THE TRADEOFF BETWEEN BREADTH AND DEPTH

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Abstract: This paper focusses on the tradeoff between breadth and depth. Matthewson & Weisberg (2008) have already put their hopes on this tradeoff to reach an account of rational idealization. I provide a generalization and historical contextualization of their framework in order to show that it is not only capable of providing such an account, but might well turn out to be the basis for the 'comprehensive model-based account' of topics such as explanation, models, theoretical virtues, realism, reduction, laws and idealization called for by Frigg & Hartmann (2006). More specifically, the first two sections aim to understand the content of the axes of the framework and why their relationship is a tradeoff. Once the framework is in place, section three puts it to work to analyse notions such as idealization, abstraction, isolation and ontological unification. The fourth section considers the role of pragmatic factors in our explanatory activity.

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Introduction

This paper addresses the tradeoff between breadth and depth. The idea of a so-called "tradeoff" in model-building has found its way into philosophy of science through Richard Levins' seminal paper 'The Strategy of Model Building in Population Biology' (1966). A recent increase in philosophical interest for models has sparked a revival of contributions around this idea with contributions such as Orzack and Sober (1993), Odenbaugh (2003), Orzack(2005), Weisberg (2006), Matthewson & Weisberg (2008) and Marchionni (2008, forthcoming). What makes Levins influential to this day is, according to Michael Weisberg, that "this analysis will help us develop an account of rational idealization." (2006, p.18) I have two comments to make on this claim. Firstly, I believe this is an understatement. The kind of framework Levins proposes can sustain a coherent view on explanation, models, theoretical virtues, realism, reduction, laws, idealization, abstraction, isolation,... Moreover, these are the kinds of problems that arise in connection with models in science (cf. Frigg & Hartmann, 2006) and interestingly Frigg & Hartmann claim that *"no comprehensive model-based accounts of any of these issues have been developed"* (ibid.). Secondly, perhaps disconcertingly, the idea of a tradeoff in explanatory virtues as a general framework for the problems just mentioned is nothing new. The framework based on a tradeoff in explanatory virtues goes back to Kant and the tension it represents was already apparent in discussions between Plato and Aristotle. In fact some claimed that this tension has constituted most of Western philosophy (see section 2).

However, what *is* new about Levins' approach is its application to scientific models. Only recently have philosophers become very interested in models and it is perhaps for this reason

that the link between this old tradeoff framework and recent debates on models has not yet resulted in a 'comprehensive model-based account'. In this paper, I will show that the tradeoff framework is able to provide the kind of account Matthewson & Weisberg are calling for and make a first step at completing the 'research programme' Frigg & Hartmann call for. More specifically, the first two sections aim to understand what the axes represent and why their relationship is a tradeoff. Once the framework[1] is in place, section three puts it to work to analyse notions such as idealization, abstraction, isolation and ontological unification. The fourth section considers the role of pragmatic factors in our explanatory activity.

1. Breadth and depth.

The aim of this section and the next is to understand what the axes represent and why their relationship is a tradeoff. Actual characterizations of the tradeoff between breadth and depth are notoriously vague (e.g. Levins does not even define them), drawing criticism from i.a. Orzack & Sober (1993), Orzack (2005) and Marchionni (2008). To counter this criticism, Matthewson & Weisberg's (2008) provided the most rigorous account of the tradeoff to date. It is this account on which this section will focus. The next section focuses on historical contextualization of the framework.

Matthewson & Weisberg discuss the tradeoff between generality and precision. "Generality is a measure of how many phenomena a model or set of models relate to." (section 4.2) Precision is narrowed down to parameter precision, which "corresponds to the fineness of specification of the model description's parameters." (4.1). They further distinguish between different tradeoffs (strict, increase and Levins tradeoffs), different kinds of generality (individual model and model set generality, a- and p-generality). Through a series of arguments, they reach three results:

> 1) "*precision and model set a-generality exhibit an increase tradeoff*"
> 2) "*precision and model set p-generality exhibit a strict tradeoff*"
> 3) "*precision and both types of individual model generality show no tradeoffs*"
> (Weisberg & Matthewson 2008, 5.2)

These results obtain ceteris paribus; other model attributes such as scope and fidelity criteria are controlled for. They do indicate that they believe

> *"that the definitions and techniques described in this paper can be used to demonstrate the existence of a number of other tradeoffs among properties such as simplicity, representational capacity, and accuracy, as well as precision and generality." (ibid., 6)*

Matthewson & Weisberg seem to envision their programme of 'rational idealization' as a long and difficult process of carefully weighing all kinds of model attributes against each other in order to establish certain relationships between the. However, given the significance of this

[1] The only reason not to use 'model' instead of 'framework' is to avoid the confusion associated with the presentation of a 'model of models'.

tension in Western philosophy (cf. infra), it is not unreasonable to expect that history contains tools for its precise characterization. Precision and generality as defined by Matthewson & Weisberg only partially cover the meaning of the axes (e.g. they explicitly leave out simplicity, representational capacity and accuracy). By contrast, I want to suggest a powerful generalization of Weisberg & Matthewson's results. This generalization is based on the idea that their results are instantiations of what has been known by logicians for centuries as the 'law of inverse variation' between intension and extension. The intension of a term is the set of *"attributes in virtue of which they may be correctly applied to a delimited set of objects"* (Cohen & Nagel 1934, 31); the extension of a term is *"the set of objects to which it is applicable"* (ibid.) In other words, the extension is those objects which a term denotes and the intension is the reason why these objects are denoted by the term.

The law of inverse variation is usually stated for concepts, but it can be extended to anything that uses something to represent something else. This means it is equally valid for individual models *and* sets of models if we follow a standard model conception which states that *"Scientists use* models *to represent aspects of the world for various purposes."* (Giere 2004, 743) In the case of individual models, the intension is the model statement, that in virtue of which the model can explain something, the explanans; the extension is the model target, that which is explained by the model, the explanandum. For a model set, the intension is the conjunction of model statements of the individual models it contains; the extension is the conjunction of the model targets of the individual models it contains.

To substantiate the claim that intension and extension are a generalized way of capturing the content of the axes, I will show first that it preserves Matthewson & Weisberg's three results and second that it is not too general so as to lose its force; this is shown by demonstrating its capacity to successfully discriminate between model attributes that are relevant for the tradeoff and those that are not but have often been mistakenly thought to be relevant.

1.1. The preservation of their results

RESULT 1: The law of inverse variation states that *"If a series of terms is arranged in order of increasing intension, the denotation* [i.e. extension] *of the terms will either remain the same or diminish."* (Cohen & Nagel 1934, 33) This relationship is precisely what Matthewson & Weisberg call an 'increase tradeoff', viz. a relation whereby *"an increase in the magnitude of one cannot be accompanied by an increase in the other."* (Matthewson & Weisberg 2008, 2.2) Matthewson & Weisberg clearly characterize precision as an element of the model statement (*"precision, which is an attribute of the equations we use to describe models"* (4.3) while generality is characterised as pertaining to the model target (*"we take generality to be a measure of how many targets the models in question apply to"* (ibid.)). Hence, an increase tradeoff between precision and a-generality immediately follows from the law of inverse variation.

RESULT 2: Their second claim is that a 'strict tradeoff', viz. *"a relation whereby an increase in the magnitude of one desideratum necessarily results in a decrease of the magnitude of the second, and vice versa"* (ibid. 2.1) obtains when generality is a measure of all *possible* targets to which a set of models applies (p-generality), rather than merely actual (a-generality). This

follows immediately from the law of inverse variation if we consider that e.g. if the intension of an attribute X is a, b and c. Whenever d is added to the intension one can imagine a set of X's that are not d that have been excluded by this addition, and vice versa. (Cf. Jevons 1874, 30, §13)

RESULT 3: The distinction between a model and a model set is meant to address Orzack & Sober's point (1993, 535) that it makes a difference for the tradeoff whether a model is instantiated or not. If it is instantiated, the tradeoff doesn't obtain. This is reflected in Matthewson & Weisberg's result that no tradeoff obtains between precision and individual model (as opposed to 'model set') generality: parameter precision is an aspect of the model statement and the precision of a model statement does not affect the number of targets of any individual model that complies to the model statement. (The precision of the set of these individual models does affect the number of models selected by the model statement and thus the generality of the model set, enabling results 1 and 2.) This perhaps somewhat surprising result can be explained by noting that parameter precision does not bear on the intension of a model and as such it shows no relationship with its extension. So Matthewson & Weisberg's result surely does not imply that no tradeoff whatsoever occurs at the level of individual models: if precision (or whatever other attribute) were to be defined in such a way that it correlates with the intension of an individual model, there would indeed be a tradeoff at the level of individual models as well. The present result is a reflection of the fact that models that lack detail are not more general but simply shallower.

1.2. Successful discrimination

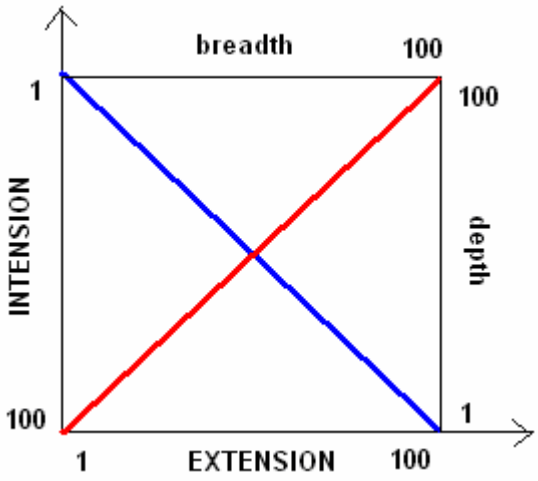
Common mistakes are to take generality for scope, depth for detail and the tradeoff for a strict tradeoff. The generalization (viz. that breadth and depth are anything correlated with intension and extension of the model respectively) provides instant clarification. Scope refers to the size of the set of targets; as such it is correlated with the extension and has nothing to do with the intension. Detail is a matter of how accurately the intension is described; as such it is indeed related with the intension, but not correlated. A strict tradeoff applies only in ideal cases; as such it is not surprising that counterexamples can be found in scientific practice (cf. Marchionni (forthcoming, 6.3)) whereby an increase in breadth does not necessarily decrease depth and vice versa. However, what does obtain is an increase tradeoff.

1.3. The framework

Uncovering the underlying source of the tradeoff as the law of inverse variation between intension and extension, it is possible to generalise Matthewson & Weisberg's results as follows:

a1) *Ceteris paribus*, a model attribute stands in an increase tradeoff with another iff one is a member of the set of attributes correlated with the intension of the model and the other a member of the set of attributes correlated with the extension of the model.
a2) An increase tradeoff between sets of attributes *always* obtains iff one is the set of all model attributes correlated with the intension and the other the set of all model attributes correlated with the extension.

The breadth and depth of a model has been identified as the model's intension and extension. The blue line indicates the tradeoff between intension and extension. The red line indicates corresponding values for breadth and depth. Breadth and depth are intension and extension, so they take the exact same values. Increasing the intension increases breadth but decreases extension. Increasing the extension increases depth but decreases intension.



Any explanation needs a combination of both intension and extension. This reflects Kant's statement that "*Thoughts without content are empty, intuitions without concepts are blind.*" (Kant 1929[1787], B75) Together with the law of inverse variation, this entails that the tradeoff line delimits the space of possible explanations the model can generate: or the explanation runs out of intension, or it runs out of extension, or it breaches the law of inverse variation. If the framework is used to represent a model set instead of an individual model, what it delimits are not explanations but individual models.

2. A contextualization of the tradeoff

Now that a more general view of the tradeoff framework has been produced, it is possible to situate it in the literature. This contextualization is an additional means to gain more understanding of the framework. I first discuss contemporary literature after which some historical depth is added.

2.1. Contemporary

Links between the axes and theories of explanation have been suggested by Marchionni (2008, 319) and Matthewson & Weisberg (2008, section 6). The Y-axis can be associated with the unification account (Kitcher 1981, 1989 and Friedman 1974); the X-axis with Salmon's causal-mechanistic account (Salmon 1984). In light of the previous section, it is to be expected that these accounts will bear on the intension and extension respectively. Indeed they do. Unification searches for patterns, theoretical laws,... to explain as much as possible

by as little as possible.[2] It strives to make the intension as small as possible (paucity of patterns) in order to max out breadth. Causal explanation searches for causal processes and interactions, empirical laws, causes, capacities,... to explain individual occurrences as complete as possible. It strives to increase the extension in order to attain high depth. Unification aims to maximize the explanandum and minimize the explanans, while causal-mechanical explanation minimizes the explanandum and maximizes the explanans. Since explanans and explanandum respectively bear on intension and extension, the inverse relation between intension and extension is again identified as the source of the explanatory tradeoff. As such these two ideals of explanation stand in an inverse relation to each other.

Just as in any tradeoff, the question is not which ideal is correct and which one is false, but how much of one needs to be sacrificed for the other. The usefulness of both accounts is expressed by Wesley Salmon himself in what is known as his complementarity thesis:

> *"Although many philosophers see a conflict between these two conceptions, I find them mutually compatible and complementary. One and the same phenomenon can often be explained in both ways, each providing a different sort of understanding." (Salmon 1998, 9-10)*

A more widespread view (e.g. Schurz & Lambert 1994; Kitcher 1989; Skipper 1999; Strevens 2004) is that both accounts are important, but that one reduces to the other.

> *"To be sure, some authors explicitly endorse Salmon's complementarity thesis [...] But many others who, at first sight, seem to agree with Salmon's diagnosis are actually merely paying lip service to it and ultimately regard one of the two aspects as truly fundamental." (De Regt 2006, 138)*

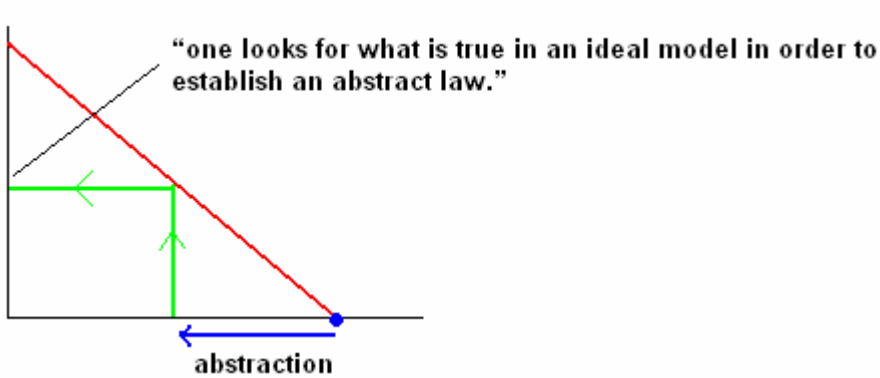
this view is probably more widespread because to give substance to an ideal of explanation is equivalent with a negation of the other ideal. Which one is then said to reduce to the other can be expressed in the framework by X=f(Y) and Y=f(X), meaning respectively that whatever happens on the X-axis is brought about by what is on the Y-axis and vice versa.

As such, Y=f(X) can be taken to stand for the view that laws are to be explained by causes. Nancy Cartwright gives a very explicit statement of this view:

> *"The laws themselves are generally pieces of science fiction, and where they do exist they are usually the objects of human construction, objects to be explained, and not ones to serve as the source of exlanation. Causes and their capacities are not to play a role alongside laws in scientific explanation, but –at least in many domains- to replace them altogether." (Cartwright 1989, 218)*

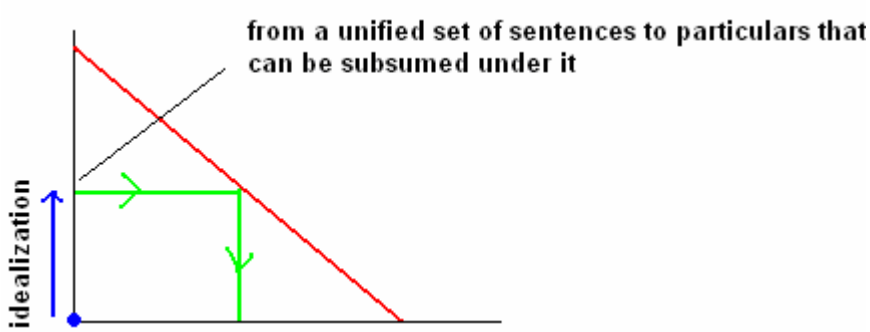
For Cartwright, *"one looks for what is true in an ideal model in order to establish an abstract law."* (1989, 191)

[2] *"science increases our understanding of the world by reducing the total number of independent phenomena that we have to accept as ultimate or given."* (Friedman 1974, 15)
"[Science] teaches us how to reduce the number of types of facts we have to accept as ultimate (or brute)." Kitcher 1994, 432



This shows the dependence of Y on the value of X in Cartwright's account. The starting point (for Cartwright and Aristotle alike) is on the side of the extension: *"we begin with a concrete particular complete with all its properties."* (1989, 197) We abstract from it to its essence; the resulting X-value is then taken as given and given X, the tradeoff determines the resulting value of Y. The best explanation then consists of maximizing the level of depth for a given amount of breadth.

On the other hand, X=f(Y) then stands for the view that laws are more fundamental than causes. Causes, if any, must be explained in terms of laws. *"Causal explanation of particular occurrences recapitulates the ordering derived from the systematization of regularities."* (Kitcher 1985, 636)



The value of Y is first determined and only then the value of X follows. This perspective takes the intension as its starting point: we begin with a set of sentences. If K is the set of accepted sentences, then

> *"The general problem I have set is that of specifying E(K), the explanatory store over K, which is the set of arguments acceptable as the basis for acts of explanation by those whose beliefs are exactly the members of K. [...] for each K, E(K) is the set of arguments which best unifies K." (Kitcher 1981, 512)*

E(K) provides a level of depth which can be taken as given. Given this value of Y, the tradeoff then determines the resulting value of X. The best explanation consists of maximizing the level of breadth for a given level of depth.

Immanuel Kant nicely rounds up the distinction between X=f(Y) and Y=f(X) as *reflective judgment* and *determinative judgment*:

> *"Judgment can be regarded either as mere[ly] an ability to reflect, in terms of a certain principle, on a given presentation so as to [make] a concept possible, or as an ability to determine an underlying concept by means of a given empirical presentation."* (Kant & Pluhar & Gregor 1987 [1790], 399)

This suggests that further historical contextualization is relevant to gain additional understanding about what exactly goes on in the framework.

2.2. Historical

This 'battle of the axes', the debate about whether to put emphasis on the intension or the extension of our knowledge, has been going on since Antiquity. It can be retraced to Aristotle's primacy of individual substance over form (Y=f(X)) versus Plato's primacy of form over substance (X=f(Y). In fact this formulation is very suggestive because X=f(Y) *literally* states Plato's claim that form maps onto substance. Throughout the Middle Ages this debate continued in the form of the opposition between realism (X=f(Y)) and nominalism (Y=f(X)). Afterwards, Descartes' rationalism preferred to start its inquiries from 'clear and distinct ideas' (X=f(Y)) while empiricism started from the senses (Y=f(X).

Francis Bacon uses the metaphor of the spider and the ant to describe the two axes.

> *"Those who have handled sciences have been either men of experiment or men of dogmas. The men of experiment are like the ant, they only collect and use; the reasoners resemble spiders, who make cobwebs out of their own substance. But the bee takes a middle course: it gathers its material from the flowers of the garden and of the field, but transforms and digests it by a power of its own." (Bacon 2000[1620], First Book of Aphorisms, XCV)*

But if the axes represent the debate between rationalism and empiricism, then a natural point to look for a comprehensive account is Immanuel Kant. Immanuel Kant is of course famous for of rationalism and empiricism. He could be the bee in Bacon's story. Kant asserts that neither of the regulative ideals of unity and completeness can provide explanation alone and in the introduction to the Critique of Pure Reason, he mocks Plato for relying too exclusively on form:

> *"The light dove, cleaving the air in her free flight, and feeling its resistance, might imagine that its flight would be still easier in empty space. It was thus that Plato left the world of the senses, as setting too narrow limits to the understanding, and ventured out beyond it on the wings of the ideas, in the empty space of the pure understanding." (ibid. A5)*

Indeed, Kant actually lays out his version of the tradeoff framework in the Appendix to the Transcendental Dialectic of the Critique of Pure Reason.

> *"Reason thus prepares the field for the understanding: (1) through a principle of the homogeneity of the manifold under higher genera; (2) through a principle of the variety of the homogeneous under lower species; and (3) in order to complete the systematic unity, a further law, that of the affinity of all concepts -- a law which prescribes that we proceed from each species to every other by gradual increase of the diversity." (Kant 1929[1787], A657)*

Kant's principle of variety and principle of homogeneity work through in the distinction introduced by Windelband between nomothetic and idiographic sciences: "*the* nomothetic, *which seek to establish abstract general laws for indefinitely repeatable events and processes; and the* ideographic, *which aim to understand the unique and the nonrecurrent."* (Nagel 1961, 547) More generally, this distinction found its way into the social sciences in the 1880's through the Methodenstreit in economics between the Austrian School of Menger and the

Historical School of von Schmoller. A 'complementarity thesis' as that of Salmon was stated here by Wilhelm Dilthey who claimed that "any science might usefully combine both methods" (Hodgson 2001, 22).

Around the time of the Methodenstreit, Ludwig Wittgenstein was born. Well aware of Kant, his Tractatus Logico-Philosophicus (Wittgenstein, 1990[1921]) came to have great influence on the Wiener Kreis. Their syntactic view of theories (X=f(Y)) contrasts with the semantic view (Y=f(X)) developed after the second World War by Patrick Suppes (1960) among others. The logical empiricist official view on explanation, the covering law model (Hempel 1965) was refined by Kitcher(1981, 1989) and attracted criticism from causalist accounts of e.g. Salmon(1984). Thus we rejoin the previous section.

2.3. A second generalization

I conclude this section by making a second generalization of Matthewson & Weisberg's paper. The tension represented goes much further than Kitcher and Salmon, but has arguably played a constitutive role throughout the entire Western philosophy.[3] This bold statement is not new; it was held by the Neo-Kantian Marburg School, e.g. by Hermann Cohen and Paul Natorp (see Cohen 1885, especially the introduction). If this point is granted, the tension has much broader implications than merely Matthewson & Weisberg's project of 'rational idealization'. The framework should then in principle be powerful enough to sustain accounts on explanation, models, theoretical virtues, realism, reduction, laws, idealization,... Interestingly, *"no comprehensive model-based accounts of any of these issues have been developed"* (Frigg & Hartmann, 2006). The current framework might well be a key to such a comprehensive model-based account.

3. Idealization, abstraction, isolation, ontological unification

The different sides in these historical debates take the Y-axis and the X-axis respectively as independent variable. This means both approaches have a different view of what the activity of explanation consists of. For the unificationist, explanation means moving on the Y-axis, providing better patterns, in order to change the position on the X-axis. The causalist's area of operation is the X-axis: *"scientific explanation consists in fathoming the causal structure of the world."* (Kitcher 1985, 632) For both views, explanatory practice (the improvement of our explanations) consists of a movement within the framework, be it along the X or the Y axis. Activities that are part of our explanatory practice (idealization, abstraction, isolation, approximation, concretization,....) can be represented in the same way. In this section I will categorize existing conceptions of these notions, appoint them to X=f(Y) or Y=f(X) and show how they fit into the framework in a coherent way.

3.1. Idealization and abstraction

[3] However, a common mistake would be to think that the micro-macro debate also maps on this framework. It does not. This has been convincingly argued by Marchionni(2008) and Potochnik(under review).

3.1.1. Idealization

Nancy Cartwright states that *"in idealization we start with a concrete object and we mentally rearrange some of its inconvenient features –some of its specific properties- before we try to write down a law for it."* (1989, 187) She mentions the frictionless plane as a paradigm example. An important difference with abstraction is that all factors that are deemed relevant need to be assigned a value: *"it must say something, albeit something idealizing, about all the factors which are relevant."* With this characterization, she is in agreement with (Nowak 1980, 28) and Mäki (1992, 324), where *"idealizations are formulated in terms of limiting concepts designated or designatable by variables with the value 0 or* $|\infty|$." Examples mentioned are full employment, zero transaction costs, infinitely elastic demand curves, perfectly divisible goods and zero cross elasticities.

An idealization 'sucks out' the content of some structure, leaving the structure itself intact (i.e. creating a black box). This amounts to a reduction of the intension: the model description becomes less restrictive; anything can happen inside the black box. Conversely, de-idealization amounts to *filling* the structure. For the model statement this means a structure is *substituted* by multiple substructures. E.g. xy -> x(a+b) The model is literally 'filled with intension'. The increased structure of what used to be the black box puts additional limits to what goes on inside of it. As a consequence, intension increases. As more of it is poured into the model, breadth is reduced (the intension becomes more restrictive) but depth is gained (more targets will actually or possible fall under the model, increased extension).

3.1.2. Abstraction

Nancy Cartwright uses an Aristotelian notion of abstraction

> *"where 'abstraction' means 'taking away' or 'subtraction'. For Aristotle, we begin with a concrete particular complete with all its properties. We then strip away –in our imagination- all that is irrelevant to the concerns of the moment to focus on some single property or set of properties, 'as if they were separate'." (1989, 197)*

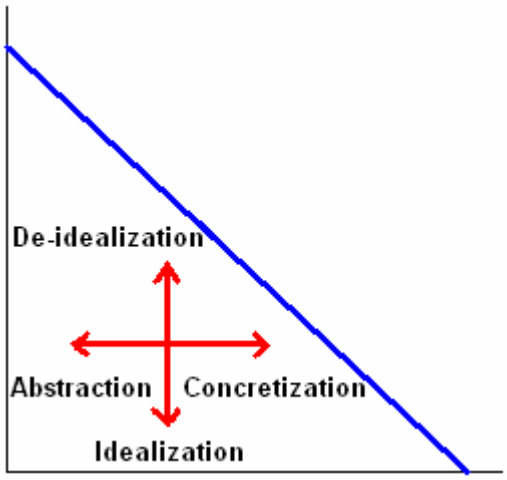
Uskali Mäki follows her in his statement that "*In an abstraction, a universal or quasi-universal is isolated from particular exemplifications."* Mäki (1992, 322)

An abstraction amounts to subtracting structure from an object (i.e. removing a box), leaving less extension for a model to explain. Conversely, concretization amounts to *adding* structure *to* something. This means complementing the initial structure with more structure. E.g. xy -> axy That which is added, is extension; it makes the model provide a more complete explanation. The more of it is added, the greater the extension (increase in depth, decrease in breadth).

3.1.3. Idealization, de-idealization, abstraction and concretization

Since idealization, de-idealization, abstraction and concretization are part of our explanatory activity, they necessarily take place within the constraints of the framework. Moreover, it has just been demonstrated that their meaning is restricted to particular axes: idealization and de-idealization bear on the intension and as such they are movements along the Y-axis;

abstraction and concretization bear on the extension and as such they are movements along the X-axis. These movements are represented in the underlying figure.



This figure demonstrates that the tradeoff framework can sustain many different notions related to models. It here relates idealization, de-idealization, abstraction and concretization with respect to each other and with respect to the tradeoff between intension and extension. In 3.2 isolation and ontological unification will be added and section 4 will connect it to 'complete explanation', the pragmatics of explanation and a notion of scientific progress. The figure explains the intuition that abstraction and idealization are somehow constitutive of explanation although they also seem to impede on it at the same time: we need to idealize in order to concretize; we need to abstract in order to de-idealize. This is not surprising in the light of what was previously stated. From their characterization as pertaining to intension and extension respectively, it follows of course that idealization and abstraction are subject to its very same tradeoff: idealization and abstraction can never be undone simultaneously.

Concretization cannot go on indefinitely. When concretization goes too far, Ockham's razor steps in: do not multiply entities without reason.[4] Immanuel Kant posited a counterprinciple which states that diversity must not be diminished without reason.[5] Together, these two principles force us on the tradeoff.[6] This is a key aspect of Cartwright's conception of abstraction. She follows Aristotle in this regard: *"Other information is not relevant to understanding what this substance is [...] Less information will be defective."* (ibid., my emphasis) The purpose of abstraction and concretization is then to cut away the inessential and reveal *"what is essential in that way of describing or treating the object."* (Cartwright 1989, 219) In framework language: given an intension, abstraction cuts away the irrelevant parts (i.e. those we think do not pertain to the essence of what we describe) of its extension, resulting in a loss of depth. This can then be 'cashed' in terms of more de-idealization, resulting in a gain in breadth. For the framework, this is the essence of abstract: the activity of identifying irrelevant breadth (e.g. we mentally cut away the properties of a heart until what is left is a pump) and allowing it to be traded for more relevant depth.

[4] *"Entia non sunt multiplicanda praeter necessitatem"* (Kant 1929 [1787], A652)
[5] *"Entium varietates non temere esse minuendas"* (Kant 1929 [1787], A656)
[6] Compare "Our theories should be as simple as possible, but no simpler." (Einstein, SEP)

In the same vein, there are limits to de-idealization. In the introduction to the Critique of Pure Reason, Kant mocks Plato for underestimating these limits:

> *"The light dove, cleaving the air in her free flight, and feeling its resistance, might imagine that its flight would be still easier in empty space. It was thus that Plato left the world of the senses, as setting too narrow limits to the understanding, and ventured out beyond it on the wings of the ideas, in the empty space of the pure understanding."* (Kant 1929[1787], A5)

In the limit, de-idealization brings us into the logical space of possibilities and all connection to reality is lost. As such, any explanation will consist of a certain amount of idealization and a certain amount of concreteness.[7] The essence of idealization then consists in the activity of identifying irrelevant depth (e.g. we do not care what goes on inside people's heads so we assume their actions are rational, making their head a 'black box') and allowing it to be traded for more relevant breadth.

3.1.4. Queries of idealization

In 'Three kinds of idealization' Michael Weisberg puts forward a number of questions surrounding idealization.

> *What exactly constitutes idealization? Is idealization compatible with realism? Are idealization and abstraction distinct? Should theorists work to eliminate idealizations as science progresses? Are there rules governing the rational use of idealization, or should a theorist's intuition alone guide the process?* (Weisberg 2007, 639)

The present framework provides a coherent answer to these questions. Idealization is a reduction of intension of a model. It is compatible with realism because it is exactly idealization which enables us to concretize. Idealization is indeed distinct from abstraction; one is strictly related to intension, the other to extension. Theorists should work to eliminate idealization whenever they judge it relevant to do so (viz. whenever they think the concretization it will buy is more relevant than the de-idealization they already have). As such, the rules for rational idealization will depend almost exclusively on scientist's judgment of the relevance of breadth versus depth in specific cases. The only general rules that can be put forth are 1) one ought always to be as close to the tradeoff line as possible and 2) no single explanation (model) will satisfy all epistemic interests simultaneously; as such explanatory pluralism (a pluralism of models) is imperative.

Idealization and concretization correspond with what Kant calls the 'faculty of distinction' and 'faculty of wit'. The faculty of wit is the ability to find similarity in diversity; the faculty of distinction is the ability to notice even the smallest differences. This distinction is reflected in the difference Jackson & Pettit (1992) make between comparative and contrastive information.[8] They use these two different kinds of information to make an argument for

[7] *Explanation is caught, and lives, in a tension between these two requirements. On the one hand, explanations are about the world and so must refer to real things. On the other hand, every explanation must have some generality, and so its objects must in some sense be abstract. (Garfinkel, 1981, p.174)*

[8] It is true that going micro and getting at smaller levels of causal grain involves getting better and better contrastive information – greater and greater detail – on causal history. But it does not follow that it involves getting better and better information tout court. On the contrary, the obvious thing to say is that while it means

explanatory pluralism (or 'ecumenism' as they call it). This is not surprising. The idea that these different abilities serve different interests is already indicated by Kant himself[9]:

> *This principle (of discriminative observation, that is, of the faculty of distinction) sets a limit to possible indiscretion in the former principle (of the faculty of wit); and reason thus exhibits a twofold, self-conflicting interest, on the one hand interest in extent (universality) in respect of genera, and on the other hand in content (determinateness) in respect of the multiplicity of the species. In the one case the understanding thinks more under its concepts, in the other more in them. This twofold interest manifests itself also among students of nature in the diversity of their ways of thinking. Those who are more especially speculative are, we may almost say, hostile to heterogeneity, and are always on the watch for the unity of the genus; those, on the other hand, who are more especially empirical, are constantly endeavouring to differentiate nature in such manifold fashion as almost to extinguish the hope of ever being able to determine its appearances in accordance with universal principles. (Kant 1929 [1787], A654-55)*

Here perhaps the most important question about idealization is reached, the question whether or not idealization is merely a reflection of mental economy; a way to cope with our cognitive limitations. McMullin (1985, 248) calls this *"the great divide between the Platonic and Aristotelian traditions"*. Batterman (forthcoming, section 2) characterizes this divide as one between a 'traditional view':

> *"according to which one aims for the most exact and detailed representation of the phenomenon of interest. On this view, the use of idealizations is, in effect, justified pragmatically: We need to introduce idealizations into our equations in order to simplify them so as to make them tractable or solvable."I*

And a 'nontraditional view':

> *This other view, which for lack of a better term I will call "nontraditional," maintains that in some cases (and actually in many cases) idealized "overly simple" model equations can better explain and characterize the dominant features of the physical phenomenon of interest.*

The current framework sees an irreducible value in idealization. Reduction of explanations or models can in the current framework be characterized as de-idealization. If all explanatory activity was brought down to the X-axis, it would cease to be explanation altogether. Completeness is only one of two regulative ideals and as such the value of idealization will not be fixed but dependent on one's epistemic interests. This matter is discussed further in section 4.

3.2. Isolation and ontological unification

In addition to abstraction and idealization, Uskali Mäki also speaks of isolation and ontological unification. These are interesting notions to complement the framework with. They are characterized as follows:

> *"In an isolation, something, a set X of entities, is "sealed off" from the involvement or influence of everything else, a set Y of entities; together X and Y comprise the universe." (Mäki 1992, 321)*

> *"Ontological unification is a matter of redescribing apparently independent and diverse phenomena as manifestations (outcomes, phases, forms, aspects) of one and the same small number of entities, powers, and processes." (Mäki 2001, 498).*

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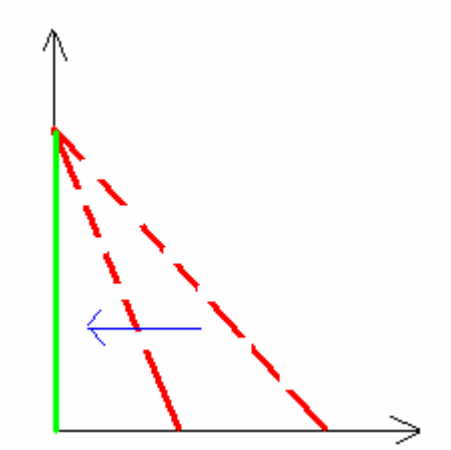
getting better and better contrastive information, it means losing information of a comparative kind. (Jackson & Pettit, 1992, p.15)

[9] Indeed, as was indicated in section 2, it is this distinction on which the difference between nomothetic and idiographic sciences is based.

Isolation is related to ontological unification. Both activities result in a change of the slope of the tradeoff line (i.e. a change in the cost of breadth in terms of depth and vice versa). Isolation is brought about by restricting the domain of our model. Ontological unification is a matter of empirical discovery and consists of discovering unity in the world where there was previously thought not to be none. Gottlob Frege's famous example is the discovery that the morning star and the evening star are in fact the same (Frege 1892). This can be stated in terms of the framework as the discovery that different intensions have the same extension. As a consequence, while the value of intension remains unchanged, the value of the intersection of the tradeoff line with the X-axis decreases: the world is more homogeneous than previously thought; the cost of breadth in terms of depth and vice versa was lower than we thought.

Although isolation is a matter of convention and ontological unification is a matter of empirical discovery, they have the same effect on the slope of the tradeoff line. In the limit, the domain is isolated such that it only comprises one object (whatever is valid for that object can be generalized to all objects at no cost) or we have discovered that the world is one.



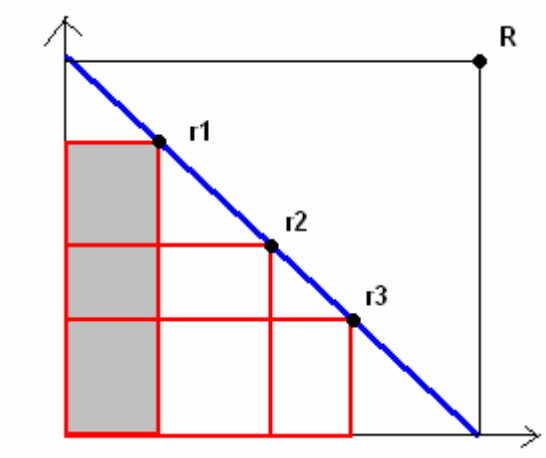
The framework neatly separates derivational unification (defined by Mäki 2001 in terms of the unification account of explanation) and ontological unification. Derivational unification is a pursuit of unification as a regulative ideal, resulting in an increase along the Y-axis of the framework. Ontological unification is the discovery that the world is indeed more unified, resulting in a change of slope. Derivational and ontological unification only coincide in case the world is indeed as unified as the regulative ideal of unity wishes.

4. The pragmatics of explanation

Idealization and abstraction are inherently pragmatic. They rely on our judgment about what is relevant in the light of what we want to explain. Idealization and abstraction enable us to manipulate the intension and the extension of a model respectively, converting whatever surplus we think we have of one into currency that can be used to de-idealize and concretize. As such they are the levers that epistemic agents can push and pull to calibrate a model to their epistemic interests.

A 'complete explanation'[10] R is an explanation where breadth and depth are maximized. The tradeoff line makes it impossible to reach.[11] An omniscient being will know R, but still the explanations it will be able to produce will necessarily be explanations that are on the tradeoff line. Moreover, it will not have the means to discriminate between those explanations: the creature has no particular perspective, no epistemic interests and as a consequence it lacks the sense of relevance necessary to trade breadth and depth (compare Van Fraassen 1980, p.130). Objectively speaking, all explanations on the tradeoff line are thus equally valid with respect to R.

Once we take a perspective, we have certain epistemic interests and a resulting notion of relevance which we can use to determine a position on the Y axis (if X=f(Y)) or on the X-axis (if Y=f(X)). Constrained by the tradeoff we then maximize breadth or depth and this corresponds with a particular value of X or Y. What results is the possibility space of valid explanations (or models in case the tradeoff framework represents a model set) given a particular perspective. This is the grey area in the figure below. In contrast to the non-perspectival case of the omniscient being, explanatory (or model-) discrimination is possible. Hence, as perspectival beings we do not face the tradeoff as long as we find the balance of abstraction and idealization which matches our interests. There is a single best explanation and it is possible to reach it: r is the point where the best combination of breadth and depth is obtained relative to our interests. All other points in the grey area are valid but suboptimal. Different interests result in different r's. Any point on the tradeoff has the potential to answer a specific interest that might arise, so all points on the tradeoff represent potentially valuable explanations. A view on explanations and models results which holds that an explanation or model is one possible answer to the question of how to trade off breadth and depth. A model or explanation then strives to optimize the combination of breadth and depth for a given epistemic interest.



In principle r can be reached, but this does not mean that our current explanations or models are already there. This yields a notion of 'scientific progress'. Scientific progress can be

[10] This would be something like the 'Ideal Explanatory Text' Railton (1978), including a complete Hempelian deductive schema as well as a complete mechanistic account.
[11] We could say here that representation is an insurmountable barrier between us and the world in itself. Within this framework, reality could be said to be transcendental.

defined as trading breadth for more relevant depth or depth for more relevant breadth such that our current explanation or model moves closer to r. For example Galileo made a move which Aristotelians had always shunned: he showed that the set of models the Aristotelians clinged to contained an unreasonably high level of depth. He demonstrated that this depth could readily be traded for more breadth and that the resulting model set was closer to r, i.e. it better served our epistemic interests. (Cf. Galilean Idealization, McMullin 1985)

In this framework, our interests exhaust the problem of explanatory diversity and model selection. Kitcher considers his position on this as equal to Kant's and describes his and Kant's position on these matters in terms of the tradeoff, so I can readily quote him here:

> *"At (A666-7/B694-5), Kant talks about the "different interests" that might incline thinkers to unity or to diversity. In my view, this is to recognize that in searching for the set of patterns that omtimally unifies our beliefs we may face choices between systematizations that trade paucity and stringency, on the one hand, for diversity in consequences, on the other. If different agents made the tradeoffs differently, then at least one must be wrong. As Kant puts it, "reason has only one single interest [the maximal unification with whatever tradeoff it involves: PK], and the conflict of its maxims is only a difference in, and a mutual limitation of, the methods whereby this interest endeavors to obtain satisfaction" (A666/B694)." (Kitcher 1986, 271 my emphasis)*

I do not follow this view.[12] The boundaries of this framework show us the boundaries of this view:

1) Interests are taken as unproblematic. However, the problem of reaching a single explanation or model is shifted to the problem of agreeing on which are our exact interests. If no method is provided to determine our epistemic interests very clearly, then no solution for the problem of explanatory diversity and model selection is provided; it is merely shifted to interests.

2) A level of abstraction and idealization can be reached in many ways. As such, for every co-ordinate in the framework there will not be one but multiple alternative explanations or models. The problem remains because not one but many explanations or models can thus occupy point r.

The extent of the problem of explanatory diversity and model selection is here reduced, but not solved. Any further advance will need means that go beyond the present framework.

5. Conclusion

If we want our representation to be complete, we might as well use the world as its own model. If we want our representation to be unified, we will crash like Kant's dove. If we want to maximize both at the same time, we face the all-pervading tradeoff between intension and extension. This is the fate of knowledge. Our explanatory practice is thus contained within the triangle of the tradeoff framework. As such it should be sufficient as a framework to provide the comprehensive model-based account Frigg & Hartmann (2006) have called for. I have

[12] I develop this point in De Langhe(forthcoming).

constructed a framework that is in two ways a generalized version of Mathewson & Weisberg (2008): first the content of the axes was made more general (but not too general); second the meaning of the content of these axes for Western philosophy was extended through a historical contextualization. In line with Mathewson & Weisberg's hope for a rational account of idealization and as a first phase of the research programme Frigg & Hartmann have outlined, this paper has then presented an account of rational idealization.

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