}$ is abnormal and $\mathcal{M}_{\Gamma}^{\mathbf{LLL}} \neq \emptyset$, then $\mathcal{M}_{\Gamma}^{\mathbf{ULL}} \subset \mathcal{M}_{\Gamma}^{m}$ and hence $Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma) \subset Cn_{\mathbf{ULL}}(\Gamma)$.
3. $\mathcal{M}_{\Gamma}^{\mathbf{ULL}} \subseteq \mathcal{M}_{\Gamma}^{m} \subseteq \mathcal{M}_{\Gamma}^{r} \subseteq \mathcal{M}_{\Gamma}^{\mathbf{LLL}}$ whence $Cn_{\mathbf{LLL}}(\Gamma) \subseteq Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma) \subseteq Cn_{\mathbf{ULL}}(\Gamma)$.
4. $\mathcal{M}_{\Gamma}^{r} \subset \mathcal{M}_{\Gamma}^{\mathbf{LLL}}$ iff $\Gamma \cup \{A\}$ is $\mathbf{LLL}$-satisfiable for some $A \in \Omega - U(\Gamma)$.
5. $Cn_{\mathbf{LLL}}(\Gamma) \subset Cn_{\mathbf{AL}^r}(\Gamma)$ iff $\mathcal{M}_{\Gamma}^{r} \subset \mathcal{M}_{\Gamma}^{\mathbf{LLL}}$.
6. $\mathcal{M}_{\Gamma}^{m} \subset \mathcal{M}_{\Gamma}^{\mathbf{LLL}}$ iff there is a (possibly infinite) $\Delta \subseteq \Omega$ such that $\Gamma \cup \Delta$ is $\mathbf{LLL}$-satisfiable and there is no $\varphi \in \Phi_{\Gamma}$ for which $\Delta \subseteq \varphi$.

Immediate in view of the definitions of a Minimal Abnormal model and $\Gamma \vDash_{\mathbf{AL}^m} A$.

Theorem 10 each of the following obtains:

1. If Γ is normal, then $\mathcal{M}_\Gamma^{\mathbf{ULL}} = \mathcal{M}_\Gamma^m = \mathcal{M}_\Gamma^r$ whence $Cn_{\mathbf{AL}^r}(\Gamma) = Cn_{\mathbf{AL}^m}(\Gamma) = Cn_{\mathbf{ULL}}(\Gamma)$.
2. If Γ is abnormal and $\mathcal{M}_\Gamma^{\mathbf{LLL}} \neq \emptyset$, then $\mathcal{M}_\Gamma^{\mathbf{ULL}} \subset \mathcal{M}_\Gamma^m$ and hence $Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma) \subset Cn_{\mathbf{ULL}}(\Gamma)$.
3. $\mathcal{M}_\Gamma^{\mathbf{ULL}} \subseteq \mathcal{M}_\Gamma^m \subseteq \mathcal{M}_\Gamma^r \subseteq \mathcal{M}_\Gamma^{\mathbf{LLL}}$ whence $Cn_{\mathbf{LLL}}(\Gamma) \subseteq Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma) \subseteq Cn_{\mathbf{ULL}}(\Gamma)$.
4. $\mathcal{M}_\Gamma^r \subset \mathcal{M}_\Gamma^{\mathbf{LLL}}$ iff $\Gamma \cup \{A\}$ is $\mathbf{LLL}$-satisfiable for some $A \in \Omega - U(\Gamma)$.
5. $Cn_{\mathbf{LLL}}(\Gamma) \subset Cn_{\mathbf{AL}^r}(\Gamma)$ iff $\mathcal{M}_\Gamma^r \subset \mathcal{M}_\Gamma^{\mathbf{LLL}}$.
6. $\mathcal{M}_\Gamma^m \subset \mathcal{M}_\Gamma^{\mathbf{LLL}}$ iff there is a (possibly infinite) $\Delta \subseteq \Omega$ such that $\Gamma \cup \Delta$ is $\mathbf{LLL}$-satisfiable and there is no $\varphi \in \Phi_\Gamma$ for which $\Delta \subseteq \varphi$.
7. If there are $A_1, \ldots, A_n \in \Omega$ $(n \geq 1)$ such that $\Gamma \cup \{A_1, \ldots, A_n\}$ is $\mathbf{LLL}$-satisfiable and, for every $\varphi \in \Phi_\Gamma$, $\{A_1, \ldots, A_n\} \not\subseteq \varphi$, then $Cn_{\mathbf{LLL}}(\Gamma) \subset Cn_{\mathbf{AL}^m}(\Gamma)$.

Suppose the antecedent is true. Every $M \in \mathcal{M}_\Gamma^m$ falsifies some A_i whereas some $M \in \mathcal{M}_\Gamma^{\mathbf{LLL}}$ (viz. an $M \in \mathcal{M}_{\Gamma \cup \{A_1,\ldots,A_n\}}^{\mathbf{LLL}}$) verifies $A_1 \sqcap \ldots \sqcap A_n$.

Theorem 10 each of the following obtains:

1. If Γ is normal, then $\mathcal{M}_{\Gamma}^{\mathbf{ULL}} = \mathcal{M}_{\Gamma}^{m} = \mathcal{M}_{\Gamma}^{r}$ whence $Cn_{\mathbf{AL}^r}(\Gamma) = Cn_{\mathbf{AL}^m}(\Gamma) = Cn_{\mathbf{ULL}}(\Gamma)$.
2. If Γ is abnormal and $\mathcal{M}_{\Gamma}^{\mathbf{LLL}} \neq \emptyset$, then $\mathcal{M}_{\Gamma}^{\mathbf{ULL}} \subset \mathcal{M}_{\Gamma}^{m}$ and hence $Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma) \subset Cn_{\mathbf{ULL}}(\Gamma)$.
3. $\mathcal{M}_{\Gamma}^{\mathbf{ULL}} \subseteq \mathcal{M}_{\Gamma}^{m} \subseteq \mathcal{M}_{\Gamma}^{r} \subseteq \mathcal{M}_{\Gamma}^{\mathbf{LLL}}$ whence $Cn_{\mathbf{LLL}}(\Gamma) \subseteq Cn_{\mathbf{AL}^r}(\Gamma) \subseteq Cn_{\mathbf{AL}^m}(\Gamma) \subseteq Cn_{\mathbf{ULL}}(\Gamma)$.
4. $\mathcal{M}_{\Gamma}^{r} \subset \mathcal{M}_{\Gamma}^{\mathbf{LLL}}$ iff $\Gamma \cup \{A\}$ is $\mathbf{LLL}$-satisfiable for some $A \in \Omega - U(\Gamma)$.
5. $Cn_{\mathbf{LLL}}(\Gamma) \subset Cn_{\mathbf{AL}^r}(\Gamma)$ iff $\mathcal{M}_{\Gamma}^{r} \subset \mathcal{M}_{\Gamma}^{\mathbf{LLL}}$.
6. $\mathcal{M}_{\Gamma}^{m} \subset \mathcal{M}_{\Gamma}^{\mathbf{LLL}}$ iff there is a (possibly infinite) $\Delta \subseteq \Omega$ such that $\Gamma \cup \Delta$ is $\mathbf{LLL}$-satisfiable and there is no $\varphi \in \Phi_{\Gamma}$ for which $\Delta \subseteq \varphi$.
7. If there are $A_1, \ldots, A_n \in \Omega$ $(n \geq 1)$ such that $\Gamma \cup \{A_1, \ldots, A_n\}$ is $\mathbf{LLL}$-satisfiable and, for every $\varphi \in \Phi_{\Gamma}$, $\{A_1, \ldots, A_n\} \not\subseteq \varphi$, then $Cn_{\mathbf{LLL}}(\Gamma) \subset Cn_{\mathbf{AL}^m}(\Gamma)$.
8. $Cn_{\mathbf{AL}^m}(\Gamma)$ and $Cn_{\mathbf{AL}^r}(\Gamma)$ are non-trivial iff $\mathcal{M}_{\Gamma}^{\mathbf{LLL}} \neq \emptyset$.

Immediate from Reassurance + no $\mathbf{LLL}$-model trivial.

Theorem 11
If $\Gamma \vdash_{\mathbf{AL}} \boldsymbol{A}$, then every $\mathbf{AL}$-proof from Γ can be extended in such a way that $\boldsymbol{A}$ is finally derived in it. (Proof Invariance)

etc.

6 Computability Matters

In view of the reasoning processes explicated by $\vdash_{\mathbf{AL}}$,

- $\vdash_{\mathbf{AL}}$ is not decidable
- there is no positive test for $\vdash_{\mathbf{AL}}$

Does the dynamics of the proofs go anywhere?

Are there criteria for final derivability?

Does the dynamics of the proofs go anywhere?

in view of the block analysis of proofs (and the block semantics):

- a stage of a proof provides a certain insight in the premises
- every step of the proof is informative or non-informative
  - if informative: more insight in the premises gained
  - if non-informative: no insight lost (sq)
- sensible proofs converge toward maximal insight
  (sensible proofs are obtained by the procedure on the next slides)

Are there criteria for final derivability?

- the block semantics
- tableau methods
- procedural criterion

Procedural criterion for Reliability

based on prospective dynamic proofs
(goal-directed + most heuristics pushed into the proof)

3 phase procedure for testing whether $\Gamma \vdash_{\mathbf{AL}^r} A$

if the procedure stops: answer is obtained (YES / NO)

(procedure at least as good as tableau methods)

`pdp2.exe` at `http://logica.ugent.be/centrum/programs/` implements procedure for propositional $\mathbf{ACLuN}^r$

$\Gamma \vdash_{\mathbf{AL}^r} \boldsymbol{G}$?

Phase 1

try to derive $\boldsymbol{G}$ on a condition

- no success: $\Gamma \nvdash_{\mathbf{ACLuN1}} \boldsymbol{G}$
- success: $\boldsymbol{G}$ derived on a condition $\boldsymbol{\Delta}$ at line $\boldsymbol{i}$
  - $\boldsymbol{\Delta} = \emptyset$: $\Gamma \vdash_{\mathbf{ACLuN1}} \boldsymbol{G}$
  - $\boldsymbol{\Delta} \neq \emptyset$:

    $\Rightarrow$ phase 2 $\Rightarrow$ phase 1

    - line $\boldsymbol{i}$ not marked: $\Gamma \vdash_{\mathbf{ACLuN1}} \boldsymbol{G}$
    - line $\boldsymbol{i}$ marked: try to derive $\boldsymbol{G}$ on a (different) condition

$\Gamma \vdash_{\mathbf{AL}^r} G$?

G derived on condition Δ ($\neq \emptyset$) at line i

Phase 2

try to derive $Dab(\Delta)$ on a condition

- no success:[1] return to phase 1 (line i is unmarked)
- success: $Dab(\Delta)$ derived on condition Θ at line j
  - $\Theta = \emptyset$: mark line i; return to phase 1
  - $\Theta \neq \emptyset$:

    $\Rightarrow$ phase 3 $\Rightarrow$ phase 2

    - line j not marked:[2] mark line i; return to phase 1
    - line j marked:[3] try to derive $Dab(\Delta)$ on a (different) condition

[1] $\Delta \cap U(\Gamma) = \emptyset$
[2] $\Theta \cap U(\Gamma) = \emptyset$ whence $\Delta \cap U(\Gamma) \neq \emptyset$
[3] so $\Gamma \vdash_{\mathbf{LLL}} Dab(\Theta)$, so *possibly* $\Delta \cap U(\Gamma) = \emptyset$

$\Gamma \vdash_{\mathbf{AL}^r} G$?

G derived on condition Δ ($\neq \emptyset$) at line i

$Dab(\Delta)$ derived on condition Θ at line j[1]

Phase 3

try to derive $Dab(\Theta)$ on a the condition $\emptyset$

- no success: return to phase 2 (line j is unmarked)[2]
- success: mark line j; return to phase 2[3]

[1] so $\Gamma \vdash_{\mathbf{LLL}} Dab(\Delta \cup \Theta)$
[2] so $\Gamma \nvdash_{\mathbf{LLL}} Dab(\Theta)$, whence $\Delta \cap U(\Gamma) \neq \emptyset$
[3] so $\Gamma \vdash_{\mathbf{LLL}} Dab(\Theta)$, so *possibly* $\Delta \cap U(\Gamma) = \emptyset$

Universal logic

the aim: characterize every reasoning form that displays the internal dynamics (including all defeasible reasoning) by an adaptive logic in SF

slotwoord slotwoord

slotwoord slotwoord

slotwoord slotwoord

slotwoord slotwoord

alarm alarm

A Further examples and applications

- Corrective
- Ampliative (+ ampliative and corrective)
- Incorporation
- Applications

Corrective

- inconsistency-adaptive logics (adapting to negation gluts): $\mathbf{ACLuN^r}$ and $\mathbf{ACLuN^m}$, those based on other paraconsistent logics, including **CLuNs** (**LP**, ...), **ANA**, Jaśkowski's **D2**, ...
- negation gaps
- gluts/gaps for all logical symbols
- ambiguity adaptive logics
- adaptive zero logic
- corrective deontic logics
- prioritized ial
- ...

Ampliative (+ ampliative and corrective)

- compatibility (characterization)
- compatibility with inconsistent premises
- diagnosis
- prioritized adaptive logics
- inductive generalization
- abduction
- inference to the best explanation
- analogies, metaphors
- erotetic evocation and erotetic inference
- changing positions in discussions
- ...

Incorporation (possibly + extension)

- flat Rescher–Manor consequence relations (+ extensions)
- partial structures and pragmatic truth
- prioritized Rescher–Manor consequence relations
- circumscription, defaults, negation as failure, ...
- dynamic characterization of $\mathbf{R}_{\rightarrow}$
- signed systems (Besnard & C°)
- ...

Applications

- scientific discovery and creativity
- scientific explanation
- diagnosis
- positions defended / agreed upon in discussions
- changing positions in discussions
- belief revision in inconsistent contexts
- inconsistent arithmetic
- inductive statistical explanation
- tentatively eliminating abnormalities
- Gricean maxims
- ...