Adaptive Logics

Handling Dynamic Reasoning

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1.1 The Problem

many reasoning processes in the sciences (and elsewhere) display

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an internal dynamics

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$\Uparrow$ absence of positive test (at predicative level)

Problem: gain technically sound control on the reasoning processes

1.2 Example 1: Process of explanation

given: $\begin{cases} \text{explanandum } \boldsymbol{E} \\ \text{theory } \boldsymbol{T} \end{cases}$

find: initial condition $\boldsymbol{I}$

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Two different steps

- find potential initial conditions
- establish one of them (other theories)

Six conditions (Hintikka–Halonen)

$\boldsymbol{T}$ and $\boldsymbol{I}$ form an explanation of $\boldsymbol{E}$
iff $\boldsymbol{T}, \boldsymbol{I} \vdash \boldsymbol{E}$ and

(i) $\boldsymbol{T}$ and $\boldsymbol{E}$: no common ind. cons.
(ii) $\boldsymbol{I}$ and $\boldsymbol{E}$: no common predicates
(iii) $\nvdash_{\mathrm{CL}} \sim\boldsymbol{I}$ — $\boldsymbol{I}$ not inconsistent
(iv) $\boldsymbol{T} \nvdash_{\mathrm{CL}} \boldsymbol{E}$ — $\boldsymbol{E}$ not implied by $\boldsymbol{T}$ alone *
(v) $\boldsymbol{I} \nvdash_{\mathrm{CL}} \boldsymbol{E}$ — $\boldsymbol{E}$ not implied by $\boldsymbol{I}$ alone
(vi) $\boldsymbol{T} \nvdash_{\mathrm{CL}} \sim\boldsymbol{I}$ — $\boldsymbol{T}$ not falsified by $\boldsymbol{I}$ *

Six conditions (Hintikka–Halonen)

T and I form an explanation of E
iff $T, I \vdash E$ and

(i) T and E: no common ind. cons.
(ii) I and E: no common predicates
(iii) $\nvdash_{\mathrm{CL}} \sim I$ — I not inconsistent
(iv) $T \nvdash_{\mathrm{CL}} E$ — E not implied by T alone *
(v) $I \nvdash_{\mathrm{CL}} E$ — E not implied by I alone
(vi) $T \nvdash_{\mathrm{CL}} \sim I$ — T not falsified by I *

Comments

no *positive test* for (iv) and (vi)
irrelevant predicates: $I[a] \wedge I'[a]$

1.3 Example 2: (Classical) Compatibility

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find: those A that (taken separately) do not make Γ inconsistent

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note: paraconsistent compatibility (?!)

1.4 Inductive generalization

given: $\left\{ \begin{array}{l} \text{a set of data } \Gamma \text{ and} \\ \\ \text{zero or more background theories} \end{array} \right.$

find: the suitable generalizations
(generalization: $\forall \boldsymbol{A}$ with $\boldsymbol{A}$ purely functional)

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$$\Downarrow$$

only those generalizations $\forall A_i$ derivable for which
no 'minimal' disjunction $\sim\forall A_1 \vee \ldots \vee \sim\forall A_i \vee \ldots \vee \sim\forall A_n$ $(n \geq 1)$
is $\mathbf{CL}$-derivable from Γ

1.5 Erotetic inferences

given: $\left\{ \begin{array}{l} \text{a set of declarative sentences } \Gamma \text{ and/or} \\ \\ \text{an initial question } Q \end{array} \right.$

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question evocation (Andrzej Wiśniewski):

a question Q is evoked by a set of declarative sentences Γ iff

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(ii) $\Gamma \nvdash A$, for any $A \in dQ$ (Q is informative with respect to Γ)

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- interpret an inconsistent theory as consistently as possible
- inductive prediction
- interpreting a person's position during an ongoing discussion
- all reasoning that involves defaults (or more or less preferred premises)
  - diagnostic reasoning
  - handling preferred sets of premises

. . .

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What is an adaptive logic?

What is a dynamic proof theory?

2 Inconsistency-Adaptive Logics

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the original problem:

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interpret in terms of paraconsistent logic?

2.2 Going Paraconsistent

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1 retain full positive logic

if $A_1, \ldots, A_n \vdash_{\mathbf{CL}} B$ and no negation occurs in A_1, . . . , A_n or B

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Replacement of Equivalents not generally valid (not valid in scope of $\sim$)

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What is lost?

DS:

$$\frac{\begin{array}{c} \boldsymbol{A} \vee \boldsymbol{B} \\ \sim\boldsymbol{A} \end{array}}{\boldsymbol{B}}$$

classical semantic reasoning for DS:

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classical

${\sim}A$	$A \vee B$	A	B	
1	1	0	1	
1	1	1	1	impossible
1	1	1	0	impossible

DS: $$\frac{A \vee B \quad \sim A}{B}$$

classical semantic reasoning for DS:

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paraconsistent

$\sim A$	$A \vee B$	A	B	
1	1	0	1	
1	1	1	1	possible
1	1	1	0	possible

note:

DS and many other rules (MT, RAA, . . .)

are invalid in **CLuN**

adding them to **CLuN** results in **CL**

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examples: $\sim\sim A \; / \; A$, de Morgan, . . .

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one wants to consider a formula of the form $A \wedge {\sim}A$ as false,
unless and until proven otherwise
(= unless the premises do not permit so)

Γ requires that $q \wedge {\sim}q$ is true, but not that $p \wedge {\sim}p$ is true

if Γ is true and $p \wedge {\sim}p$ is false, r is true !

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Can this be explicated formally, and how?

2.3 Going Adaptive: Dynamic Proofs

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6	r	1, 3; RC	$\{p \wedge {\sim}p\}$	
7	s	2, 4; RC	$\{q \wedge {\sim}q\}$	✓
8	$q \wedge {\sim}q$	2, 5; RU	$\emptyset$	

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nothing interesting happens when the proof is continued

no mark will be removed or added

Can marked lines become unmarked?

1	$(p \wedge q) \wedge t$	PREM	$\emptyset$
2	$\sim p \vee r$	PREM	$\emptyset$
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6	r	1, 2; RC	$\{p \wedge \sim p\}$	$\checkmark$
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8	$(p \wedge \sim p) \vee (q \wedge \sim q)$	1, 4; RU	$\emptyset$	

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minimal $\boldsymbol{Dab}$-formula at stage $\boldsymbol{s}$:

at stage $\boldsymbol{s}$:
$\boldsymbol{Dab(\Delta)}$ derived on the empty condition
for every $\boldsymbol{\Delta' \subset \Delta}$, $\boldsymbol{Dab(\Delta')}$ not derived on the empty condition

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Dab-formula: disjunction of inconsistencies, $Dab(\Delta)$

minimal Dab-formula at stage s:

at stage s:
$Dab(\Delta)$ derived on the empty condition
for every $\Delta' \subset \Delta$, $Dab(\Delta')$ not derived on the empty condition

where $Dab(\Delta_1),\ \ldots, Dab(\Delta_n)$ are the minimal Dab-formulas at stage s,

$U_s(\Gamma) = \Delta_1 \cup \ldots \cup \Delta_n$

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minimal $\boldsymbol{Dab}$-formula at stage $\boldsymbol{s}$:

at stage $\boldsymbol{s}$:
$\boldsymbol{Dab(\Delta)}$ derived on the empty condition
for every $\boldsymbol{\Delta' \subset \Delta}$, $\boldsymbol{Dab(\Delta')}$ not derived on the empty condition

where $\boldsymbol{Dab(\Delta_1), \ldots, Dab(\Delta_n)}$ are the minimal $\boldsymbol{Dab}$-formulas at stage $\boldsymbol{s}$,

$$\boldsymbol{U_s(\Gamma) = \Delta_1 \cup \ldots \cup \Delta_n}$$

where $\boldsymbol{\Theta}$ is the condition of line $\boldsymbol{i}$, line $\boldsymbol{i}$ is marked iff $\boldsymbol{\Theta \cap U_s(\Gamma) \neq \emptyset}$

Final derivability

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$$\Updownarrow$$

line i will not be marked in any extension of the proof

2.4 Going Adaptive: Semantics

consider the **CLuN**-models of the premise set Γ

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a $\mathbf{CLuN}$-model M of Γ is reliable iff $Ab(M) \subseteq U(\Gamma)$

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▼

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$\Gamma \vDash_{\mathbf{ACLuN}^r} A$ iff all reliable models of Γ verify A

it is provable that $\Gamma \vdash_{\mathbf{ACLuN}^r} A$ iff $\Gamma \vDash_{\mathbf{ACLuN}^r} A$

2.5 Strategies

naive approach:

Simple strategy: take $A \wedge \sim A$ to be false, unless $\Gamma \vdash_{\mathbf{CLuN}} A \wedge \sim A$

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before, we used the Reliability strategy

there are other strategies, each suitable for specific applications

3 The Standard Format

3.1 The Problem

many adaptive logics seem to have a common structure

others can be given this structure under a translation

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whence the plan:
- describe the structure: the SF (standard format)
- define the proof theory and semantics from the SF
- prove as many properties as possible by relying on the SF only

3.2 The Format

- *lower limit logic*

- *set of abnormalities* $\mathbf{\Omega}$

- *strategy*

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upper limit logic:

$\mathbf{ULL} = \mathbf{LLL} +$ axiom/rule that trivializes abnormalities

semantically: the $\mathbf{LLL}$-models that verify no abnormality

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flip-flop

Example 1: $\mathbf{ACLuN}^r$

- *lower limit logic*: $\mathbf{CLuN}$
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upper limit logic: $\mathbf{CL} = \mathbf{CLuN} + (A \wedge {\sim}A) \supset B$

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Example 2: $\mathbf{ACLuN}^m$

- *lower limit logic*: $\mathbf{CLuN}$
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Example 2: $\mathbf{ACLuN}^m$

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Example 3: $\mathbf{IL}^m$

- *lower limit logic*: $\mathbf{CL}$

- *set of abnormalities*: $\boldsymbol{\Omega} = \{\exists \boldsymbol{A} \wedge \exists{\sim}\boldsymbol{A} \mid \boldsymbol{A} \in \mathcal{F}^{\circ}\}$

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Example 3: $\mathbf{IL}^m$

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upper limit logic: $\mathbf{UCL} = \mathbf{CL} + \exists\alpha A(\alpha) \supset \forall\alpha A(\alpha)$

semantically: the $\mathbf{CL}$-models that verify no abnormality (are uniform)

Example 4: $\mathbf{AT}^{1\,m}$ (extension with plausible statements)

- *lower limit logic*: $\mathbf{T}$ (a certain predicative version)

- *set of abnormalities*: $\boldsymbol{\Omega} = \{\Diamond \boldsymbol{A} \wedge \sim \boldsymbol{A} \mid \boldsymbol{A} \in \boldsymbol{\mathcal{W}}^{\boldsymbol{p}}\}$

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semantically: $\mathbf{T}$-models that verify no abnormality (nothing contingent)
(includes the one world models)

the SF provides $\mathbf{AL}$ with:

- a dynamic proof theory
- a semantics
- most of the metatheory

3.3 Annotated Dynamic Proofs: Reliability

rules of inference and marking definition

a *line* consists of

- a line number
- a formula
- a justification (line numbers + rule)
- a condition (finite subset of Ω)

3.3 Annotated Dynamic Proofs: Reliability

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- a condition (finite subset of Ω)

for all adaptive logics of the described kind:

A is derivable on the condition Δ (in the dynamic proof)

iff

$A \vee Dab(\Delta)$ is derivable (on the condition $\emptyset$) (in the dynamic proof)

iff

$\Gamma \vdash_{\mathbf{LLL}} A \vee Dab(\Delta)$

Rules of inference (depend on $\mathbf{LLL}$ and $\boldsymbol{\Omega}$, *not* on the strategy)

PREM If $A \in \Gamma$:

$$\frac{\begin{array}{ll} \ldots & \ldots \end{array}}{\begin{array}{ll} A & \emptyset \end{array}}$$

RU If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B$:

$$\frac{\begin{array}{ll} A_1 & \Delta_1 \\ \ldots & \ldots \\ A_n & \Delta_n \end{array}}{\begin{array}{ll} B & \Delta_1 \cup \ldots \cup \Delta_n \end{array}}$$

RC If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B \vee Dab(\Theta)$

$$\frac{\begin{array}{ll} A_1 & \Delta_1 \\ \ldots & \ldots \\ A_n & \Delta_n \end{array}}{\begin{array}{ll} B & \Delta_1 \cup \ldots \cup \Delta_n \cup \Theta \end{array}}$$

Marking Definition for Reliability

where $\boldsymbol{Dab}(\boldsymbol{\Delta_1}), \ldots, \boldsymbol{Dab}(\boldsymbol{\Delta_n})$ are the minimal $\boldsymbol{Dab}$-formulas derived on the condition $\emptyset$ at stage s, $\quad \boldsymbol{U_s(\Gamma) = \Delta_1 \cup \ldots \cup \Delta_n}$

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Definition
where $\boldsymbol{\Delta}$ is the condition of line $\boldsymbol{i}$, line $\boldsymbol{i}$ is marked (at stage $\boldsymbol{s}$) iff
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Marking Definition for Minimal Abnormality: later

Derivability at a stage vs. final derivability

idea: A derived on line i and the proof is stable with respect to i

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stability concerns a specific consequence and a specific line !

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Definition

A is *finally derived* from Γ on line i of a proof at stage s iff

(i) A is the second element of line i,

(ii) line i is unmarked at stage s, and

(iii) any extension of the proof may be further extended in such a way that line i is unmarked.

Derivability at a stage vs. final derivability

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Definition

$\Gamma \vdash_{\mathbf{AL}} A$ (A is *finally* $\mathbf{AL}$-*derivable* from Γ) iff A is finally derived on a line of a proof from Γ.

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Even at the predicative level, there are criteria for final derivability.

LLL invalidates certain rules of **ULL**

AL invalidates certain applications of rules of **ULL**

LLL invalidates certain rules of **ULL**

AL invalidates certain applications of rules of **ULL**

ULL extends **LLL** by validating some further rules

AL extends **LLL** by validating some applications of some further rules

example

adaptive logic: $\mathbf{IL}$

- *lower limit logic*: $\mathbf{CL}$
- *set of abnormalities*: $\Omega = \{\exists A \wedge \exists{\sim}A \mid A \in \mathcal{F}^{\circ}\}$
- *strategy*: Reliability

$$\Gamma = \{(Pa \wedge {\sim}Qa) \wedge {\sim}Ra, {\sim}Pb \wedge (Qb \wedge Rb), Pc \wedge Rc, Qd \wedge {\sim}Pe\}$$

1	$(Pa \wedge {\sim}Qa) \wedge {\sim}Ra$	PREM	$\emptyset$
2	${\sim}Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge {\sim}Pe$	PREM	$\emptyset$

number of data of each form immaterial
$\Rightarrow$ same generalizations derivable from $\{Pa\}$ and from $\{Pa, Pb\}$

in conditions and "Dab"-expressions, $A(x)$ abbreviates
$\exists x A(x) \wedge \exists{\sim}x A(x)$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
5	$\forall x(Qx \supset Rx)$	2; RC	$\{Qx \supset Rx\}$

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3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
5	$\forall x(Qx \supset Rx)$	2; RC	$\{Qx \supset Rx\}$
6	Rd	4, 5; RU	$\{Qx \supset Rx\}$

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3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
5	$\forall x(Qx \supset Rx)$	2; RC	$\{Qx \supset Rx\}$
6	Rd	4, 5; RU	$\{Qx \supset Rx\}$
7	$\forall x(\sim Px \supset Qx)$	2; RC	$\{\sim Px \supset Qx\}$
8	Qe	4, 7; RU	$\{\sim Px \supset Qx\}$

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3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge {\sim}Pe$	PREM	$\emptyset$

$\ldots$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
9	$\forall x(Px \supset \sim Rx)$	1; RC	$\{Px \supset \sim Rx\}$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
9^{L10}	$\forall x(Px \supset \sim Rx)$	1; RC	$\{Px \supset \sim Rx\}$
10	$Dab(Px \supset \sim Rx)$	1, 3; RU	$\emptyset$

1	$(Pa \wedge {\sim}Qa) \wedge {\sim}Ra$	PREM	$\emptyset$
2	${\sim}Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
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...

1	$(Pa \wedge {\sim}Qa) \wedge {\sim}Ra$	PREM	$\emptyset$
2	${\sim}Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge {\sim}Pe$	PREM	$\emptyset$
...			
11	$\forall x(Px \supset {\sim}Qx)$	1; RC	$\{Px \supset {\sim}Qx\}$
12	${\sim}Qc$	3, 11; RU	$\{Px \supset {\sim}Qx\}$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
11	$\forall x(Px \supset \sim Qx)$	1; RC	$\{Px \supset \sim Qx\}$
12	$\sim Qc$	3, 11; RU	$\{Px \supset \sim Qx\}$
13	$\forall x(Rx \supset Qx)$	2; RC	$\{Rx \supset Qx\}$
14	Qc	3, 13; RU	$\{Rx \supset Qx\}$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
11	$\forall x(Px \supset \sim Qx)$	1; RC	$\{Px \supset \sim Qx\}$
12	$\sim Qc$	3, 11; RU	$\{Px \supset \sim Qx\}$
13	$\forall x(Rx \supset Qx)$	2; RC	$\{Rx \supset Qx\}$
14	Qc	3, 13; RU	$\{Rx \supset Qx\}$
15	$\exists x \sim (Px \supset \sim Qx) \vee \exists x \sim (Rx \supset Qx)$	3; RU	$\emptyset$
16	$\exists x(Px \supset \sim Qx) \wedge \exists x(Rx \supset Qx)$	1, 2; RU	$\emptyset$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
11^{L17}	$\forall x(Px \supset \sim Qx)$	1; RC	$\{Px \supset \sim Qx\}$
12^{L17}	$\sim Qc$	3, 11; RU	$\{Px \supset \sim Qx\}$
13^{L17}	$\forall x(Rx \supset Qx)$	2; RC	$\{Rx \supset Qx\}$
14^{L17}	Qc	3, 13; RU	$\{Rx \supset Qx\}$
15	$\exists x \sim(Px \supset \sim Qx) \vee \exists x \sim(Rx \supset Qx)$	3; RU	$\emptyset$
16	$\exists x(Px \supset \sim Qx) \wedge \exists x(Rx \supset Qx)$	1, 2; RU	$\emptyset$
17	$Dab\{Px \supset \sim Qx, Rx \supset Qx\}$	15, 16; RU	$\emptyset$

1	$(Pa \wedge {\sim}Qa) \wedge {\sim}Ra$	PREM	$\emptyset$
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$\dots$

1	$(Pa \wedge {\sim}Qa) \wedge {\sim}Ra$	PREM	$\emptyset$
2	${\sim}Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge {\sim}Pe$	PREM	$\emptyset$
...			
18	$\forall x(Px \supset Sx)$	4; RC	$\{Px \supset Sx\}$
19	Sa	1, 18; RU	$\{Px \supset Sx\}$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
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4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
18	$\forall x(Px \supset Sx)$	4; RC	$\{Px \supset Sx\}$
19	Sa	1, 18; RU	$\{Px \supset Sx\}$
20	$\exists x{\sim}(Px \supset Sx) \vee \exists x{\sim}(Px \supset \sim Sx)$	3; RU	$\emptyset$
21	$\exists x(Px \supset Sx) \wedge \exists x(Px \supset \sim Sx)$	4; RU	$\emptyset$

1	$(Pa \wedge {\sim}Qa) \wedge {\sim}Ra$	PREM	$\emptyset$
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3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge {\sim}Pe$	PREM	$\emptyset$
...			
18^{L22}	$\forall x(Px \supset Sx)$	4; RC	$\{Px \supset Sx\}$
19^{L22}	Sa	1, 18; RU	$\{Px \supset Sx\}$
20	$\exists x{\sim}(Px \supset Sx) \vee \exists x{\sim}(Px \supset {\sim}Sx)$	3; RU	$\emptyset$
21	$\exists x(Px \supset Sx) \wedge \exists x(Px \supset {\sim}Sx)$	4; RU	$\emptyset$
22	$Dab\{Px \supset Sx, Px \supset {\sim}Sx\}$	20, 21; RU	$\emptyset$

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Definition (repetition)
A is *finally derived* from Γ on line i of a proof at stage s iff
(i) A is the second element of line i,
(ii) line i is not marked at stage s, and
(iii) any extension of the proof may be further extended in such a way that line i is unmarked.

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pace Leon Horsten (transfinite proofs) example:

proponent provides proof P in which A is derived at an unmarked line i

Game theoretic approaches to final derivability

example:

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can be extended (by the proponent) into a P$''$ in which i is unmarked example:

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any extension (by the opponent) of P into a P$'$ in which i is marked

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the proponent has an ‘answer’ to any ‘attack’

3.4 Semantics

$\boldsymbol{Dab}(\boldsymbol{\Delta})$ is a minimal $\boldsymbol{Dab}$-consequence of $\boldsymbol{\Gamma}$:
$\boldsymbol{\Gamma} \vDash_{\mathbf{LLL}} \boldsymbol{Dab}(\boldsymbol{\Delta})$ and, for all $\boldsymbol{\Delta'} \subset \boldsymbol{\Delta}$, $\boldsymbol{\Gamma} \nvDash_{\mathbf{LLL}} \boldsymbol{Dab}(\boldsymbol{\Delta'})$

where $\boldsymbol{M}$ is a $\mathbf{LLL}$-model: $\boldsymbol{Ab}(\boldsymbol{M}) = \{\boldsymbol{A} \in \boldsymbol{\Omega} \mid \boldsymbol{M} \vDash \boldsymbol{A}\}$

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a $\mathbf{LLL}$-model M of Γ is minimally abnormal

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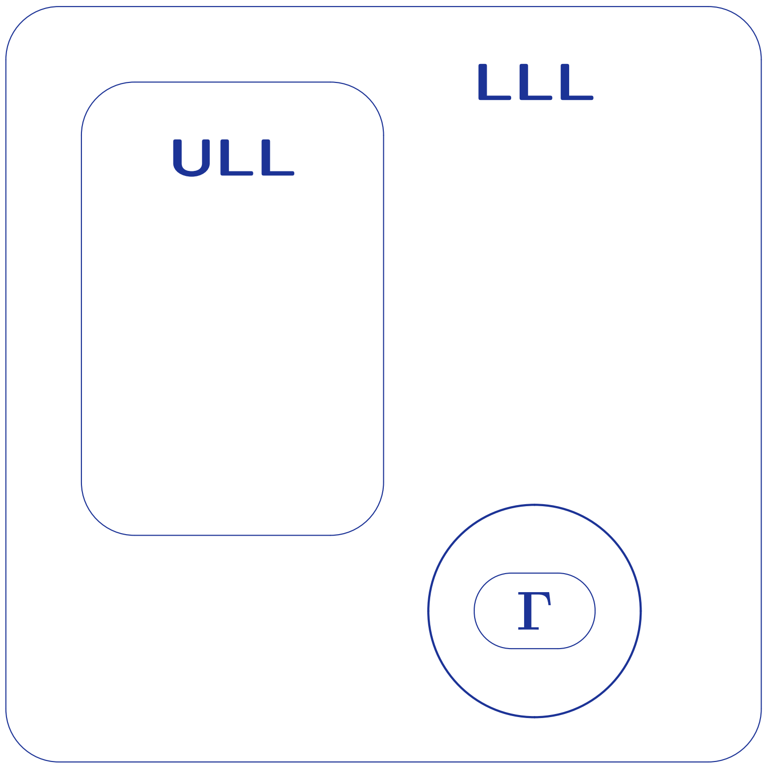
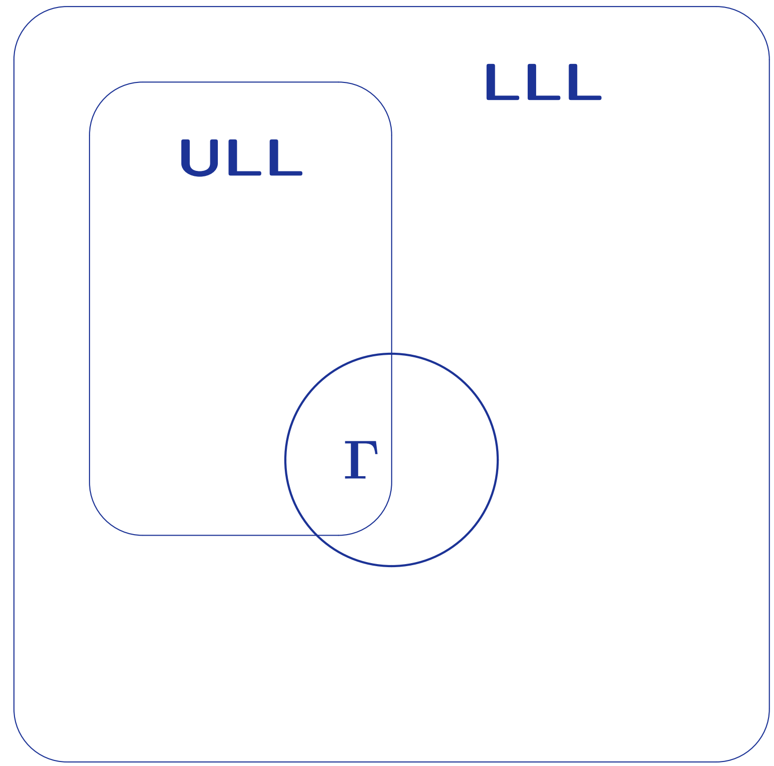


Abnormal Γ



Normal Γ

there are no $\mathbf{AL}$-models, but only $\mathbf{AL}$-models of some $\mathbf{\Gamma}$

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the **AL**-semantics selects some **LLL**-models of Γ as **AL**-models of Γ

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rules (as for Reliability) and marking definition

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Definition
where A is the formula and Δ is the condition of line i,
line i is marked at stage s iff,

(i) there is no $\varphi \in \Phi_s(\Gamma)$ such that $\varphi \cap \Delta = \emptyset$,
or
(ii) for some $\varphi \in \Phi_s(\Gamma)$, there is no line at which A is derived on a condition Θ for which $\varphi \cap \Theta = \emptyset$

example: $\mathbf{\Gamma} = \{\sim\boldsymbol{p},\ \sim\boldsymbol{q},\ \boldsymbol{p} \vee \boldsymbol{q},\ \boldsymbol{p} \vee \boldsymbol{r},\ \boldsymbol{q} \vee \boldsymbol{s}\}$

$\mathbf{\Gamma} \vdash_{\mathbf{ACLuN^m}} \boldsymbol{r} \vee \boldsymbol{s}$

$\mathbf{\Gamma} \nvdash_{\mathbf{ACLuN^r}} \boldsymbol{r} \vee \boldsymbol{s}$

example: $\Gamma = \{\sim p,\ \sim q,\ p \vee q,\ p \vee r,\ q \vee s\}$

$\Gamma \vdash_{\mathbf{ACLuN^m}} r \vee s$

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⋮	⋮	⋮	
•	$r \vee s$		$\{p \wedge \sim p\}$
•	$r \vee s$		$\{q \wedge \sim q\}$
•	$(p \wedge \sim p) \vee (q \wedge \sim q)$		$\emptyset$

3.6 Some Properties

Soundness: if $\mathbf{\Gamma} \vdash_{\mathbf{AL}} \boldsymbol{A}$ then $\mathbf{\Gamma} \models_{\mathbf{AL}} \boldsymbol{A}$

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Derivability Adjustment Theorem:

$\boldsymbol{A} \in \boldsymbol{Cn}_{\mathbf{ULL}}(\boldsymbol{\Gamma})$ iff $\boldsymbol{A} \vee \boldsymbol{Dab}(\boldsymbol{\Delta}) \in \boldsymbol{Cn}_{\mathbf{LLL}}(\boldsymbol{\Gamma})$ for some $\boldsymbol{\Delta} \subset \boldsymbol{\Omega}$.

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4 Combining Adaptive Logics

4.1 By Union

required:

combined adaptive logics share lower limit and strategy

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example: any adaptive logic + plausibility extension

handling inconsistency + plausibility extension

inductive generalization + plausibility extension

. . .

4.2 By Intersection and Union

required:

- common strategy
- intersection of lower limits is a (compact and monotonic) logic

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LLL: intersection of the lower limit logics

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example: gluts/gaps with respect to several logical symbols

note: combination of all gluts and gaps with ambiguity (zero logic)

4.3 Sequential Combination

required:

apparently only that the combination is meaningful
(e.g. that it does not lead to triviality)

example: sequential combination of the (infinitely many) $\mathbf{AT}^{i\,r}$

- *lower limit logic*: $\mathbf{T}$

- *set of abnormalities*: $\Omega^i = \{\Diamond^i A \wedge \sim A \mid A \in \mathcal{W}\}$
  (abnormality is falsehood of an expectancy)

- *strategy*: Reliability

upper limit logic: $\mathbf{Triv} = \mathbf{T} + \Diamond A \supset A$

$\Diamond^0 A$: A
$\Diamond^1 A$: $\Diamond A$
$\Diamond^2 A$: $\Diamond\Diamond A$
...

we want $$Cn_{\mathbf{Pref}}(\Gamma) = \ldots\ Cn_{\mathbf{AT}^3}(Cn_{\mathbf{AT}^2}(Cn_{\mathbf{AT}^1}(\Gamma))) \qquad (1)$$

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Notwithstanding (1), some criteria warrant final derivability after finitely many steps.

other examples

handling (different kinds) of background knowledge

$+$

inductive generalization

other examples

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diagnosis $+$ inductive generalization

other examples

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5 Decidability and Decisions

5.1 The Challenge

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note: some decidable inference relations can be characterized by adaptive logics (example: $\mathbf{R}_{\rightarrow}$)

given that there is no positive test for the inference relation

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- the block semantics
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- procedural criterion

(2) What if no criterion applies?

Can one sensibly decide on the basis of derivability at a stage?

5.2 Tableaux

idea: construct tableau for $\boldsymbol{A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B}$ as follows

· start by writing $\boldsymbol{\cdot TA_1,\ \ldots, \cdot TA_n,\ FB}$

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- start by writing $\cdot TA_1,\ \ldots, \cdot TA_n,\ FB$

- apply rules: descendants of labelled formulas are labelled

rules for negation $\dfrac{F{\sim}A}{TA}$ $\dfrac{T{\sim}A}{TA \mid FA}$

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- each branch: set of abnormalities, set of labelled abnormalities

  abnormality: $[\cdot]TA$ and $[\cdot]T{\sim}A$ (no, one or two labels)

  labelled abnormality: $\cdot TA$ and $\cdot T{\sim}A$

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- mark the unsuitable branches (in function of the strategy)

  Minimal abnormality:
  mark branch iff its set of abnormalities is a proper subset of the set of labelled abnormalities of another branch

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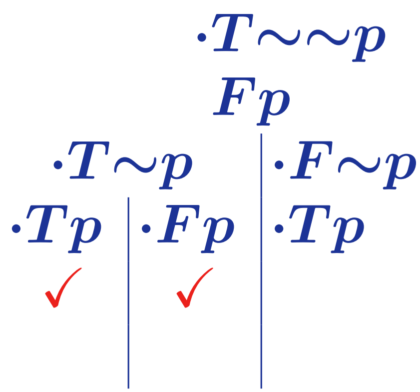
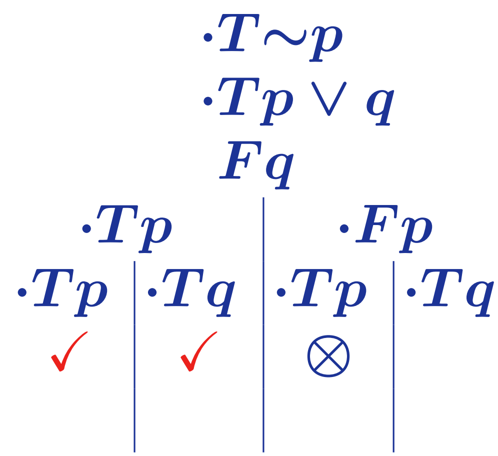
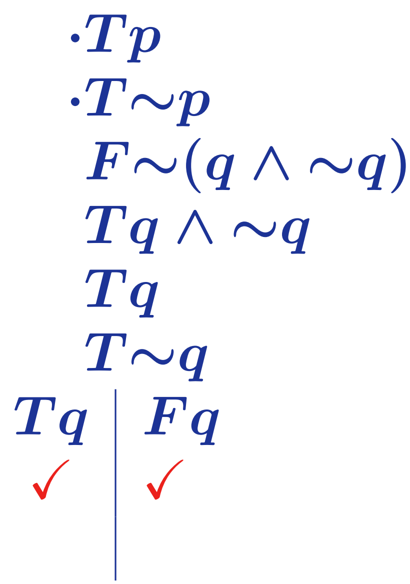
- each branch: set of abnormalities, set of labelled abnormalities
- mark the unsuitable branches (in function of the strategy)
- in the predicative case: apply finishing procedure
- tableau closes iff all branches are marked or closed

  branch closed: $[\cdot]TA$ and $[\cdot]FA$

some elementary illustrations:







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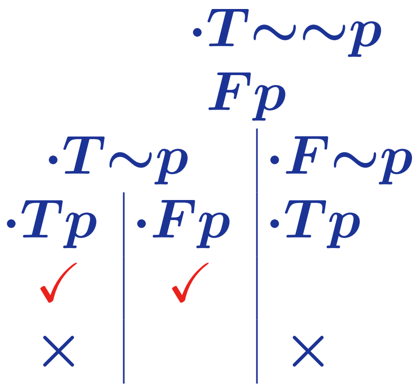

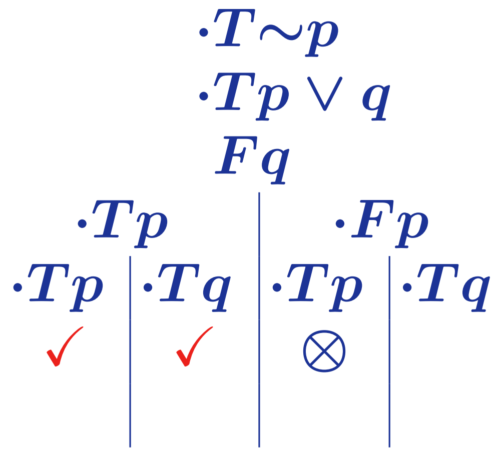

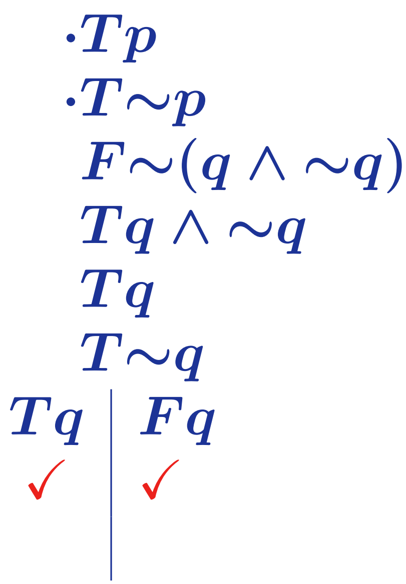

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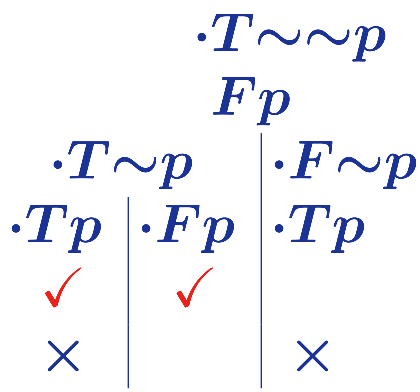

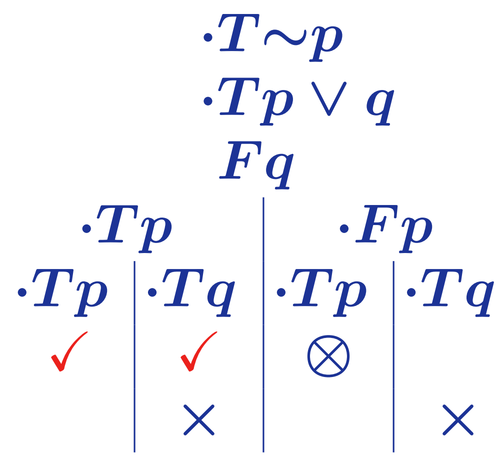

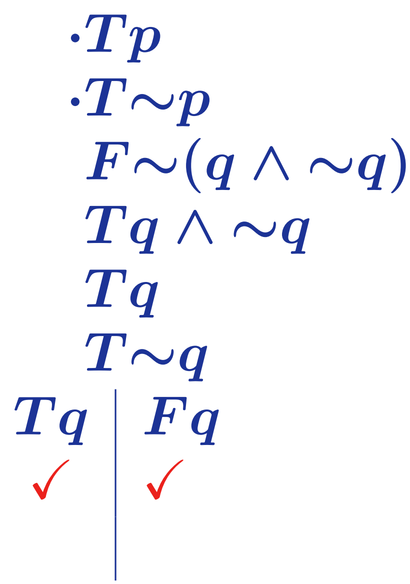

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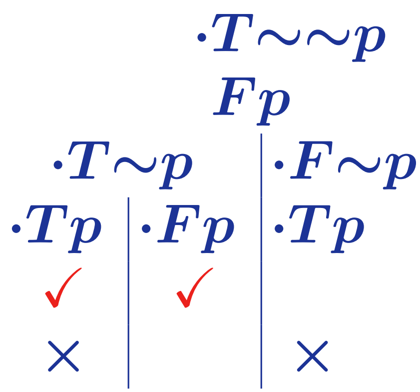

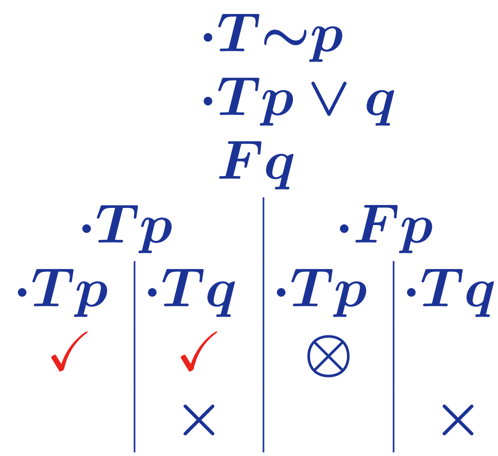

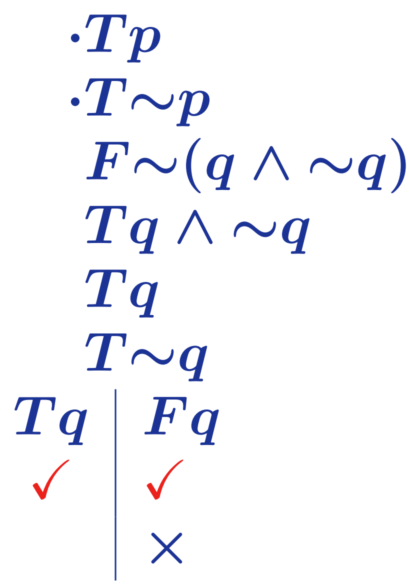

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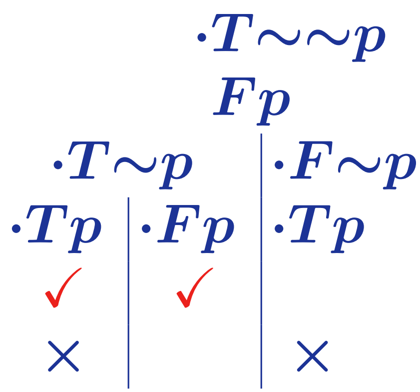

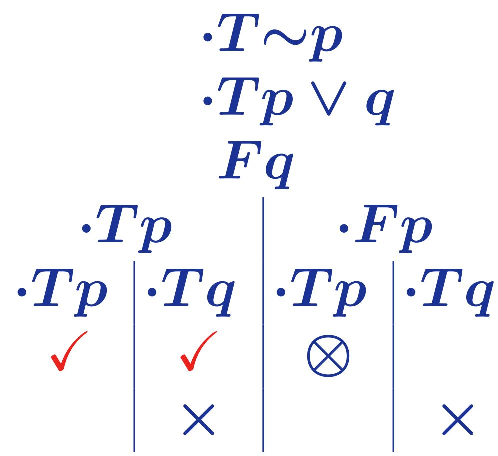

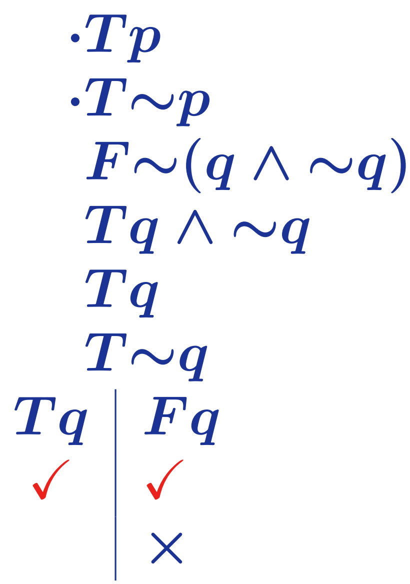

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prospective proofs

- contain most of the proof heuristics
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propositional examples:

$\sim q, p \vee q, \sim p \nvdash p$ pdp2 80

$p \vee q, \sim q, p \vee r, \sim r, p \vee s, \sim s, q \vee r \vdash p$ pdp2 81

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decision procedure at propositional level

criteria at predicative level

5.4 What If No Criterion Applies

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- economical considerations
  (cost of proceeding, possible cost of wrong decision, . . .)

6 Further examples and applications

6.1 Corrective

- $\mathbf{ACLuN^r}$ and $\mathbf{ACLuN^m}$ (negation gluts)
- other paraconsistent logics as **LLL**, including **ANA**
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- gluts/gaps for all logical symbols
- ambiguity adaptive logics
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- corrective deontic logics
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- compatibility (characterization)
- compatibility with inconsistent premises
- diagnosis
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- inductive generalization
- abduction
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- flat Rescher–Manor consequence relations (+ extensions)
- partial structures and pragmatic truth
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- circumscription, defaults, negation as failure, . . .
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