Adaptive Logics

Handling Dynamic Reasoning

Diderik Batens   Joke Meheus

Centre for Logic and Philosophy of Science
Ghent University, Belgium

{diderik.batens,joke.meheus}@ugent.be

http://logica.ugent.be/dirk/

http://logica.ugent.be/joke/

http://logica.ugent.be/centrum/writings/

http://logica.ugent.be/adlog/

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1 Dynamic reasoning patterns

1.1 The Problem

▼

many reasoning processes in the sciences (and elsewhere) display

an external dynamics

non-monotonic

an internal dynamics

revise conclusions as insights in premises grow

$\Uparrow$ absence of positive test (at predicative level)

Problem: gain technically sound control on the reasoning processes

1.2 Example 1: Process of explanation

given: $\left\{ \begin{array}{l} \text{explanandum } \boldsymbol{E} \\ \\ \text{theory } \boldsymbol{T} \end{array} \right.$

find: initial condition $\boldsymbol{I}$

Two different steps

- find potential initial conditions
- establish one of them (other theories)

Six conditions (Hintikka–Halonen)

$\boldsymbol{T}$ and $\boldsymbol{I}$ form an explanation of $\boldsymbol{E}$
iff $\boldsymbol{T}, \boldsymbol{I} \vdash \boldsymbol{E}$ and

(i) $\boldsymbol{T}$ and $\boldsymbol{E}$: no common ind. cons.		
(ii) $\boldsymbol{I}$ and $\boldsymbol{E}$: no common predicates		
(iii) $\nvdash_{\mathrm{CL}} \sim\boldsymbol{I}$	$\boldsymbol{I}$ not inconsistent	
(iv) $\boldsymbol{T} \nvdash_{\mathrm{CL}} \boldsymbol{E}$	$\boldsymbol{E}$ not implied by $\boldsymbol{T}$ alone	*
(v) $\boldsymbol{I} \nvdash_{\mathrm{CL}} \boldsymbol{E}$	$\boldsymbol{E}$ not implied by $\boldsymbol{I}$ alone	
(vi) $\boldsymbol{T} \nvdash_{\mathrm{CL}} \sim\boldsymbol{I}$	$\boldsymbol{T}$ not falsified by $\boldsymbol{I}$	*

Comments

no *positive test* for (iv) and (vi)
irrelevant predicates: $\boldsymbol{I}[\boldsymbol{a}] \wedge \boldsymbol{I}'[\boldsymbol{a}]$

1.3 Example 2: (Classical) Compatibility

given: a (consistent) set Γ

find: those A that (taken separately) do not make Γ inconsistent

plays a central role in:

partial structures approach of da Costa and associates
belief revision
ampliative reasoning
extending a theory
...

A is compatible with Γ iff $\Gamma \nvdash_{\mathbf{CL}} \sim A$ (no positive test)

note: paraconsistent compatibility (?!)

1.4 Inductive generalization

given: $\begin{cases} \text{a set of data } \Gamma \text{ and} \\ \\ \text{zero or more background theories} \end{cases}$

find: the suitable generalizations
(generalization: $\forall A$ with A purely functional)

natural restriction:

the generalizations should be jointly compatible with Γ

$$\Downarrow$$

only those generalizations $\forall A_i$ derivable for which
no 'minimal' disjunction $\sim\forall A_1 \vee \ldots \vee \sim\forall A_i \vee \ldots \vee \sim\forall A_n$ $(n \geq 1)$
is $\mathbf{CL}$-derivable from Γ

1.5 Erotetic inferences

given: $\begin{cases} \text{a set of declarative sentences } \Gamma \text{ and/or} \\ \text{an initial question } Q \end{cases}$

find: the questions that 'arise' from Γ and/or Q

question evocation (Andrzej Wiśniewski):

a question Q is evoked by a set of declarative sentences Γ iff

(i) $\Gamma \vdash \bigvee(dQ)$ (Q is sound with respect to Γ)
(ii) $\Gamma \nvdash A$, for any $A \in dQ$ (Q is informative with respect to Γ)

erotetic impliation (Andrzej Wiśniewski)

1.6 Some further examples

- interpret an inconsistent theory as consistently as possible
- inductive prediction
- interpreting a person's position during an ongoing discussion
- all reasoning that involves defaults (or more or less preferred premises)
  - diagnostic reasoning
  - handling preferred sets of premises

...

1.7 Adaptive logics and dynamic proof theories

no positive test for $\Gamma \vdash A$

$\vdash \quad \swarrow \qquad \searrow \quad$ reasoning

adaptive logic $\qquad$ internal dynamics

$\uparrow$ explicates

dynamic proof theory
of the adaptive logic

What is an adaptive logic?

What is a dynamic proof theory?

2 Inconsistency-Adaptive Logics

2.1 An Application Type

▼

the original problem:

T, intended as consistent, turns out to be inconsistent.

reason from T in order to *find consistent replacement*.

interpret T 'as consistently as possible'
= **adapt** to the specific inconsistencies of T

examples: Frege's set theory, thermodynamics around 1840, . . .

cannot be interpreted in terms of $\mathbf{CL}$: triviality

interpret in terms of paraconsistent logic?

2.2 Going Paraconsistent

the basic paraconsistent logic $\mathbf{CLuN}$

1 retain full positive logic

if $A_1, \ldots, A_n \vdash_{\mathbf{CL}} B$ and no negation occurs in $A_1, \ldots, A_n$ or B

then $A_1, \ldots, A_n \vdash_{\mathbf{CLuN}} B$

2 retain *Excluded Middle*: $A \vee \sim A$ (or $(A \supset \sim A) \supset \sim A$)

notes:

Replacement of Equivalents not generally valid (not valid in scope of $\sim$)

Replacement of Identicals not generally valid (not valid in scope of $\sim$)

What is lost?

DS: $\dfrac{A \vee B \quad \sim A}{B}$

classical semantic reasoning for DS:

$A \vee B$ is true, so A is true or B is true
$\sim A$ is true, so A is false
B is true

paraconsistent semantic reasoning for DS:

$A \vee B$ is true, so A is true or B is true
$\sim A$ is true (but A may be true together with $\sim A$)
B may be true as well as false

paraconsistent

$\sim A$	$A \vee B$	A	B	
1	1	0	1	
1	1	1	1	possible
1	1	1	0	possible

note:

DS and many other rules (MT, RAA, . . .)

are invalid in **CLuN**

adding them to **CLuN** results in **CL**

other rules

are invalid in **CLuN**

adding them to **CLuN** results in a (richer) paraconsistent logic

examples: $\sim\sim A \ / \ A$, de Morgan, . . .

interpreting a premise set paraconsistently delivers

a sensible (= non-trivial) interpretation

not an interpretation that is as consistent as possible

simplistic example: $\Gamma = \{p,\ q,\ {\sim}p \vee r,\ {\sim}q \vee s,\ {\sim}q\}$

$\Gamma \nvdash_{\mathbf{CLuN}} s$ and $\Gamma \nvdash_{\mathbf{CLuN}} r$

one wants to consider a formula of the form $A \wedge {\sim}A$ as false,
unless and until proven otherwise
(= unless the premises do not permit so)

Γ requires that $q \wedge {\sim}q$ is true, but not that $p \wedge {\sim}p$ is true

if Γ is true and $p \wedge {\sim}p$ is false, r is true !

put differently:

- the theory was intended to be consistent, but turned out inconsistent
- one searches for a consistent replacement of 'the theory'

'the theory'
=
'the theory in its full richness,
except for the pernicious consequences of its inconsistency'

put differently:
the theory, interpreted as consistently as possible

= consider inconsistencies as false,
except where the theory prevents so

Can this be explicated formally, and how?

2.3 Going Adaptive: Dynamic Proofs

simplistic example: $\Gamma = \{p,\ q,\ \sim p \vee r,\ \sim q \vee s,\ \sim q\}$

1	p	Prem	$\emptyset$	
2	q	Prem	$\emptyset$	
3	$\sim p \vee r$	Prem	$\emptyset$	
4	$\sim q \vee s$	Prem	$\emptyset$	
5	$\sim q$	Prem	$\emptyset$	
6	r	1, 3; RC	$\{p \wedge \sim p\}$	
7	s	2, 4; RC	$\{q \wedge \sim q\}$	$\checkmark$
8	$q \wedge \sim q$	2, 5; RU	$\emptyset$	

nothing interesting happens when the proof is continued

no mark will be removed or added

Can marked lines become unmarked?

1	$(p \wedge q) \wedge t$	PREM	$\emptyset$
2	$\sim p \vee r$	PREM	$\emptyset$
3	$\sim q \vee s$	PREM	$\emptyset$
4	$\sim p \vee \sim q$	PREM	$\emptyset$
5	$t \supset \sim p$	PREM	$\emptyset$

Can marked lines become unmarked?

1	$(p \wedge q) \wedge t$	PREM	$\emptyset$
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3	$\sim q \vee s$	PREM	$\emptyset$
4	$\sim p \vee \sim q$	PREM	$\emptyset$
5	$t \supset \sim p$	PREM	$\emptyset$
6	r	1, 2; RC	$\{p \wedge \sim p\}$
7	s	1, 3; RC	$\{q \wedge \sim q\}$

Can marked lines become unmarked?

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3	$\sim q \vee s$	PREM	$\emptyset$	
4	$\sim p \vee \sim q$	PREM	$\emptyset$	
5	$t \supset \sim p$	PREM	$\emptyset$	
6	r	1, 2; RC	$\{p \wedge \sim p\}$	$\surd$
7	s	1, 3; RC	$\{q \wedge \sim q\}$	$\surd$
8	$(p \wedge \sim p) \vee (q \wedge \sim q)$	1, 4; RU	$\emptyset$	

Can marked lines become unmarked?

1	$(p \wedge q) \wedge t$	PREM	$\emptyset$	
2	$\sim p \vee r$	PREM	$\emptyset$	
3	$\sim q \vee s$	PREM	$\emptyset$	
4	$\sim p \vee \sim q$	PREM	$\emptyset$	
5	$t \supset \sim p$	PREM	$\emptyset$	
6	r	1, 2; RC	$\{p \wedge \sim p\}$	$\checkmark$
7	s	1, 3; RC	$\{q \wedge \sim q\}$	
8	$(p \wedge \sim p) \vee (q \wedge \sim q)$	1, 4; RU	$\emptyset$	
9	$p \wedge \sim p$	1, 5; RU	$\emptyset$	

nothing interesting happens when the proof is continued

no mark will be removed or added

Making marking precise

the dynamic proofs need to explicate the dynamic reasoning

at the level of the proofs, the dynamics needs to be controlled

- the conditions
- the marking definition

Which lines are marked?

Which lines are marked?

$\boldsymbol{Dab}$-formula: disjunction of inconsistencies, $\boldsymbol{Dab(\Delta)}$

minimal $\boldsymbol{Dab}$-formula at stage $\boldsymbol{s}$:

at stage $\boldsymbol{s}$:
$\boldsymbol{Dab(\Delta)}$ derived on the empty condition
for every $\boldsymbol{\Delta' \subset \Delta}$, $\boldsymbol{Dab(\Delta')}$ not derived on the empty condition

where $\boldsymbol{Dab(\Delta_1), \ldots, Dab(\Delta_n)}$ are the minimal $\boldsymbol{Dab}$-formulas at stage $\boldsymbol{s}$,

$$\boldsymbol{U_s(\Gamma) = \Delta_1 \cup \ldots \cup \Delta_n}$$

where $\boldsymbol{\Theta}$ is the condition of line $\boldsymbol{i}$, line $\boldsymbol{i}$ is marked iff $\boldsymbol{\Theta \cap U_s(\Gamma) \neq \emptyset}$

Final derivability

derivability seems to be unstable: it changes from stage to stage

next to derivability at a stage,

one wants a stable notion of derivability: final derivability: $\Gamma \vdash_{\mathbf{ACLuN}^r} A$

idea behind final derivability:

A is derived at an unmarked line i

and

the proof is stable with respect to i

$$\Updownarrow$$

line i will not be marked in any extension of the proof

2.4 Going Adaptive: Semantics

consider the **CLuN**-models of the premise set Γ

$Dab(\Delta)$ is a minimal Dab-consequence of Γ:

$\Gamma \vDash_{\mathbf{CLuN}} Dab(\Delta)$ and for all $\Delta' \subset \Delta$, $\Gamma \nvDash_{\mathbf{CLuN}} Dab(\Delta')$

where $Dab(\Delta_1), \ldots, Dab(\Delta_n)$ are the minimal Dab-consequences of Γ,

$U(\Gamma) = \Delta_1 \cup \ldots \cup \Delta_n$

$Ab(M) = \{\exists(A \wedge \sim A) \mid M \models \exists(A \wedge \sim A)\}$

a **CLuN**-model M of Γ is reliable iff $Ab(M) \subseteq U(\Gamma)$

$\Gamma \vDash_{\mathbf{ACLuN}^r} A$ iff all reliable models of Γ verify A

it is provable that $\Gamma \vdash_{\mathbf{ACLuN}^r} A$ iff $\Gamma \vDash_{\mathbf{ACLuN}^r} A$

2.5 Strategies

naive approach:
Simple strategy: take $A \wedge \sim A$ to be false, unless $\Gamma \vdash_{\mathbf{CLuN}} A \wedge \sim A$

the Simple strategy is inadequate (in this case) because, for some Γ, $Dab(\Delta)$ is a minimal Dab-consequence of Γ and Δ is not a singleton.

before, we used the Reliability strategy

there are other strategies, each suitable for specific applications

3 The Standard Format

3.1 The Problem

many adaptive logics seem to have a common structure

others can be given this structure under a translation

the structure is central for the metatheoretic proofs

whence the plan:
- describe the structure: the SF (standard format)
- define the proof theory and semantics from the SF
- prove as many properties as possible by relying on the SF only

3.2 The Format

- *lower limit logic*
  monotonic and compact logic

- *set of abnormalities* $\mathbf{\Omega}$:
  characterized by a (possibly restricted) logical form

- *strategy*:
  Reliability, Minimal Abnormality, . . .

upper limit logic:
$\mathbf{ULL} = \mathbf{LLL} +$ axiom/rule that trivializes abnormalities

semantically: the $\mathbf{LLL}$-models that verify no abnormality

flip-flop

Example 1: $\mathbf{ACLuN}^r$

- *lower limit logic*: $\mathbf{CLuN}$
- *set of abnormalities*: $\Omega = \{\exists(A \wedge {\sim}A) \mid A \in \mathcal{F}\}$
- *strategy*: Reliability

upper limit logic: $\mathbf{CL} = \mathbf{CLuN} + (A \wedge {\sim}A) \supset B$

semantically: the $\mathbf{CLuN}$-models that verify no inconsistency

Example 2: $\mathbf{ACLuN}^m$

- *lower limit logic*: $\mathbf{CLuN}$
- *set of abnormalities*: $\Omega = \{\exists(A \wedge \sim A) \mid A \in \mathcal{F}\}$
- *strategy*: Minimal Abnormality

upper limit logic: $\mathbf{CL} = \mathbf{CLuN} + (A \wedge \sim A) \supset B$

semantically: the $\mathbf{CLuN}$-models that verify no inconsistency

Example 3: $\mathbf{IL}^m$

- *lower limit logic*: $\mathbf{CL}$
- *set of abnormalities*: $\Omega = \{\exists A \wedge \exists{\sim}A \mid A \in \mathcal{F}^\circ\}$
- *strategy*: Minimal Abnormality

upper limit logic: $\mathbf{UCL} = \mathbf{CL} + \exists\alpha A(\alpha) \supset \forall\alpha A(\alpha)$

semantically: the $\mathbf{CL}$-models that verify no abnormality (are uniform)

Example 4: $\mathbf{AT}^{1\,m}$ (extension with plausible statements)

- *lower limit logic*: $\mathbf{T}$ (a certain predicative version)

- *set of abnormalities*: $\Omega = \{\Diamond A \wedge \sim A \mid A \in \mathcal{W}^p\}$

- *strategy*: Minimal Abnormality

upper limit logic: $\mathbf{Triv} = \mathbf{T} + \Diamond A \supset A$

semantically: $\mathbf{T}$-models that verify no abnormality (nothing contingent)
(includes the one world models)

the SF provides **AL** with:

- a dynamic proof theory
- a semantics
- most of the metatheory

3.3 Annotated Dynamic Proofs: Reliability

rules of inference and marking definition

a *line* consists of

- a line number
- a formula
- a justification (line numbers + rule)
- a condition (finite subset of Ω)

for all adaptive logics of the described kind:

A is derivable on the condition Δ (in the dynamic proof)

iff

$A \vee Dab(\Delta)$ is derivable (on the condition $\emptyset$) (in the dynamic proof)

iff

$\Gamma \vdash_{\mathbf{LLL}} A \vee Dab(\Delta)$

Rules of inference (depend on $\mathbf{LLL}$ and $\mathbf{\Omega}$, *not* on the strategy)

PREM If $A \in \Gamma$:

$$\begin{array}{ll} \ldots & \ldots \\ \hline A & \emptyset \end{array}$$

RU If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B$:

$$\begin{array}{ll} A_1 & \Delta_1 \\ \ldots & \ldots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \end{array}$$

RC If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B \vee Dab(\Theta)$

$$\begin{array}{ll} A_1 & \Delta_1 \\ \ldots & \ldots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \cup \Theta \end{array}$$

Marking Definition for Reliability

where $Dab(\Delta_1)$, ..., $Dab(\Delta_n)$ are the minimal Dab-formulas derived on the condition $\emptyset$ at stage s, $U_s(\Gamma) = \Delta_1 \cup \ldots \cup \Delta_n$

Definition
where Δ is the condition of line i, line i is marked (at stage s) iff $\Delta \cap U_s(\Gamma) \neq \emptyset$

$\Rightarrow$ idea for consequence set applied to stage of proof

Marking Definition for Minimal Abnormality: later

Derivability at a stage vs. final derivability

idea: A derived on line i and the proof is stable with respect to i

stability concerns a specific consequence and a specific line !

Definition

A is *finally derived* from Γ on line i of a proof at stage s iff

(i) A is the second element of line i,

(ii) line i is unmarked at stage s, and

(iii) any extension of the proof may be further extended in such a way that line i is unmarked.

Definition

$\Gamma \vdash_{\mathbf{AL}} A$ (A is *finally* $\mathbf{AL}$-*derivable* from Γ) iff A is finally derived on a line of a proof from Γ.

Even at the predicative level, there are criteria for final derivability.

LLL invalidates certain rules of **ULL**

AL invalidates certain applications of rules of **ULL**

ULL extends **LLL** by validating some further rules

AL extends **LLL** by validating some applications of some further rules

example

adaptive logic: $\mathbf{IL}$

- *lower limit logic*: $\mathbf{CL}$
- *set of abnormalities*: $\Omega = \{\exists A \wedge \exists{\sim}A \mid A \in \mathcal{F}^{\circ}\}$
- *strategy*: Reliability

$$\Gamma = \{(Pa \wedge {\sim}Qa) \wedge {\sim}Ra, {\sim}Pb \wedge (Qb \wedge Rb), Pc \wedge Rc, Qd \wedge {\sim}Pe\}$$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
5	$\forall x(Qx \supset Rx)$	2; RC	$\{Qx \supset Rx\}$
6	Rd	4, 5; RU	$\{Qx \supset Rx\}$
7	$\forall x(\sim Px \supset Qx)$	2; RC	$\{\sim Px \supset Qx\}$
8	Qe	4, 7; RU	$\{\sim Px \supset Qx\}$

number of data of each form immaterial
$\Rightarrow$ same generalizations derivable from $\{Pa\}$ and from $\{Pa, Pb\}$

in conditions and "Dab"-expressions, $A(x)$ abbreviates
$\exists x A(x) \wedge \exists{\sim}x A(x)$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
9^{L10}	$\forall x(Px \supset \sim Rx)$	1; RC	$\{Px \supset \sim Rx\}$
10	$Dab(Px \supset \sim Rx)$	1, 3; RU	$\emptyset$

1	$(Pa \wedge \sim Qa) \wedge \sim Ra$	PREM	$\emptyset$
2	$\sim Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge \sim Pe$	PREM	$\emptyset$
...			
11^{L17}	$\forall x(Px \supset \sim Qx)$	1; RC	$\{Px \supset \sim Qx\}$
12^{L17}	$\sim Qc$	3, 11; RU	$\{Px \supset \sim Qx\}$
13^{L17}	$\forall x(Rx \supset Qx)$	2; RC	$\{Rx \supset Qx\}$
14^{L17}	Qc	3, 13; RU	$\{Rx \supset Qx\}$
15	$\exists x\sim(Px \supset \sim Qx) \vee \exists x\sim(Rx \supset Qx)$	3; RU	$\emptyset$
16	$\exists x(Px \supset \sim Qx) \wedge \exists x(Rx \supset Qx)$	1, 2; RU	$\emptyset$
17	$Dab\{Px \supset \sim Qx, Rx \supset Qx\}$	15, 16; RU	$\emptyset$

1	$(Pa \wedge {\sim}Qa) \wedge {\sim}Ra$	PREM	$\emptyset$
2	${\sim}Pb \wedge (Qb \wedge Rb)$	PREM	$\emptyset$
3	$Pc \wedge Rc$	PREM	$\emptyset$
4	$Qd \wedge {\sim}Pe$	PREM	$\emptyset$
...			
18^{L22}	$\forall x(Px \supset Sx)$	4; RC	$\{Px \supset Sx\}$
19^{L22}	Sa	1, 18; RU	$\{Px \supset Sx\}$
20	$\exists x{\sim}(Px \supset Sx) \vee \exists x{\sim}(Px \supset {\sim}Sx)$	3; RU	$\emptyset$
21	$\exists x(Px \supset Sx) \wedge \exists x(Px \supset {\sim}Sx)$	4; RU	$\emptyset$
22	$Dab\{Px \supset Sx, Px \supset {\sim}Sx\}$	20, 21; RU	$\emptyset$

Some theoretical stuff

a stage (of a proof) is a sequence of lines

a proof is a chain of (1 or more) stages

a subsequent stage is obtained by adding a line to the stage

the marking definition determines which lines of the stage are marked
(marks may come and go with the stage)

an extension of a proof P is a proof P′ that has P as its initial fragment

Definition (repetition)
A is *finally derived* from Γ on line i of a proof at stage s iff
(i) A is the second element of line i,
(ii) line i is not marked at stage s, and
(iii) any extension of the proof may be further extended in such a way that line i is unmarked.

for some logics (esp. Minimal Abnormality strategy), premise sets and conclusions, stability (final derivability) is reached only after infinitely many stages

if a stage has infinitely many lines, the next stage is reached by inserting a line (variant)

pace Leon Horsten (transfinite proofs) example:

proponent provides proof P in which A is derived at an unmarked line i

A is finally derived at i

iff

any extension (by the opponent) of P into a P′ in which i is marked

can be extended (by the proponent) into a P″ in which i is unmarked

the proponent has an ‘answer’ to any ‘attack’

3.4 Semantics

$Dab(\Delta)$ is a minimal Dab-consequence of Γ:
$\Gamma \vDash_{\mathbf{LLL}} Dab(\Delta)$ and, for all $\Delta' \subset \Delta$, $\Gamma \nvDash_{\mathbf{LLL}} Dab(\Delta')$

where M is a **LLL**-model: $Ab(M) = \{A \in \Omega \mid M \models A\}$

Reliability

where $Dab(\Delta_1)$, $Dab(\Delta_2)$, $\ldots$ are the minimal Dab-consequences of Γ,
$U(\Gamma) = \Delta_1 \cup \Delta_2 \cup \ldots$

a **LLL**-model M of Γ is reliable iff $Ab(M) \subseteq U(\Gamma)$

$\Gamma \vDash_{\mathbf{AL}} A$ iff all reliable models of Γ verify A

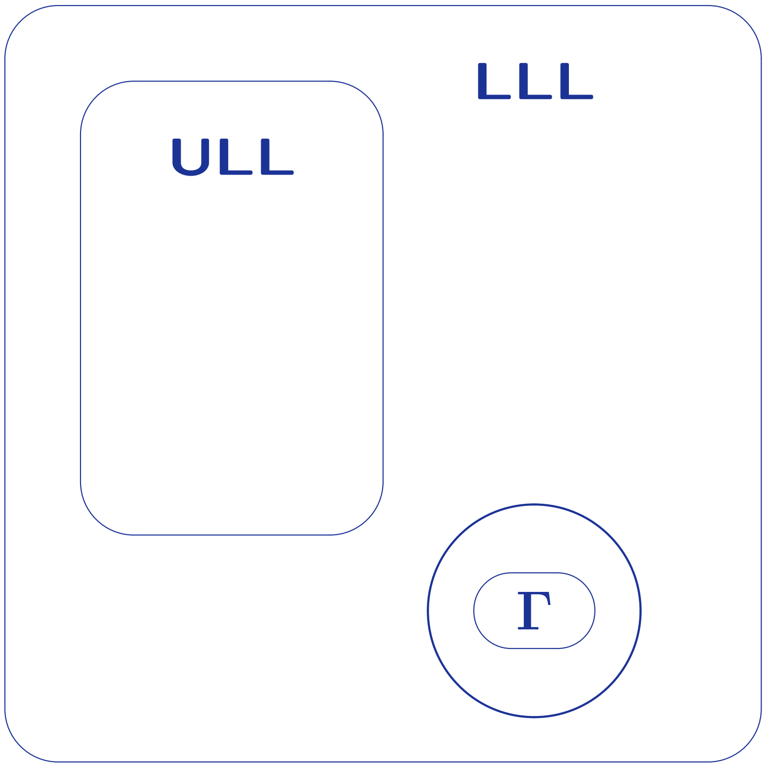
Minimal Abnormality

a $\mathbf{LLL}$-model M of Γ is minimally abnormal

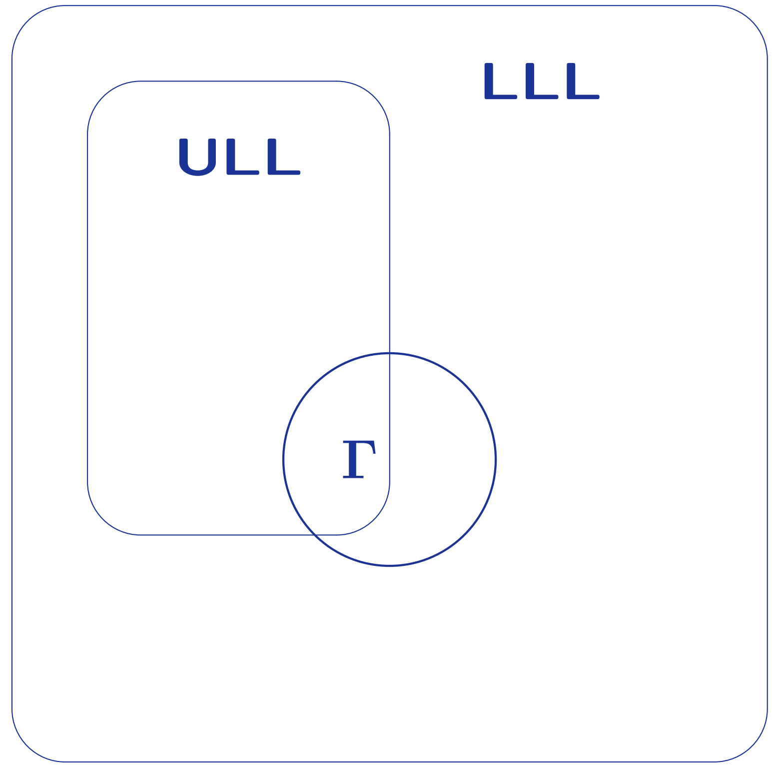
iff

there is no $\mathbf{LLL}$-model M' of Γ for which $Ab(M') \subset Ab(M)$

$\Gamma \models_{\mathbf{AL}} A$ iff all minimally abnormal models of Γ verify A



Abnormal Γ



Normal Γ

there are no **AL**-models, but only **AL**-models of some Γ

all **LLL**-models are **AL**-models of some Γ

the **AL**-semantics selects some **LLL**-models of Γ as **AL**-models of Γ

rules (as for Reliability) and marking definition

where $Dab(\Delta_1), \ldots, Dab(\Delta_n)$ are the minimal Dab-formulas derived on the condition $\emptyset$ at stage s

$\Phi^{\circ}_s(\Gamma)$: the set of all sets that contain one member of each Δ_i

$\Phi^{\star}_s(\Gamma)$: contains, for any $\varphi \in \Phi^{\circ}_s(\Gamma)$, $Cn_{\mathbf{LLL}}(\varphi) \cap \Omega$

$\Phi_s(\Gamma)$: $\varphi \in \Phi^{\star}_s(\Gamma)$ that are not proper supersets of a $\varphi' \in \Phi^{\star}_s(\Gamma)$

Definition
where A is the formula and Δ is the condition of line i,
line i is marked at stage s iff,

(i) there is no $\varphi \in \Phi_s(\Gamma)$ such that $\varphi \cap \Delta = \emptyset$,
or
(ii) for some $\varphi \in \Phi_s(\Gamma)$, there is no line at which A is derived on a condition Θ for which $\varphi \cap \Theta = \emptyset$

example: $\Gamma = \{\sim p,\ \sim q,\ p \vee q,\ p \vee r,\ q \vee s\}$

$\Gamma \vdash_{\mathbf{ACLuN^m}} r \vee s$

$\Gamma \nvdash_{\mathbf{ACLuN^r}} r \vee s$

$\vdots$	$\vdots$	$\vdots$	
$\cdot$	$r \vee s$		$\{p \wedge \sim p\}$
$\cdot$	$r \vee s$		$\{q \wedge \sim q\}$
$\cdot$	$(p \wedge \sim p) \vee (q \wedge \sim q)$		$\emptyset$

3.6 Some Properties

Soundness: if $\Gamma \vdash_{\mathbf{AL}} A$ then $\Gamma \vDash_{\mathbf{AL}} A$

Completeness: if $\Gamma \vDash_{\mathbf{AL}} A$ then $\Gamma \vdash_{\mathbf{AL}} A$

Derivability Adjustment Theorem:

$A \in Cn_{\mathbf{ULL}}(\Gamma)$ iff $A \vee Dab(\Delta) \in Cn_{\mathbf{LLL}}(\Gamma)$ for some $\Delta \subset \Omega$.

Reassurance: if $Cn_{\mathbf{LLL}}(\Gamma)$ is not trivial, then $Cn_{\mathbf{AL}}(\Gamma)$ is not trivial (if Γ has $\mathbf{LLL}$-models, then it has $\mathbf{AL}$-models)

Strong Reassurance: if a $\mathbf{LLL}$-model M of Γ is not a $\mathbf{AL}$-model of Γ, then $Ab(M') \subset Ab(M)$ for some $\mathbf{AL}$-model M' of Γ.

Proof Invariance: if $\Gamma \vdash_{\mathbf{AL}} A$, then every $\mathbf{AL}$-proof from Γ can be extended in such a way that A is finally derived in it.

$Cn_{\mathbf{LLL}}(\Gamma) \subseteq Cn_{\mathbf{AL}}(\Gamma) \subseteq Cn_{\mathbf{ULL}}(\Gamma)$ ($\subset$ and $=$ where justifiable)

. . .

4 Combining Adaptive Logics

4.1 By Union

4.2 By Intersection and Union

4.3 Sequential Combination

4.1 By Union

required:

combined adaptive logics share lower limit and strategy

$$\Omega = \Omega_1 \cup \Omega_2$$

example: inductive generalization + abduction

example: any adaptive logic + plausibility extension

handling inconsistency + plausibility extension

inductive generalization + plausibility extension

...

4.2 By Intersection and Union

required:

- common strategy
- intersection of lower limits is a (compact and monotonic) logic

LLL: intersection of the lower limit logics

$$\boldsymbol{\Omega = \Omega_1 \cup \Omega_2}$$

example: gluts/gaps with respect to several logical symbols

note: combination of all gluts and gaps with ambiguity (zero logic) example: sequential combination of the (infinitely many) $\mathbf{AT}^{i\,r}$

- *lower limit logic*: $\mathbf{T}$

- *set of abnormalities*: $\Omega^i = \{\Diamond^i A \wedge \sim A \mid A \in \mathcal{W}\}$
  (abnormality is falsehood of an expectancy)

- *strategy*: Reliability

upper limit logic: $\mathbf{Triv} = \mathbf{T} + \Diamond A \supset A$

$\Diamond^0 A$: A
$\Diamond^1 A$: $\Diamond A$
$\Diamond^2 A$: $\Diamond\Diamond A$
...

the combination

we want $$Cn_{\mathbf{Pref}}(\Gamma) = \ldots\ Cn_{\mathbf{AT}^3}(Cn_{\mathbf{AT}^2}(Cn_{\mathbf{AT}^1}(\Gamma))) \qquad (1)$$

seems superposition of supertasks

Proofs: (skipping a couple of details)

at every stage:

- apply rules of $\mathbf{AT^1}$, $\mathbf{AT^2}$, ... in any order
- Marking definition: mark first for $\mathbf{AT^1}$, next for $\mathbf{AT^2}$, ... up to the highest $\Diamond^i$ that occurs in the proof

finite stage may contain applications of every $\mathbf{AT}^i$

Notwithstanding (1), some criteria warrant final derivability after finitely many steps.

other examples

handling (different kinds) of background knowledge
+
inductive generalization

diagnosis + inductive generalization

handling inconsistency + abduction
(abduction from inconsistent knowledge)

handling inconsistency + compatibility

paraconsistent compatibility

pragmatic truth in terms of partial structures (da Costa *et al.*)

handling inconsistency + question evocation

. . .

5 Decidability and Decisions

5.1 The Challenge

the reasoning patterns explicated by adaptive logics

- are undecidable
- there is no positive test for them

same *should* obtain for the explications

note: not all are non-monotonic
(example: Rescher's Weak Consequence Relation)

note: some decidable inference relations can be characterized by adaptive logics (example: $\mathbf{R}_{\rightarrow}$)

given that there is no positive test for the inference relation

(1) one may still search for criteria for final derivability

- the block semantics
- tableau methods
- procedural criterion

(2) What if no criterion applies?

Can one sensibly decide on the basis of derivability at a stage?

5.2 Tableaux

idea: construct tableau for $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B$ as follows

- start by writing $\cdot TA_1,\ \ldots,\ \cdot TA_n,\ FB$

- apply rules: descendants of labelled formulas are labelled

rules for negation $$\frac{F{\sim}A}{TA} \qquad \frac{T{\sim}A}{TA \mid FA}$$

5.2 Tableaux

idea: construct tableau for $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B$ as follows

- start by writing $\cdot TA_1,\ \ldots,\ \cdot TA_n,\ FB$

- apply rules: descendants of labelled formulas are labelled

- each branch: set of abnormalities, set of labelled abnormalities

  abnormality: $[\cdot]TA$ and $[\cdot]T{\sim}A$ (no, one or two labels)

  labelled abnormality: $\cdot TA$ and $\cdot T{\sim}A$

5.2 Tableaux

idea: construct tableau for $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B$ as follows

- start by writing $\cdot TA_1,\ \ldots,\ \cdot TA_n,\ FB$
- apply rules: descendants of labelled formulas are labelled
- each branch: set of abnormalities, set of labelled abnormalities
- mark the unsuitable branches (in function of the strategy)

  Minimal abnormality:
  mark branch iff its set of abnormalities is a proper subset
  of the set of labelled abnormalities of another branch

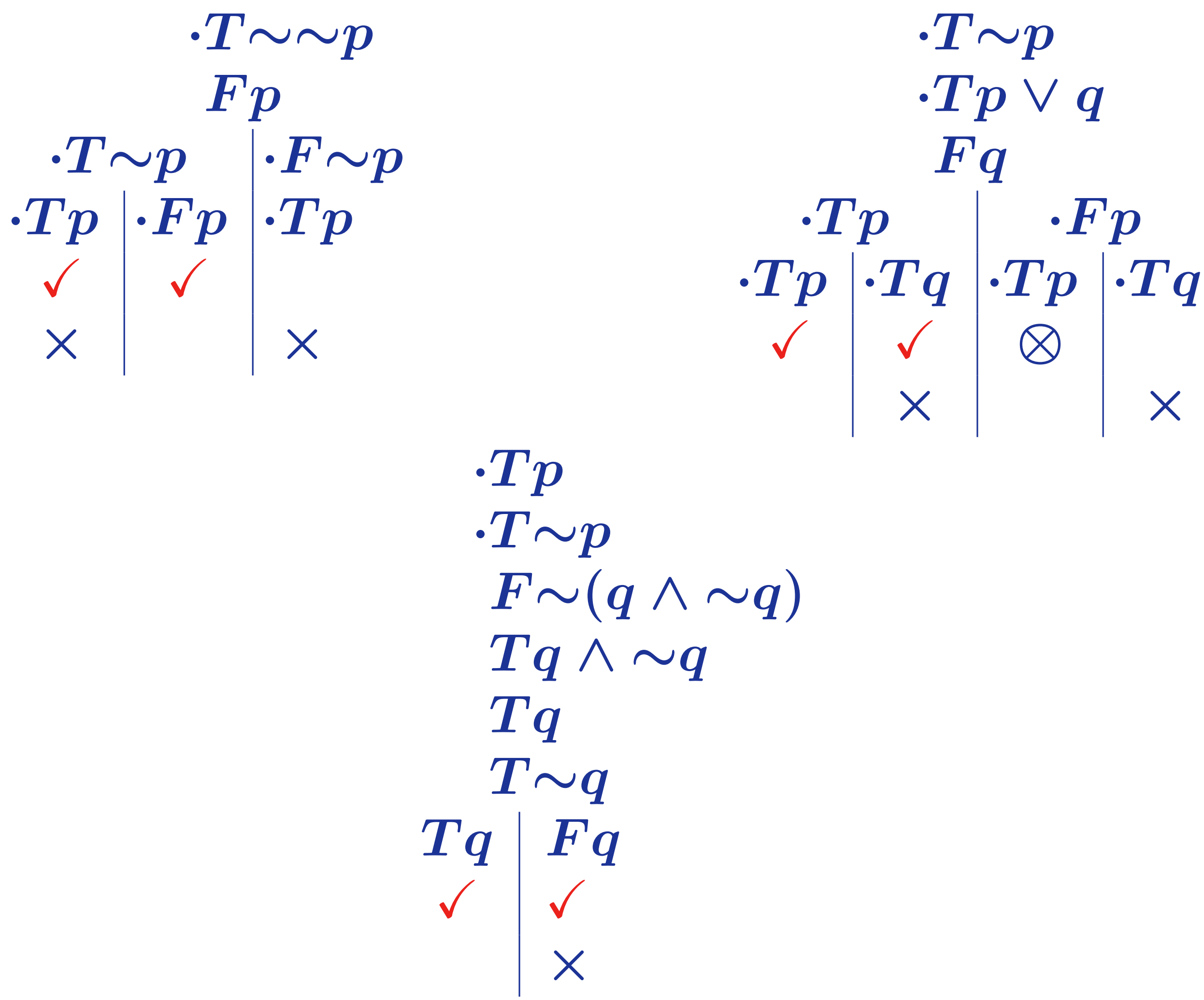
5.2 Tableaux

idea: construct tableau for $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B$ as follows

- start by writing $\cdot TA_1, \ \ldots, \cdot TA_n, \ FB$
- apply rules: descendants of labelled formulas are labelled
- each branch: set of abnormalities, set of labelled abnormalities
- mark the unsuitable branches (in function of the strategy)
- in the predicative case: apply finishing procedure
- tableau closes iff all branches are marked or closed

  branch closed: $[\cdot]TA$ and $[\cdot]FA$

some elementary illustrations:



5.3 Procedural Criterion

prospective proofs
- contain most of the proof heuristics
- enable one to define a procedure

applied to $\mathbf{ACLuN}^r$ and can be generalized

if the (three stage) procedure is applied to $A_1, \ldots, A_n \vdash B$ and stops, we can read off whether the expression is true or false

propositional examples:
$\sim q, p \vee q, \sim p \nvdash p$ pdp2 80
$p \vee q, \sim q, p \vee r, \sim r, p \vee s, \sim s, q \vee r \vdash p$ pdp2 81

decision procedure at propositional level
criteria at predicative level

5.4 What If No Criterion Applies

Given the presupposition that abnormalities are false until and unless proven otherwise, the derivability of A on a condition Δ of which no member is shown to be unreliable is a good reason to consider A as derivable.

The block analysis shows:

as the proof proceeds, one *may obtain more insights* in the premises (and *cannot lose insights* in the premises)

- derivability at a stage converges towards final derivability

- economical considerations
  (cost of proceeding, possible cost of wrong decision, . . .)

6 Further examples and applications

6.1 Corrective

6.2 Ampliative (+ ampliative and corrective)

6.3 Incorporation

6.4 Applications

6.1 Corrective

- $\mathbf{ACLuN^r}$ and $\mathbf{ACLuN^m}$ (negation gluts)
- other paraconsistent logics as **LLL**, including **ANA**
- negation gaps
- gluts/gaps for all logical symbols
- ambiguity adaptive logics
- adaptive zero logic
- corrective deontic logics
- prioritized ial
- ...

6.2 Ampliative (+ ampliative and corrective)

- compatibility (characterization)
- compatibility with inconsistent premises
- diagnosis
- prioritized adaptive logics
- inductive generalization
- abduction
- inference to the best explanation
- analogies, metaphors
- erotetic evocation and erotetic inference
- discussions
- ...

6.3 Incorporation

- flat Rescher–Manor consequence relations (+ extensions)
- partial structures and pragmatic truth
- prioritized Rescher–Manor consequence relations
- circumscription, defaults, negation as failure, . . .
- dynamic characterization of $\mathbf{R}_{\rightarrow}$
- signed systems (Besnard & C°)
- . . .

6.4 Applications

- scientific discovery and creativity
- scientific explanation
- diagnosis
- positions defended / agreed upon in discussions
- changing positions in discussions
- belief revision in inconsistent contexts
- inconsistent arithmetic
- inductive statistical explanation
- tentatively eliminating abnormalities
- Gricean maximes
- ...